ON THE MODELLING OF BUCKET BRIGADES

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Abstract: This paper deals with the modelling of a particular class of manufacturing lines, governed by a rule imposing the self-balancing. Bucket brigades were first studied as stochastic dynamic systems. Thus, a sufficient condition for self-balancing was proved. Under some particular conditions, bucket brigades may be regarded as nonlinear dynamic systems. Simulations have shown that their state trajectories are piecewise continuous in between occurrences of certain discrete events. Bucket brigades may therefore be modelled as hybrid dynamic systems, more precisely, with autonomous switching and autonomous jumps. A stability analysis is possible in the frame of hybrid dynamic systems with discontinuous motions.

Keywords: production systems, self-organising systems, stochastic modelling, hybrid dynamic systems.

1. INTRODUCTION

The idea of bucket brigades was first implemented in the 70's, in order to increase flexibility of different production lines. These lines are also known as "TSS lines", because the Toyota Sewn Management System (TSS), registered trademark of Aisin Seiki Co. Ltd., a subsidiary of Toyota, was the first case putting into practice a way of organising workers on a flow line so that the line balances itself. Workers, fewer than workstations, are allowed to walk to adjacent workstations to continue work on an item, each of them independently following a simple rule – the *TSS Rule* – that determines what to do next.

TSS lines have offered a new way of organising work on a production line, as an alternative to the traditional point of view, such as classical assembly line, where the station with the greatest work content determines the production rate. Opposite to the strict assignment of equipment and tasks to workstations, the concept of *"workstation"* is given a new meaning, as the equipment are, this time, human operators, which are not strictly assigned to certain workstations, but can move among them.

Flow manufacturing lines can be found wherever "products" may be imagined to move along, from worker to worker. An assembly line is an example. When following the TSS Rule, a flow line is spontaneously maximally productive and autonomously maintains its optimal production rate. The self-balancing is an intrinsic property of the line, avoiding a solution based on the assembly line balancing (ALB) classical problem, which is NP-complete. The cycle time of the line – the most used optimisation criterion in ALB approach – is in this case implicitly minimised.

An instance of the product is called an item. Each worker has an index, as higher as he is closer to the end of the line. A workstation can process at most one item at a time, requiring precisely one worker to do that. Workers move according to the *TSS Rule*:

Forward part – Remain devoted to a single item and process it on successive workstations (where at any station the worker of higher index has priority). If your item is taken by your successor (or if you are the last worker and you finish processing the item), then relinquish the item and begin to follow the *backward part*.

Backward part – Walk back and take over the item of your predecessor (or, if you are the first worker, pick up raw materials to start a new item). Begin to follow the *forward part*.

Note that a worker can be blocked during the forward phase, if trying to enter an occupied station. Also, during the backward phase, each worker interrupts his predecessor and takes over his work.

Different approaches of particular flexible lines, quite close to the logic of a TSS line, can be encountered in the literature. TSS lines, regarded as dynamic systems, can have very complicated and even chaotic behaviour. That is why they were primarily treated under some simplifying but quite realistic assumptions. Human implications – such as motivation, mentality, responsibility of workers – have been ignored in a first modelling approach, considering that this would pointlessly complicate the model. Some simulation studies use a simple model of workers, where all are identical and work at a single common velocity across all stations. Assuming also that the processing time at each station is normally distributed about its mean, the average behaviour of the line is linear deterministic; the state trajectories – as defined below – converge to a fixed point (Schroer, *et al*, 1991).

More realistic is to consider each worker as a working velocity function depending on his position on the line. Under "The Normative Model", bucket brigades were viewed as *stochastic dynamic systems* and a sufficient condition for obtaining a stationary behaviour, that is a steady production rate, was established (Bartholdi and Eisenstein, 1996a). Thus, a bucket brigade production line is spontaneously maximally productive if workers are sequenced from slowest to fastest. This yields a stable partition of work, corresponding to a fixed point in the system's state space. The same authors studied some practical implications of bucket brigades (Bartholdi and Eisenstein, 1996b), and the asymptotic behaviour of lines with two or three workers (Bartholdi, *et al.*, 1999c).

In a recent work, simulations carried out on *nonlinear block diagrams* have shown the discontinuity of the state trajectories (Bratcu and Mînzu, 1999). Two *hybrid phenomena* were identified. Therefore, it was necessary to regard the bucket brigades as *hybrid dynamic systems*. More specific, they are *hybrid dynamic systems with autonomous switching and autonomous jumps*, in view of the classification of Branicky, *et al.* (1998) (see also Flaus, 1998). Using concepts within a unitary *hybrid model with discontinuous motions* (Ye, *et al.*, 1995), the existence of the stationary behaviour may be related to the stability analysis of hybrid dynamic systems (Bratcu and Mînzu, 2001).

This paper first presents the basic assumptions for modelling bucket brigades. Then it will be summarised the main approach from the literature: the modelling as stochastic dynamic systems. A nonlinear model of a bucket brigade will be presented next, leading to the necessity of modelling in the frame of the hybrid dynamic systems theory. Finally, a conclusion will be listed.

2. BASIC MODELLING ASSUMPTIONS

Bucket brigades are self-organising, therefore the need for centralised planning and management is reduced. Their operation is simple: each worker carries an item towards completion; when the last worker finishes his item, he sends it off and then walks back upstream to take over the work of his predecessor, who walks back and takes over the work of his predecessor and so on until, after relinquishing his item, the first worker walks back to the start to begin a new item. A worker might catch up to his successor and be blocked from proceeding until the station becomes available. The last worker is never blocked and he determines the line productivity.

Let m be the number of workstations and n be the number of workers. The Normative Model is the simplest model of bucket brigades (Bartholdi and Eisenstein, 1996a):

a) *insignificant walk-back time*: the handoff is instantaneous and simultaneous for all workers;

b) *total ordering of workers by velocity*: each worker i is modelled by a velocity function, $v_i(x)$, giving his instantaneous work velocity at position $x \in [0;1]$;

c) *smoothness and predictability of work*: the standard work content for an item, normalised to 1, is spread continuously and uniformly along the flow line (see figure 1).

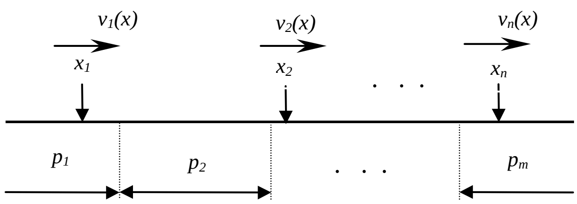

Fig. 1. The standard work content split into workstations and the workers positions ($\sum_{j=1}^{m} p_j = 1$).

x_i gives the position of the *i*-th worker, denoting the cumulative fraction of work content completed on his item at a given moment. The vector $\underline{x} = (x_1, x_2, ...x_n)$ represents the state of the system at any time. An *iteration* is the time elapsed between two successive handoff moments (also called *reset* moments).

The attention in the state space is restricted to the sequence $\{\underline{x}^{(0)}, \underline{x}^{(1)}, \underline{x}^{(2)}, ... \underline{x}^{(k)}, ...\}$ of positions immediately after reset (where $x_1^{(k)}=0$, $x_{n+1}^{(k)}=1$). This sequence is called the *orbit* beginning at $\underline{x}^{(0)}$. Let *f* be the function that maps the vector of workers reset positions so that $\underline{x}^{(k+1)}=f(\underline{x}^{(k)})$. The orbits $\{\underline{x}^{(k+1)}=f^k(\underline{x}^{(0)})\}_{k=0,1,...}$, with the initial conditions $\underline{x}^{(0)}$, describe the line behaviour. The *stationary behaviour* is defined by the *fixed points* of *f*, which are points of balancing.

3. BUCKET BRIGADES AS STOCHASTIC DYNAMIC SYSTEMS

The main modelling and analysis approach of bucket brigades from the literature is hereafter resumed. Proofs have been skipped below (see Bartholdi and Eisenstein, 1996a).

The first result states the *existence* of fixed points, $\underline{x}^*=f(\underline{x}^*)$, for any bucket brigade, meaning that there exist worker positions $\underline{x}^*$ such that, if workers start at positions $\underline{x}^*$, then they will always reset to $\underline{x}^*$. So, balancing is always at least theoretically possible.

It is said that worker *j* is faster than worker *i* – written as $v_i \prec v_j$ – if

$$\sup_{x\in[0,1]}\left(\frac{v_i(x)}{v_j(x)}\right)<1,$$

meaning that *j* is faster than *i* for any operation from the line. The *uniqueness* of a fixed point of a bucket brigade is guaranteed if workers are sequenced *from slowest to fastest*. If not, then the line may have multiple fixed points. The well order being kept, it has been proved that any orbit of worker positions, $\{x^{(t+1)}=f^t(x^{(0)})\}$, converges to the unique fixed point.

The *production rate* of a production line may be defined as the number of items completed in a time unit, as well as the time for processing an item. If there exists a stationary regime, the second definition corresponds to the *cycle time* of the line.

Let $\tau_i(x,x')$ be the time required to worker *i*, if not blocked, for walking from position *x* to position *x'*:

$$\tau_i(x,x')=\int_x^{x'}\frac{dz}{v_i(z)}$$

Let P_k be the cumulative amount of work performed on an item when it has just left the station *k*:

$$P_0=0,\quad P_k=\sum_{j=0}^{k}p_j,\quad k=1,2,...m \tag{1}$$

The interval $(P_{k-1};P_k)$ is the work content assigned to station *k*. Only one worker can use a certain station at a given time, therefore no two *xi*'s can assume values within the same interval $(P_{k-1};P_k)$. There are defined:

$$\underline{x}=P_{k-1},\ \text{if } x\in[P_{k-1};P_k)$$
$$\overline{x}=P_k,\quad \text{if } x\in(P_{k-1};P_k]$$

The interval $[x_i^{(t)};x_{i+1}^{(t)}]$ may be regarded like a dynamic partition of the work content assigned to worker *i* during iteration *t*. Let $a_i^{(t)}$ be the time that would take to worker *i* to complete his suggested share of work, including both effective work time and possible delays because of blocking. This time is called *allocation*. Thus, one can write:

$$\begin{cases}a_n^{(t)}=\tau_n(x_n^{(t)},1)\\ a_i^{(t)}=\tau_i(x_i^{(t)},x_{i+1}^{(t)})+\\ \quad +max\left\{0,\tau_{i+1}(x_{i+1}^{(t)},\overline{x_{i+1}^{(t)}})-\tau_i(x_i^{(t)},\underline{x_{i+1}^{(t)}})\right\}\\ \quad i=1,2,...n-1\end{cases}$$

Pure work time allocations are called *simple*, whereas the other ones are *delayed* allocations. They are respectively expressed by the two relations below:

$$a_i^{(t)}=\tau_i(x_i^{(t)},x_{i+1}^{(t)}),\ \ i=1,2,...n$$
$$a_i^{(t)}=\tau_i(\underline{x_{i+1}^{(t)}},x_{i+1}^{(t)})+\tau_{i+1}(x_{i+1}^{(t)},\overline{x_{i+1}^{(t)}}),\ \ i=1,2,...n$$

The main result states that, if workers' velocities are constant, with $v_1<v_2<...<v_n$, and if workers are never blocked, then the line converges *exponentially fast* to the unique fixed point:

$$\underline{x}^*=\begin{bmatrix}0\\ \frac{v_1}{\sum_{j=1}^{n}v_j}\\ ...\\ \frac{\sum_{j=1}^{i-1}v_j}{\sum_{j=1}^{n}v_j}\\ ...\\ \frac{\sum_{j=1}^{n-1}v_j}{\sum_{j=1}^{n}v_j}\end{bmatrix} \tag{2}$$

where the production rate is the largest possible: $\sum_{j=1}^{n} v_j$. The line behaviour is described by:

$$\begin{cases} a_1^{(t+1)} = a_n^{(t)} \\ a_i^{(t+1)} = \frac{v_{i-1}}{v_i} \cdot a_{i-1}^{(t)} + (1 - \frac{v_{i-1}}{v_i}) \cdot a_n^{(t)}, \quad i=1,2,...n \end{cases},$$

representing a *linear dynamic system*. By rewriting these equations in the form:

$$a^{(t+1)} = T\, a^{(t)} \qquad (3)$$

where T denotes the *transition matrix* of a *finite state Markov chain* that is irreducible and aperiodic (Resnick, 1992), the convergence of iterates $\{a_i^{(t)}\}_{t=0}^{\infty}$ is guaranteed for any $i=1,2,...n$, yielding the convergence of the bucket brigade orbit $\{x^{(t)}\}_{t=0}^{\infty}$. This is called *stationary behaviour*. The production rate improves to a limit not depending on the starting positions of the workers. The well order is only sufficient for the stationary behaviour, as there are cases of other than slowest-to-fastest sequences when the line converges to a fixed point not depending on the workers' initial positions (Bratcu and Mînzu, 1999).

4. A NONLINEAR MODEL OF BUCKET BRIGADES

Simulation has shown typical nonlinear dynamics – such as limit cycles or behaviour depending of the initial positions of workers, or either on the partition of work among workstations – occurring when the "proper" sequence is not respected or/and it happens that workers be blocked (the *complicated behaviour*). To understand it, the simulation study has been extended in between the reset moments, by building a nonlinear model of bucket brigades, which reflects the hybrid dynamics of such lines (Bratcu and Mînzu, 1999).

Notice that the system exhibits *piecewise linear dynamics*, since any worker can have *two states* during an iteration: either he moves with a nonzero velocity, or he is blocked. Two *hybrid phenomena* (Flaus, 1998) may be identified. The *autonomous switching* appears at the transition between these two states. The state trajectory remains continuous in this case. It exhibits discontinuities only in the reset moments, when the last worker reaches the line end. The reset may be modelled as the second hybrid phenomenon, called *autonomous jump*. The jump results from the insignificant walk-back time assumption. Thus, it is ensured a *single moment of reset* for all workers.

Notation:
tr_1, tr_2, $\dots tr_p$, $\dots$ = the sequence of successive moments of reset;
tr_p^{-0} = the moment immediately *before* the *p*-th reset;
tr_p^{+0} = the moment immediately *after* the *p*-th reset;

$$sign(y(t)) = \begin{cases} 1, & y(t) > 0 \\ -1, & y(t) < 0 \end{cases}$$

The evolution between any two consecutive moments of reset is characterised by: $x_n^{(p)}(t)<1$, $\forall\, t \in [tr_p, tr_{(p+1)})$, $\forall p \in \mathbf{N}^*$.

The nonlinear model of a bucket brigade, with the initial condition $\underline{x}^{(0)}=\underline{x}(0)=x_0=[x_{10}\ x_{20}\ \dots\ x_{n0}]$, is given in relation (4). It was implemented by a nonlinear block Simulink diagram, where workers were modelled by *reset integrators*.

$$If\ \ x_n(t)<1, \begin{cases} \dot{x}_1(t) = \frac{v_1}{2} + \frac{v_1}{2} \cdot sign\left\{ \frac{1}{2} \cdot \sum_{k=1}^{m} \left\{ p_k \cdot \left(sign\left(x_2(t) - P_{k-1}\right) - sign\left(x_1(t) - P_{k-1}\right) \right) \right\} \right\} \\ \dot{x}_2(t) = \frac{v_2}{2} + \frac{v_2}{2} \cdot sign\left\{ \frac{1}{2} \cdot \sum_{k=1}^{m} \left\{ p_k \cdot \left(sign\left(x_3(t) - P_{k-1}\right) - sign\left(x_2(t) - P_{k-1}\right) \right) \right\} \right\} \\ \dots \\ \dot{x}_{n-1}(t) = \frac{v_{n-1}}{2} + \frac{v_{n-1}}{2} \cdot sign\left\{ \frac{1}{2} \cdot \sum_{k=1}^{m} \left\{ p_k \cdot \left(sign\left(x_n(t) - P_{k-1}\right) - sign\left(x_{n-1}(t) - P_{k-1}\right) \right) \right\} \right\} \\ \dot{x}_n(t) = v_n \end{cases} \qquad (4)$$

$$otherwise(\ jump: x_n(t) = 1 \Leftrightarrow t \equiv t_{rp}\ ), \begin{cases} x_1(t_{rp}) = 0 \\ x_2(t_{rp}) = x_1(t_{rp}^{-0}) \\ x_3(t_{rp}) = x_2(t_{rp}^{-0}) \\ \dots \\ x_n(t_{rp}) = x_{n-1}(t_{rp}^{-0}) \end{cases}, \qquad p=1,2,3,...$$

5. BUCKET BRIGADES AS HYBRID DYNAMIC SYSTEMS

This section shows how the tools provided by the theory of hybrid dynamic systems may be used to approach the modelling of bucket brigades. Both continuous-valued and discrete-valued variables determine the evolution of a hybrid dynamic system. The continuous and discrete dynamics interact when the continuous state hits certain sets in the continuous state space. In view of the taxonomy proposed by Branicky, *et al.* (1998), bucket brigades can be regarded as *hybrid systems with autonomous switching and autonomous jumps*.

The *autonomous switching* is modelled as follows:

$$\begin{cases} \dot{\underline{x}}(t) = h\left(\underline{x}(t), \underline{q}_w(t)\right) \\ \underline{q}_w^+(t) = \nu\left(\underline{x}(t), \underline{q}_w(t), \underline{q}_s(t)\right) \\ \underline{q}_s^+(t) = \mu\left(\underline{x}(t), \underline{q}_s(t)\right) \end{cases} \tag{5}$$

where: $\dot{\underline{x}}(t) = [x_1(t)\, x_2(t) .. x_n(t)]^T \in [0;1]^n$ is the continuous state space vector, $\underline{q}_w(t) \in \{0,1\}^{n-1} \times \{1\}$ denotes the discrete states of workers (*1*-working, *0*-blocked,), $\underline{q}_s(t) \in \{0,1\}^m$ represents the discrete states of workstations (*1*-occupied, *0*-available). Notation $(\cdot)^+$ denotes the following (discrete) state.

Let $\underline{v} = [v_1\ v_2 ... v_n]^T$ be the vector of work velocities. Relation (6) gives the function *h*:

$$\dot{\underline{x}}(t) = \underbrace{\begin{bmatrix} q_{w_1}(t) & & & 0 \\ & q_{w_2}(t) & & \\ & & ... & \\ 0 & & & q_{w_n}(t) \end{bmatrix}}_{A_q = diag\{\underline{q}_w(t)\}} \cdot \underline{v} \tag{6}$$

Taking into account notation (1), functions ν and μ may be respectively expressed by (7) and (8):

$$q_{w_i}^+(t) = \begin{cases} 0, q_{w_i}(t) = 1, \exists j : (x_i(t) = P_{j-1}) \wedge (q_{s_j} = 1) \\ 1, q_{w_i}(t) = 0, \exists j : (x_i(t) = P_{j-1}) \wedge (q_{s_j} = 0), \\ q_{w_i}(t), otherwise \end{cases}$$
$$i = \overline{1, n\text{-}1} \tag{7}$$
$$q_{w_n}^+(t) = 1$$

$$q_{s_j}^+(t) = \begin{cases} 0, q_{s_j}(t) = 1, \exists i : x_i(t) = P_j \\ 1, q_{s_i}(t) = 0, \exists i : x_i(t) = P_{j-1}, \\ q_{s_j}(t), otherwise \end{cases}$$
$$j = \overline{1, m} \tag{8}$$
$$q_{w_n}^+(t) = 1$$

The *autonomous jumps* may be modelled by:

$$\begin{cases} \dot{\underline{x}}(t) = h\left(\underline{x}(t), \underline{q}_w(t)\right), & \underline{x}(t) \notin M \\ \underline{x}^+(t) = \mathrm{J}\left(\underline{x}(t)\right), & \underline{x}(t) \in M \end{cases} \tag{9}$$

where $M = \{m_1, m_2, \dots, m_{n-1}, 1\} \in [0;1]^{n-1} \times \{1\}$ (jumps describe the handoff). Function *h* is given by (6) and function J is detailed in:

$$\mathrm{J}\left(\underline{x}(t)\right) = \underbrace{\begin{bmatrix} 0 & 0 & ... & 0 & 0 \\ 1 & 0 & 0 & ... & 0 \\ 0 & 1 & 0 & ... & 0 \\ ... & ... & ... & 0 & 0 \\ 0 & ... & 0 & 1 & 0 \end{bmatrix}}_{P} \cdot \underline{x}(t) \tag{10}$$

The model of a bucket brigade viewed as a hybrid dynamic system with autonomous switching and autonomous jumps is obtained by coupling the two models above (given in (5) and in (9)):

$$\begin{cases} \dot{\underline{x}}(t) = h\left(\underline{x}(t), \underline{q}_w(t)\right) = A_q \cdot \underline{v}, \\ \qquad \underline{x}(t) \notin M \quad (x_n(t) \neq 1) \\ \underline{x}^+(t) = \mathrm{J}\left(\underline{x}(t)\right) = P \cdot \underline{x}(t), \\ \qquad \underline{x}(t) \in M \quad (x_n(t) = 1) \\ \underline{q}_w^+(t) = \nu\left(\underline{x}(t), \underline{q}_w(t), \underline{q}_s(t)\right) \\ \underline{q}_s^+(t) = \mu(\underline{x}(t), \underline{q}_s(t)) \end{cases} \tag{11}$$

with the initial condition $0 \le x_{1_0} < x_{2_0} < \ldots < x_{n_0} < 1$.

An unitary approach of hybrid dynamic systems, which allows formulating the stability analysis problem, was proposed by Ye, *et al.* (1995). Concepts such as time space, motion, invariant set and equilibrium, are extended in order to embed special features of hybrid dynamic systems. They may be used in the case of bucket brigades.

A bucket brigade may be described as a 5-uple $\{T, X, A, S, T_0\}$ – which is a hybrid dynamic system:

$T = \mathbf{R}^+$ (the system evolves continuously in time);
$T_0 = \{ t_0 | t_0 \in T \}$ (the set of *initial moments*);
$X = \mathbf{R}^n$ (the set of *states*);
$X \supset A = \left\{ a \middle| a = [x_{1_0}\ x_{2_0} ... x_{n_0}]^T \right\}$ (the *initial states*);
$S \subset \{ p(t, a, t_0) = \underline{x}(t, a, t_0) = [x_1(t, a, t_0)\ x_2(t, a, t_0) \dots x_n(t, a, t_0)]^T \}$,
where $p(t_0, a, t_0) = a$, are the *motions*.

If all workers from a bucket brigade are of constant velocity along the line, when being sequenced from slowest to fastest ($v_1 < v_2 < \ldots < v_n$) and never blocked, motions are implicitly given as the solutions of the differential equation:

$$\dot{\underline{x}}(t,a,t_0) = [v_1\ v_2 \ldots v_n]^T \qquad (12)$$

with the initial condition $0 \leq x_{1_0} < x_{2_0} < \ldots < x_{n_0} < 1$.

Definition 1 - invariant set (Ye, *et al.*, 1995):
Let $\{T, X, A, S, T_0\}$ be a hybrid dynamic system. A set $M \subset A$ is called invariant of system S if

$$\forall t \in T_{a,t_0}, \forall t_0 \in T_0, \forall p(\cdot,a,t_0) \in S:$$
$$a \in M \Rightarrow p(t,a,t_0) \in M$$

It is also said that (S, M) is invariant.

Definition 2 - equilibrium (Ye, *et al.*, 1995):
$x_0 \in A$ is called an equilibrium of a hybrid dynamic system $\{T, X, A, S, T_0\}$ if (S, $\{x_0\}$) is invariant.

In other words, any motion starting from a state of an invariant remains within that invariant. An equilibrium is a one element invariant set. The TSS Rule allows proving the existence of an invariant for a bucket brigade (Bratcu and Mînzu, 2001).

Proposition:
If the system S, described by relation (12), is reinitialised every time when $x_n(t)=1$ according to

$$\begin{cases} x_1(t) = 0 \\ x_i(t) = x_{i-1}(t^-),\ i = \overline{2,n} \end{cases} \qquad (13)$$

then (S, $[0;1]^n$) is invariant.

One may superficially think that the balancing point of the line – expressed by relation (2) – might be characterised as an equilibrium of the system, in view of definition 2. This is not true, because the motions do not converge to a single point of the state space, but to *a pattern* of moving, where each worker i repeats the execution of a work content in the interval $\left[\frac{\sum_{j=1}^{i-1} v_j}{\sum_{j=1}^{n} v_j}; \frac{\sum_{j=1}^{i} v_j}{\sum_{j=1}^{n} v_j}\right]$. The stationary behaviour is a *periodical* one.

In the particular case of constant work velocities, the convergence to a stationary behaviour of a well ordered bucket brigade was proved in the theory of discrete dynamic systems (Bratcu and Mînzu, 2001).

6. CONCLUSION

As self-organising systems, able to autonomously reach a periodical stationary behaviour, expressed by the optimal production rate, the bucket brigades were first treated as stochastic dynamic systems. This paper has shown that they may be modelled as nonlinear dynamic systems and, more correctly, as hybrid dynamic systems, namely with autonomous switching and autonomous jumps. Another modelling approach, emphasising discontinuous motions, has opened a view to the stability analysis.

The results of modelling and analysis of TSS lines have used some simplifying assumptions. A further development of this work would be to treat the general case, probably in the frame of chaos theory. On the other hand, the actual trend of implementing bucket brigades on flow lines is oriented to using robots, obviously more easily to control.

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