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A SUPERVISORY ROBUST ADAPTIVE FUZZY CONTROLLER

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Abstract: A fuzzy controller equipped with an adaptive algorithm and two supervisors is developed in this work to achieve tracking performances for a class of uncertain non-linear single input single output (SISO) systems with external disturbances. The convergence of the training algorithm is guarantied by a gradient projection law. The effect of both the approximation errors and the external disturbances is attenuated to a prescribed level thanks to $H\infty$ control. The convergence of the tracking error toward zero is guarantied by a supervisor where linguistic rules are used to accelerate the convergence speed. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Fuzzy logic control, as one of the most useful approaches for utilising expert knowledge, has been a subject of intense research in recent years (King and Mamdani, 1977; Mamdani and Assilian, 1975; Sugeno, 1985; Tong *et al*., 1980). Fuzzy logic control is generally applicable to plants that are mathematically poorly modelled and where experienced operators are available for providing qualitative guiding. Although achieving many practical success, fuzzy control has not been viewed as a rigorous science, because most of the fuzzy logic algorithms are proposed without analytical tools to guarantee basic performance criteria.

According to the universal approximation theorem (Wang, 1996), many important adaptive fuzzy-based control schemes have been developed to incorporate the expert information directly and systematically, and various stable performance criteria are guaranteed by theoretical analysis (Marino and Tomei, 1995; Spooner and Passino, 1996; Wang, 1996). The major advantages in all these fuzzy-based control schemes are that the developed controllers can be implemented without any precise knowledge about the structure of the entire dynamic model. However, the influence of both fuzzy logic approximation errors and the external disturbances can not be eliminated with these approaches (Chang, 2001). In this sense, Hamzaoui, *et al*. (2000) have proposed a fuzzy logic controller equipped with a training algorithm to approximate the system and a H_∞ control to attenuate the effect of both fuzzy approximation errors and external disturbances. However, only a good choice of the initial parameters of the fuzzy approximator can guarantee the convergence of the algorithm. Chen, *et al*. (1996) proposed a similar approach with a gradient projection law to assure the convergence of the adaptive fuzzy logic system. But, the attenuation level can not be systematically determined because it depends on the control signal (Kang, *et al*., 1998). Furthermore, no constraints are imposed to keep the system in a forced region ($g(x)\neq 0$).

In order to alleviate these problems, we propose in this work a new fuzzy adaptive algorithm equipped with a gradient projection law and two supervisors. The first supervisor, u_s, forces the system to remain in a given controllability zone. Thus, the controller's parameters are bounded and the quadratic integral of both the minimal approximation error and the tracking error is bounded, i.e, the tracking error converges to zero. The second supervisor, u_a attenuates the effect of both the approximation errors and the external disturbances to a prescribed level, ρ, using H∞ approach. The performances of the resulting controller can be improved by incorporating some linguistic rules describing the dynamic behaviour of the plant. The classical example of inverted pendulum, as presented in (Wang, 1996), is used to illustrate this approach. We show that the proposed algorithm is robust and the control signal is smooth compared to (Hamzaoui, *et al.*, 2000) and (Wang, *et al.*, 2001).

Section 2 presents the problem statement. Section 3 gives, in a constructive manner, the steps for constructing the robust adaptive fuzzy controller, and how to use the two supervisors to meet the control objectives. A pendulum tracking control example is given in section 4 for illustration.

2. PROBLEM STATEMENT

We consider the following *n*th order non linear dynamic Single Input Single Output (SISO) system in the canonical form:

$$\begin{cases} x^{(n)} = f(x,\dot{x},\ldots,x^{(n-1)}) + g(x,\dot{x},\ldots,x^{(n-1)})u + d \\ y = x \end{cases} \quad (1)$$

where *f* and *g* are unknown (uncertain) but bounded continuous functions; $u \in \Re$ and $y \in \Re$ are the input and output of the system, respectively. *d* denotes the external disturbances (due to system load, external noise, etc) which is assumed to be unknown but bounded. It should be noted that more general classes of non linear control problem can be transformed into this structure (Slotine and Li, 1991; Chen, *et al.*, 1996). Let $\mathrm{X} = \left(x,\dot{x},\ldots,x^{(n-1)}\right)^{\mathrm{T}} \in \Re^{\mathrm{n}}$ be the state vector of the system which is assumed to be available for measurement. We require the system (1) to be controllable, thus the condition $g(\mathrm{X}) \neq 0$ must be satisfied in a given controllability region $U_c \subset \Re^{\mathrm{n}}$. Without loss of generality we assume that $g(\mathrm{X}) > 0$, but the analysis throughout this paper can easily be tailored to systems with $g(\mathrm{X}) < 0$.

The control objective is to force *y* to follow a given bounded reference signal, y_r, under the constraint that all the parameters (u,y,X) are bounded and the closed loop system is globally stable and robust.

If the system is well-known and free of external disturbances, feedback linearization (Isiodori, 1989) can be used to synthesis a control law of the form:

$$u = \frac{1}{g(\mathrm{X})}\left(-f(\mathrm{X}) + y_r^{(n)} + \mathrm{K}^{\mathrm{T}}\mathrm{E}\right) \quad (2)$$

where $\mathrm{E} = [e,\dot{e},\ldots,e^{(n-1)}]^{\mathrm{T}}$ is the error vector, $e = y_r - y$, and $\mathrm{K} = [k_n,k_{n-1},\ldots,k_1]^{\mathrm{T}}$ is the dynamic error coefficient vector such that all the roots of the polynomial $\mathrm{H}(s) = s^n + k_1 s^{n-1} + \ldots + k_n$ are located in the open left half plane.

The control signal (2) gives the following dynamic error: $e^{(n)} + k_1 e^{(n-1)} + \ldots + k_n e = 0$ (3)

which implies that $\lim_{t\to\infty} e(t) = 0$. However, it is impossible to obtain such a control algorithm if *f* and *g* are unknown and the system is perturbed. A fuzzy logic approximation, as described in section 3, is therefore employed to treat this tracking control design problem. The following control law, proposed by (Hamzaoui, *et al*, 2000), can be applied to the system:

$$u = \frac{1}{\hat{g}(\mathrm{X})}\left(-\hat{f}(\mathrm{X}) + y_r^{(n)} + \mathrm{K}^{\mathrm{T}}\mathrm{E} - u_a\right) \quad (4)$$

where $\hat{f}$ and $\hat{g}$ are the approximations of *f* and *g*, respectively, and u_a is the control signal which attenuates the effect of both the approximation errors and the external disturbances.

3. ROBUST TRACKING PERFORMANCE DESIGN IN ADAPTIVE FUZZY SYSTEM

The objective of this work is to guarantee the convergence of both the estimation algorithm and the tracking error. For the first requirement, we impose the convergence of the adjustable parameters using the projection technique. For the second, we add a supervisor control signal, u_s, to guarantee the stability, in the sense of Lyapunov, of the system.

3.1 Adaptive fuzzy algorithm

The approximation $\hat{f}$ and $\hat{g}$ (in (4)), can be given by the universal fuzzy systems $\hat{f}\left(\mathrm{X}\middle|\theta_f\right)$ and $\hat{g}\left(\mathrm{X}\middle|\theta_g\right)$ (Wang,1996):

$$\hat{f}\left(\mathrm{X}\middle|\theta_f\right) = \theta_f^{\mathrm{T}}\zeta(\mathrm{X}) \quad , \quad \hat{g}\left(\mathrm{X}\middle|\theta_g\right) = \theta_g^{\mathrm{T}}\zeta(\mathrm{X}) \quad (5)$$

where θ_f and θ_g are the vectors of the tuneable parameters and $\zeta(\mathrm{X}) = [\zeta_1(\mathrm{X}),\ldots,\zeta_h(\mathrm{X})]$ is a regressive vector as given in (Wang, 1996).

The tracking error dynamic equation resulting from (4) can be written as:

$$\dot{\mathrm{E}} = \mathrm{AE} + \mathrm{B}\left[\left(\hat{f} - f\right) + \left(\hat{g} - g\right)u + u_a + d\right] \quad (6)$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & 1 & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & 1 \\ -k_n & -k_{n-1} & \ldots & \ldots & \ldots & \ldots & -k_1 \end{bmatrix}, \quad \mathrm{B} = [0 \ldots 0\ 1]^{\mathrm{T}}$$

A is a stable matrix, thus it can be associated with the following algebraic Riccati equation which has a unique positive definite solution, $\mathrm{P} = \mathrm{P}^{\mathrm{T}}$, if and only if $\frac{2}{\mathrm{r}} - \frac{1}{\rho^2} \geq 0$, i.e., if $2\rho^2 \geq \mathrm{r}$:

$$\mathrm{AP}^{\mathrm{T}} + \mathrm{PA} + \mathrm{Q} - 2\mathrm{PB}\left(\frac{1}{\mathrm{r}} - \frac{1}{2\rho^2}\right)\mathrm{B}^{\mathrm{T}}\mathrm{P} = 0 \quad (7)$$

where Q is a positive definite matrix given by the designer.

According to the universal approximation theorem (Wang, 1996), there exists optimal approximation parameters θ_f^* and θ_g^* such that $\hat{f}\left(\mathrm{X}\middle|\theta_f^*\right)$ and

$\hat{g}\left(X\middle|\theta_g^*\right)$ can, respectively, approximate $f(X)$ and $g(X)$ as closely as possible. The minimum approximation error is defined as:

$$w_e = \left(f\left(X\middle|\theta_f^*\right) - f(X)\right) + \left(g\left(X\middle|\theta_g^*\right) - g(X)\right)u \qquad (8)$$

and the tracking error dynamic equation (6) can be rewritten as:

$$\begin{aligned}\dot{E} &= AE + B\left(\hat{f}\left(X\middle|\theta_f\right) - \hat{f}\left(X\middle|\theta_f^*\right)\right) \\ &+ B\left(\hat{g}\left(X\middle|\theta_g\right) + \hat{g}\left(X\middle|\theta_g^*\right)\right)u + B\left[u_a + w_e - d\right]\end{aligned} \qquad (9)$$

From (5), (9) can be rewritten as:

$$\dot{E} = AE + B\left[\Phi_f^T\zeta(X) + \Phi_g^T\zeta(X) + u_a + w_e - d\right] \qquad (10)$$

where $\Phi_f = \theta_f - \theta_f^*$, $\Phi_g = \theta_g - \theta_g^*$.

Let's choose the Lyapunov function as:

$$V = \frac{1}{2}E^T PE + \frac{1}{2\gamma_f}\Phi_f^T\Phi_f + \frac{1}{2\gamma_g}\Phi_g^T\Phi_g \qquad (11)$$

where γ_f and γ_g are positive constants.

The time derivative of V along the error trajectory (10) is given by:

$$\begin{aligned}\dot{V} &= \frac{1}{2}E^T QE - \frac{1}{\rho^2}E^T PBB^T PE \\ &+ E^T PB\left(\frac{1}{r}B^T PE + u_a - d + w_e\right) \\ &+ \frac{1}{\gamma_f}\Phi_f^T\left(\dot{\theta}_f + \gamma_f E^T PB\zeta(X)\right) \\ &+ \frac{1}{\gamma_g}\Phi_g^T\left(\dot{\theta}_g + \gamma_g E^T PB\zeta(X)u\right)\end{aligned} \qquad (12)$$

Then by using the following control and adaptation laws proposed in (Hamzaoui, *et al.*, 2000):

$$\begin{cases} u_a = \frac{1}{r}E^T PB \\ \dot{\theta}_f = -\gamma_f E^T PB\zeta(X) \\ \dot{\theta}_g = -\gamma_g E^T PB\zeta(X)u \end{cases} \qquad (13)$$

we obtain:

$$\dot{V} \le -\frac{1}{2}E^T QE + \frac{\rho^2(w_e - d)^2}{2} \qquad (14)$$

In this case, the parameters θ_f and θ_g are not guaranteed to be bounded, which means that w_e is not bounded. The modified algorithm, proposed in the following subsection, has therefore been developed to obtain a stable system.

3.2. *Modified adaptive fuzzy algorithm*

Let the constraint sets Ω_f and Ω_g be defined as:

$$\begin{aligned}\Omega_f &= \left\{\theta_f \,\middle|\, \left|\theta_f\right| \le M_f < \infty\right\} \\ \Omega_g &= \left\{\theta_g \,\middle|\, 0 < \varepsilon \le \left|\theta_g\right| \le M_g < \infty\right\}\end{aligned} \qquad (15)$$

where M_f, ε, and M_g are constants. Since $\hat{g} \neq 0$, $|\theta_g|$ must be bounded from below by ε>0.

We therefore propose the modified adaptation law where u_a is the same as in (13), but θ_f and θ_g are calculated, using the projection technique (Goodwin and Mayne, 1987), as follows:

$$\dot{\theta}_f = \begin{cases} -\gamma_f E^T PB\zeta(X) & \text{if } \left(\left|\theta_f\right| < M_f\right) \\ & \text{or } \left(\left|\theta_f\right| = M_f \text{ and } E^T PB\zeta(X) \ge 0\right) \\ \gamma_f \Pr\left\{-E^T PB\zeta(X)\right\} & \text{if } \left(\left|\theta_f\right| = M_f \text{ and } E^T PB\zeta(X) < 0\right) \end{cases} \qquad (16)$$

where the projection operator $\Pr\{*\}$ is defined as:

$$\Pr\left\{-E^T PB\zeta(X)\right\} = -E^T PB\zeta(X) + E^T PB\frac{\theta\theta^T\zeta(X)}{|\theta|^2}$$

If an element θ_{gi} of θ_g is equal to ε, then:

$$\dot{\theta}_{gi} = \begin{cases} -\gamma_g E^T PB\zeta_i(X)u & \text{if } E^T PB\zeta_i(X)u < 0 \\ 0 & \text{if } E^T PB\zeta_i(X)u \ge 0 \end{cases} \qquad (17)$$

where $\zeta_i(X)$ is ith component of $\zeta(X)$.

Otherwise:

$$\dot{\theta}_g = \begin{cases} -\gamma_g E^T PB\zeta(X)u & \text{if } \left(\left|\theta_g\right| < M_g\right) \\ & \text{or } \left(\left|\theta_g\right| = M_g \text{ and } E^T PB\zeta(X)u \ge 0\right) \\ \gamma_g \Pr\left\{-E^T PB\zeta(X)\right\}u & \text{if } \left(\left|\theta_g\right| = M_g \text{ and } E^T PB\zeta(X)u < 0\right) \end{cases} \qquad (18)$$

3.3 Stability and robustness analysis

Since the convergence of the parameters θ_f and θ_g is guaranteed by this modified adaptation law, the next step is to guarantee the convergence of the tracking error toward zero.

From (12), the control law (13) gives:

$$\begin{aligned}\int_0^t\left|e(\tau)\right|^2 d\tau &\le \frac{2}{\lambda_{\min}(Q)-1}\left(V(0) - \sup_{t\ge0}\left|V(t)\right|\right) \\ &+ \frac{1}{\lambda_{\min}(Q)-1}|PB|^2\int_0^t\left|w_e(\tau) - d\right|^2 d\tau\end{aligned} \qquad (19)$$

where $\lambda_{min}(Q)$ is the minimum eigenvalue of Q. Let's choose Q such that $\lambda_{min}(Q) > 1$. The existence of the integral $\int_0^t\left|e(\tau)\right|^2 d\tau$ implies that $\lim_{t\to\infty} e(t) = 0$. So, the convergence of the tracking error toward zero depends only on the term $\left(V(0) - \sup_{t\ge0}\left|V(t)\right|\right)$. We require that $V = \frac{1}{2}E^T PE \le \overline{V}$, where $\overline{V}$ is a constant specified by the designer. Then, after some straightforward manipulations, we obtain:

$$\begin{aligned}\dot{V} &= -\frac{1}{2}E^T QE - \frac{1}{2}\left(\frac{E^T PB}{\rho} + \rho d\right)^2 + \frac{\rho^2|d|^2}{2} \\ &+ E^T PB\left(\left(\hat{f} - f\right) + \left(\hat{g} - g\right)u + u_a + \frac{1}{r}B^T PE\right)\end{aligned} \qquad (20)$$

Using the control law (13), (20) can be rewritten as:

$$\begin{aligned}\dot{V} &= -\frac{1}{2}E^T QE - \frac{1}{2}\left(\frac{E^T PB}{\rho} + \rho d\right)^2 \\ &+ \frac{\rho^2|d|^2}{2} + E^T PB\left(\left(\hat{f} - f\right) + \left(\hat{g} - g\right)u\right)\end{aligned} \qquad (21)$$

Since the first term is negative, a good choice of the attenuation factor, ρ, results in a small value for the term $(\rho|d|)^2/2$. Since the sign of

$E^T PB\left(\left(\hat{f}-f\right)+\left(\hat{g}-g\right)u\right)$ is unknown, we append a supervisor, u_s, to obtain the overall control signal:

$$u_t=u+u_s \tag{22}$$

We now show how to determine u_s such that $\dot{V}\le 0$ when $V>\overline{V}$.

Substituting (22) into (1) and after some manipulations, the new error equation becomes:

$$\dot{E}=AE+B\left[\left(\hat{f}-f\right)+\left(\hat{g}+g\right)u+gu_s+u_a-d\right] \tag{23}$$

using (23) and (7), we obtain:

$$\dot{V}=-\frac{1}{2}E^T QE-\frac{1}{2}\left(\frac{E^T PB}{\rho}+\rho d\right)^2+\frac{\rho^2|d|^2}{2}$$
$$+E^T PB\left(\left(\hat{f}-f\right)+\left(\hat{g}-g\right)u-gu_s\right)$$

and therefore,

$$\dot{V}\le-\frac{1}{2}E^T QE+\left|E^T PB\right|\left(\left(\left|\hat{f}\right|+|f|\right)+\left(|\hat{g}u|+|gu|\right)\right) - E^T PBgu_s \tag{24}$$

In order to design u_s such that the last term of (24) is nonpositive, we need to know the bounds of f and g, i.e., we have to determine the functions $f^M(X)<\infty$, $g^M(X)<\infty$, and $g_m(X)>0$ such that $|f(X)|\le f^M(X)$ and $g_m(X)\le|g(X)|\le g^M(X)$ for $X\subset U_c$.

Consequently, the supervisory control, u_s, is chosen as follows:

$$u_s=I\,sgn\left(E^T PB\right)\frac{1}{g_m}\left[\left|\hat{f}\right|+f^M+\left|\hat{g}u_c\right|+\left|g^M u_c\right|\right] \tag{25}$$

where I=1 if $V>\overline{V}$, I=0 if $V\le\overline{V}$, and sgn(y)=1 (respectively, -1) if $y\ge 0$, (respectively, <0). Substituting (25) into (24) and considering the case $V>\overline{V}$, we have:

$$\dot{V}\le-\frac{1}{2}E^T QE+\left|E^T PB\right|\left(\left(\left|\hat{f}\right|+|f|\right)+\left(|\hat{g}u|+|gu|\right)\right)$$
$$-\frac{g}{g_m}\left|E^T PB\right|\left(\left(\left|\hat{f}\right|+\left|f^M\right|\right)+\left(|\hat{g}u|+\left|g^M u\right|\right)\right)\le 0 \tag{26}$$

which guarantees that $\dot{V}<0$.

From (12), (16)-(18) and (7), we obtain the same inequality given in (14).

Integrating (14) from t=0 to T yields:

$$V(T)-V(0)\le-\frac{1}{2}\int_0^T E^T QE+\int_0^T\frac{\rho^2\left(w_e-d\right)^2}{2} \tag{27}$$

Since $V(T)\ge 0$, the above inequality implies that:

$$\frac{1}{2}\int_0^T E^T QE\le V(0)+\int_0^T\frac{\rho^2\left(w_e-d\right)^2}{2} \tag{28}$$

From (14), the inequality (28) is equivalent to:

$$\frac{1}{2}\int_0^T E^T QE\,dt\le\frac{1}{2}E^T(0)PE(0)+\frac{1}{2\gamma_f}\Phi_f^T(0)\Phi_f(0)$$
$$+\frac{1}{2\gamma_g}\Phi_g^T(0)\Phi_g(0)+\frac{1}{2}\rho^2\int_0^T\left(w_e-d\right)^2 dt \tag{29}$$

This is our H_∞ criterion.

3.4 Design procedure

In order to minimise the on-line computing time of our algorithm, the design of the robust adaptive fuzzy controller implies an off-line processing step, and an on-line during control execution as shown bellow:

- Off-line processing
  - Specify M_f, M_g, ε and $\overline{V}$.
  - Determine K and Q satisfying (3) and (19), respectively.
  - Solve the algebraic Ricatti equation.
  - Choosing the initial parameters.
- On-line processing
  - Apply $u_t=u+u_s$, where u is given by (4) and u_s by (25).
  - Use the adaptation law, given by (16)-(18), to adjust the parameters.

4. SIMULATION EXAMPLE

To validate our approach, we consider the inverted pendulum depicted in fig. 1.

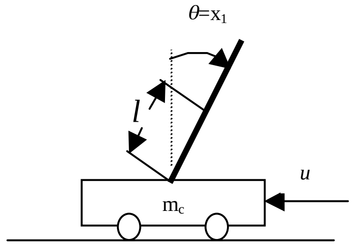

Fig. 1. The inverted pendulum system.

Let $x_1=\theta$ and $x_2=\dot{\theta}$. The dynamic equation of the inverted pendulum as is given by (Wang, 1996):

$$\dot{x}_1=x_2$$
$$\dot{x}_2=\left[\frac{g\sin x_1-\dfrac{mlx_2^2\cos(x_1)\sin(x_1)}{m_c+m}}{l\left(\dfrac{4}{3}-\dfrac{m\cos^2(x_1)}{m_c+m}\right)}\right]+\frac{\dfrac{\cos(x_1)}{m_c+m}}{l\left(\dfrac{4}{3}-\dfrac{m\cos^2(x_1)}{m_c+m}\right)}u_t+d$$
$$y=x_1 \tag{30}$$

where g is the acceleration due to gravity, m_c is the mass of the cart, m is the mass of the pole, l is the half-length of the pole, the force u_t represents the control signal, and d is the external disturbance. We choose m_c=1Kg, m=0.1Kg and l=0.5m in the following simulations. The reference signal is assumed here to be $y_r(t)=(\pi/30)\sin(t)$, and the system is subject to two disturbances:

- A structural disturbance on the mass of both the cart and the pole, in the form: $dm=0.01.m.\sin(t)$
- An external disturbance: $d(t)=0.1.\sin(t)$

If we require:

$$|X|\le\frac{\pi}{6}\quad,\quad|u_t|\le 180 \tag{31}$$

and substituting the functions sin(.), and, cos(.) by their limited development we can determine the bounds:

$$f^M(x_1,x_2)=15.78+0.366x_2^2$$
$$g^M(x_1,x_2)=1.46,\ g_m(x_1,x_2)=1.12 \tag{32}$$

To satisfy (3) and (19), we choose, for example, k_1=2, k_2=1 and Q=diag(10,10). Furthermore to simplify the calculation, we choose $r=2\rho^2$. So, the solution of the algebraic Riccati equation is: $P=\begin{bmatrix}15&5\\5&5\end{bmatrix}$, and $\lambda_{min}(P)=2.93$. To satisfy the

constraint related to $|x|$, we choose $\overline{V}=\frac{\lambda_{\min}}{2}\left(\frac{2\pi}{15}\right)^2=0.267$, $M_f=16$, $M_g=1.6$ and $\varepsilon=0.48$.
We select 5 Gaussian membership functions for both x_1 and x_2 (i=1,2) to cover the whole universe of discourse:

$$\mu F_i^1(x_i)=\exp\left(-\left(\frac{x_i-\pi/6}{\pi/24}\right)^2\right)\quad \mu F_i^2(x_i)=\exp\left(-\left(\frac{x_i-\pi/12}{\pi/24}\right)^2\right)$$

$$\mu F_i^3(x_i)=\exp\left(-\left(x_i\big/\pi/24\right)^2\right)\quad \mu F_i^4(x_i)=\exp\left(-\left(\frac{x_i+\pi/12}{\pi/24}\right)^2\right)$$

$$\mu F_i^5(x_i)=\exp\left(-\left(\frac{x_i+\pi/6}{\pi/24}\right)^2\right)\qquad (33)$$

After trial and errors, $\gamma_f=50$ and $\gamma_g=1$ are chosen. The MATLAB command "ode23s" is used to simulate the overall control system with step size 0.01.
The initial position of the pendulum is chosen as far as possible ($\theta(0)=x_1=\pi/12$) to improve the efficiency of our algorithm.
Two cases are considered to show the influence of the incorporation of the linguistic rules in the control law:
Case one: the initial values of θ_f and θ_g are chosen arbitrarily.
Case two: the initial values of θ_f and θ_g are deduced from the fuzzy rules describing the dynamic behaviour of the system. For example, if we consider the unforced system, i.e., $u_t=0$, the acceleration is equal to $f(x_1,x_2)$. So, intuitively, we state that:

"The bigger is x_1, the larger is $f(x_1,x_2)$"

Our task now is to transform this fuzzy information into a fuzzy rule. We obtain the rule:

$R_f^{(1)}$ **:** IF x_1 is F_1^5 and x_2 is F_2^5

THEN $f(x_1,x_2)$ is Positive Big

where "Positive Big" is a fuzzy set whose membership function is $\mu F_i^1(x_i)$ given in (33). The acceleration is proportional to the gravity, i.e. $f(x_1,x_2)\cong\alpha.\sin(x_1)$, where α is a constant. Since $f(x_1,x_2)$ acheives its maximum at $x_1=\pi/2$; thus based on (32), we have $\alpha\cong16$. Therefore, we the final fuzzy rules characterizing $f(x_1,x_2)$ as shown in fig. 2, which comprises 25 rules.

$f(x_1,x_2)$			x_1				
			F_1^1	F_1^2	F_1^3	F_1^4	F_1^5
			-π/6	-π/12	0	π/12	π/6
x_2	F_2^1	-π/6	-8	-4	0	4	8
	F_2^2	-π/12	-8	-4	0	4	8
	F_2^3	0	-8	-4	0	4	8
	F_2^4	π/12	-8	-4	0	4	8
	F_2^5	π/6	-8	-4	0	4	8

Fig. 2. Linguistic rules for $f(x_1,x_2)$

Now, to determine the fuzzy rules for $g(x_1,x_2)$, we use the following observation:

"The smaller is x_1, the larger is $g(x_1,x_2)$"

Similary to the case of $f(x_1,x_2)$ and based on the bounds (32), this observation can be quantified into the 25 fuzzy rules given in fig. 3.

$g(x_1,x_2)$			x_1				
			F_1^1	F_1^2	F_1^3	F_1^4	F_1^5
			-π/6	-π/12	0	π/12	π/6
x_2	F_2^1	-π/6	1.6	1.36	1.46	1.36	1.26
	F_2^2	-π/12	1.26	1.36	1.46	1.36	1.26
	F_2^3	0	1.26	1.36	1.46	1.36	1.26
	F_2^4	π/12	1.26	1.36	1.46	1.36	1.26
	F_2^5	π/6	1.26	1.36	1.46	1.36	1.26

Fig. 3. Linguistic rules for $g(x_1,x_2)$

To obtain the same tracking performances, the attenuation level, ρ, was equal to 0.2, in the first case and to 0.8 in the second.
For both the two cases fig. 4 illustrates the tracking performance for a sinusoidal trajectory; the pendulum reaches the reference trajectory in 3.14s. The quadratic error is given by the fig. 5.

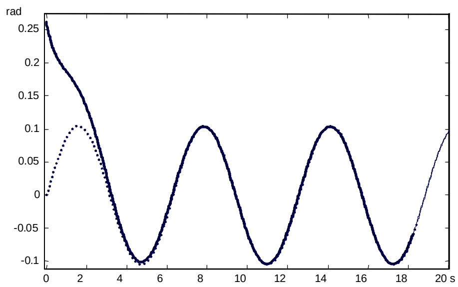

Fig. 4. The state x_1 (solid line) and its desired value $y_r(t)$ (dashed line) for $X(0)=(\pi/12,0)^T$

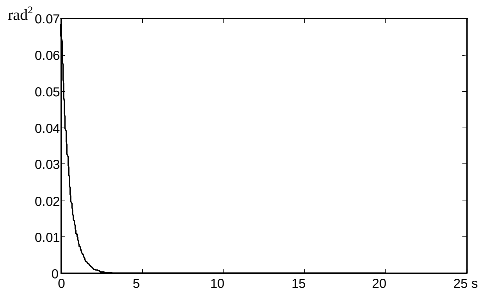

Fig. 5. The quadratic error

Figures 6 and 7 show the difference between the control signal u_s and u in the two cases, respectively. As shown in fig.8 , when we incorporate the linguistic rules in the controller, and with a high level of attenuation, the initial global control is much smaller than the control signals proposed in (Hamzaoui, *et al.*, 2000) and (Chen , *et al.*, 1996).

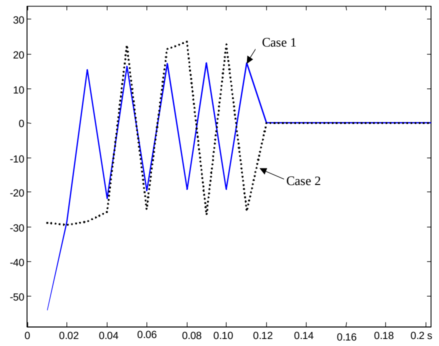

Fig. 6. The supervisory control u_s

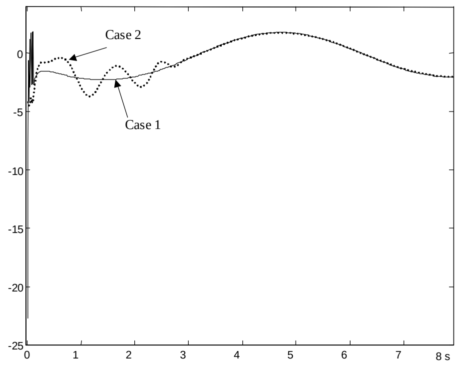

Fig. 7. The control signal u

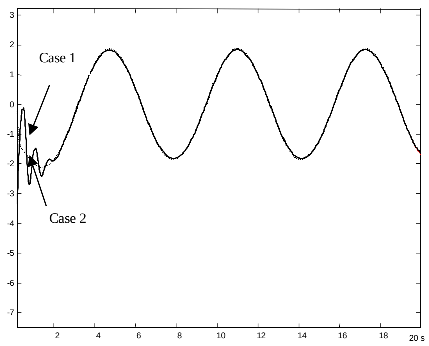

Fig. 8. The global control signal u_t

5. CONCLUSION

An adaptive fuzzy controller with two supervisors is proposed for the control of a class of nonlinear systems subject to large uncertainties or to unknown variations in the parameters and the structure of the plant. The projection theorem is used to guarantee the convergence of the adaptation laws coresponding to fuzzy approximators. The first supervisor, u_s, ensures the global stability of the system, in the sense of lyapunov. The second supervisor, u_a, uses H∞ technique to attenuate the effect of both external disturbances and approximation errors to a prescribed level. The stability and the robustness are demonstrated analytically, and an illustrative example has been used to show the efficiency of the proposed method. The performances of the approch can be improved by incorporating some linguistic rules. However, the design of the control algorithm needs a good knowledge of the dynamic behaviour of the system in order to determine both the bounds and the linguistic rules of the functions f and g. Futher work is under investigation to apply the proposed robust adaptive algorithm to mlti-input multi-output systems.

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