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ROBUST DISTURBANCE SUPPRESSION OF A MAGNETIC LEVITATION SYSTEM WITH INPUT CONSTRAINT [1]

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Abstract: In this paper the problem of noise suppression for a magnetic levitation system is addressed. The problem is formulated as a nonlinear regulation problem; a state feedback and an error feedback internal model-based regulators are designed, able to offset the noise in spite of the presence of unknown parameters affecting the model of the system. In both cases a saturated control law is designed in order to fulfill physically constraint characterizing the system. Moreover a general result in the context of output regulation of linear systems with constant uncertain parameters affecting the inputs channel and with input constraints is addressed.

Keywords: Nonlinear output regulation, Magnetic levitation system, Small gain theorem, Saturated control

1. INTRODUCTION

Several works have been recently devoted to design a control law able to stabilize the dynamics of a nonlinear magnetic levitation system (see, besides others, (Trumper *et al.*, 1997), (Alleyne, 2000)). The typical experimental setup able to reproduce all the peculiarity of a magnetic levitation system, is given by a mass moving within a magnetic field produced by a coil current.

In particular denoting by x_1 the position of the mass, x_2 its velocity, g the acceleration of gravity and u (the control input) the coil current, the nonlinear model of the mechanical subsystem is described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g + \alpha \left(\dfrac{u}{g_0 - x_1} \right)^2 + \delta(t) \end{cases} \tag{1}$$

where $\delta(t)$ is a disturbance force acting on the mass, and α and g_0 are physical parameters dependent on the mass, the permeability of the space, the reluctance of the core etc. (for a detailed discussion on how this model can be derived see (Woodson and Melcher, 1968)).

The relevant aspect of this model is the nonlinearity which characterizes the control input, namely the fact that the control input, entering squared in the model, can just enforce a positive “push”. From a physical point of view it means that, while to push up the mass (namely the steer it towards the magnetic coil) a suitably coil current value can be used, to eventually steer the mass on the opposite direction the only allowed control action is to decrease the coil current in order to make the force of gravity predominant. As the disturbance $\delta(t)$ is concerned, it is worth to note that its presence is typical in applications where magnetic levitation principle is used for low-friction rotating machinery (Alleyne, 2000). Usually any radial imbalance in the rotating components will set up a periodic disturbance whose magnitude phase and frequency are directly related to the severity of imbalance and rotational speed.

In this paper we address the problem of designing a control law for system (1) able to steer the position

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of the mass to a desired (known) position while rejecting the disturbance $\delta(t)$ robustly with respect to the uncertain value of the physical constant α. The disturbance is modeled as a period signal obtained as sum of a finite number of sinusoidal functions of time of known frequencies and unknown phases and amplitudes. The problem is set up as a *nonlinear regulation problem* (see (Byrnes *et al.*, 1997*b*)). The disturbance is thought as output of an autonomous *neutrally stable* system (usually known as *exosystem*) and an internal model-based regulator is designed able to reject any disturbance generated by the exosystem.
In particular we propose different solutions to the above problem. First of all in the next section we show that the problem can be cast as a *linear robust output regulation problem* in *presence of input constraint*. After some preliminary positions, in section 3 we address the design of a regulator in case of complete knowledge of the state (x_1, x_2) of the system. We show in particular how to design the *lowest order* controller able to achieve tracking of the reference position while rejecting the disturbance for any initial state of the system and for any initial state of the exosystem in a certain set. Then, in section 4 we relax the assumption on the knowledge of the ball velocity x_2 looking for a controller which, processing the tracking error, yields regulation for any initial state of the plant and of the exosystem belonging to given compact sets. We also point out how the latter design method could be reformulated as a general result in the context of output regulation of linear systems with constant uncertain parameters affecting the inputs channel and with inputs constraint. Simulations are given in section 5, and in section 6 we conclude with some remarks.

2. PROBLEM STATEMENT AND PRELIMINARY POSITIONS

The problem usually addressed for system (1) is that of computing a control law u in order to steer the position x_1 of the mass to a certain reference position $x_1^{\rm des}$ in spite of the presence of the disturbance $\delta(t)$ and of the uncertain parameter α.
We assume that the disturbance δ belongs to the class of signals generated by the linear autonomous system (*exosystem*)

$$\begin{aligned} \dot{\omega}(t) &= S\omega(t) \\ \delta(t) &= Q\omega(t) \end{aligned} \tag{2}$$

where S is a $\mathbb{R}^s \times \mathbb{R}^s$ matrix and Q is a vector.
In this discussion the matrix S is supposed to be perfectly known and to be “neutrally stable”. For instance any $\delta(t)$ obtained by linear combination of sinusoidal signals with known frequencies and unknown amplitudes and phases can be modeled in this way.

Defining $x_1^{\rm des}$ the reference position, α_0 the nominal value of the uncertain parameter α and $e_1 := x_1 - x_1^{\rm des}$ the tracking error, it turns out that setting the preliminary control law

$$u^2 = \frac{(g_0 - x_1)^2}{\alpha_0}(g + v) \tag{3}$$

where v is an additional control law, the error system to deal with reads as

$$\begin{cases} \dot{e}_1 = x_2 \\ \dot{x}_2 = \mu v + \delta_1 + \delta(t) \end{cases} \tag{4}$$

with

$$\mu := \frac{\alpha}{\alpha_0} \qquad \delta_1 := \frac{\alpha - \alpha_0}{\alpha_0}\, g = (\mu - 1)\, g \; .$$

In particular in view of the expression of $\delta(t)$ in (2) it turns out that the problem can be framed as a classical robust nonlinear regulation problem (see (Byrnes *et al.*, 1997*b*)) in which the exogenous terms to offset are δ_1, the residual force of gravity which can not be compensated by feedforward actions due to the presence of the uncertain parameter μ, and the time-varying disturbance $\delta(t)$. In particular the overall exogenous disturbance $\delta_1 + \delta(t)$ can be thought as generated by an *augmented* exosystem

$$\begin{aligned} \dot{w}_{\rm a} &= S_{\rm a} w_{\rm a} \\ \delta_1 + \delta &= Q_{\rm a} w_{\rm a} \end{aligned} \tag{5}$$

where

$$S_{\rm a} := \begin{pmatrix} S & 0 \\ 0 & 0 \end{pmatrix} \qquad Q_{\rm a} := \begin{pmatrix} Q & 1 \end{pmatrix} \; .$$

Moreover, bearing in mind (3), it is easy to realize the the regulation problem is complicated by the *presence of input constraint*. As a matter of fact, in order to reconstruct the real control input u, the new control variable v must satisfy

$$g + v(t) > 0 \quad \text{namely} \quad -g \le v(t) \le +g \tag{6}$$

for all $t \geq 0$ (note that condition $v(t) \geq -g$ is imposed only to obtain a symmetric saturation). Indeed if the design procedure is able to guarantee the condition (6), it follows that the original control input can be reconstructed as

$$u = (g - x_1)\sqrt{\frac{g + v}{\alpha_0}} \; .$$

With the above considerations we have recast the problem in question as a problem of linear output regulation in presence of constraint regarding the amplitude of the control input and in presence of uncertain parameters (in the specific case μ) affecting the model of the system.
In particular this problem will be solved under the assumption that the uncertain parameter μ (and hence α) ranges within a compact set with positive lower and upper bounds, namely

$$0 < \mu^{\rm L} \le \mu \le \mu^{\rm U} \; .$$

Moreover, as far as the initial state $w_{\rm a}(0)$ of the exosystem is concerned, we assume that it satisfies

$$w_{\rm a}(0) \in \mathcal{W} := \{ w_{\rm a} \in \mathbb{R}^{r+1} : \frac{\|Q_{\rm a} w_{\rm a}\|_\infty}{\mu^{\rm L}} < g - \beta \} \tag{7}$$

for some positive $\beta < g$. Clearly this constraint is due to the fact that the maximum amplitude of the disturbance must be lower than the maximum control value which, in the worst case, is equal to $\mu^{\mathrm{L}} g$.

3. STATE FEEDBACK DESIGN

Following the *nonlinear output regulation* theory ((Byrnes *et al.*, 1997*b*)), the regulator which solves the problem is given by an *internal model*, needed to generate the control input which offsets the disturbance, and a stabilizer which is devoted to stabilize the "extended system" given by (4) and the internal model.

The first step in the design of the internal model amounts in computing the solution of the so-called *regulator equations* (see (Byrnes *et al.*, 1997*b*)), namely the steady state control input $c(w_{\mathrm{a}}, \mu)$ and the associated steady state behaviour $\pi(w_a, \mu)$ compatible with the regulation objective. For system (4)-(5) it turns out that

$$\pi(w_{\mathrm{a}}, \mu) = (0, 0) \qquad c(w_{\mathrm{a}}, \mu) = -\frac{1}{\mu} Q_{\mathrm{a}} w_{\mathrm{a}} \ . \tag{8}$$

The internal model is then designed according to the procedure proposed in (Nikiforov, 1998). In particular call (Φ, Γ) the observable pair such that the following *immersion condition*

$$\begin{aligned} \frac{\partial \tau(w_{\mathrm{a}}, \mu)}{\partial w_{\mathrm{a}}} S_{\mathrm{a}} w_{\mathrm{a}} &= \Phi \tau(w_{\mathrm{a}}, \mu) \\ c(w_{\mathrm{a}}, \mu) &= \Gamma \tau(w_{\mathrm{a}}, \mu) \end{aligned}$$

is satisfied via a nonlinear map $\tau(w_{\mathrm{a}}, \mu)$. Moreover given any Hurwitz matrix F and any vector G such that (F, G) is controllable, denote by M the unique matrix solution of the Sylvester equation

$$M\Phi - FM = G\Gamma$$

and define $\Psi := \Gamma M^{-1}$. Simple computations show that, defining $\bar{\tau}(w_{\mathrm{a}}, \mu) := M\tau(w_{\mathrm{a}}, \mu)$,

$$\begin{aligned} \frac{\partial \bar{\tau}(w_{\mathrm{a}}, \mu)}{\partial w_{\mathrm{a}}} S_{\mathrm{a}} w_{\mathrm{a}} &= (F + G\Psi)\bar{\tau}(w_{\mathrm{a}}, \mu) \\ c(w_{\mathrm{a}}, \mu) &= \Psi \bar{\tau}(w_{\mathrm{a}}, \mu) \ . \end{aligned} \tag{9}$$

This suggests to choose as internal model-based regulator the system

$$\begin{aligned} \dot{\xi} &= (F + G\Psi)\xi + N(x_1, x_2, \xi) \\ v &= \mathrm{sat}_g\, (\Psi \xi + v_{\mathrm{st}}) \end{aligned} \tag{10}$$

where $\mathrm{sat}_g(s)$ is the classical saturation function defined as $\mathrm{sgn}(s) \min\{|s|, g\}$ introduced in order to explicitly fulfill the constraint $|v| \le g$. In (10) $N(\cdot, \cdot, \cdot)$ is a vector and v_{st} is a new control variable, both to be designed in order to asymptotically stabilize the "extended" system given by (4) and (10). To this end define the change of coordinates

$$\xi \ \to \ \chi := \xi - \bar{\tau}(\omega) - \frac{1}{\mu} G x_2 \tag{11}$$

and choose $N(x_1, x_2, \xi)$ as

$$N(x_1, x_2, \xi) = G\,(\, v_{\mathrm{st}} + \varphi_g(\Psi\xi + v_{\mathrm{st}})\,)$$

where

$$\varphi_g(s) := \mathrm{sat}_g(s) - s = \begin{cases} 0 & |s| \le g \\ g - s & \text{otherwise.} \end{cases}$$

Simple algebraic computations show that system (2),(4),(10) in the new coordinates reads as

$$\begin{aligned} \dot{e}_1 &= x_2 \\ \dot{x}_2 &= \mu\, \mathrm{sat}_g \left(v_{\mathrm{st}} + \frac{1}{\mu}\Psi G x_2 + \Psi\chi + \Psi\bar{\tau}(w_{\mathrm{a}}, \mu) \right) + \\ &\quad -\mu\, \Psi \bar{\tau}(w_{\mathrm{a}}, \mu) \\ \dot{\chi} &= F\chi + \frac{1}{\mu} F G x_2 \ . \end{aligned} \tag{12}$$

Now note that, due to (8), (9) and to assumption (7), the origin $(e_1, x_2, \chi) = (0, 0, 0)$ of (12) is an equilibrium point in case $v_{\mathrm{st}} = 0$. Hence the original problem is re-formulated as that of globally asymptotically stabilizing the origin of (12) by suitably designing v_{st}.

To begin with note that, by virtue of assumption (7), it is possible to rewrite the derivative of x_2 as (here $\theta = v_{\mathrm{st}} + \frac{\Psi G x_2}{\mu} + \Psi\chi$)

$$\begin{aligned} \dot{x}_2 &= \mu\, \mathrm{sat}_g(\theta + \Psi\bar{\tau}(w_{\mathrm{a}}(t), \mu)) - \mu\, \bar{\tau}(w_{\mathrm{a}}(t), \mu) \\ &= \phi(t, \theta)\, \mathrm{sat}_g(\theta) \end{aligned}$$

where

$$\phi(t, \theta) := \mu\ (\ g - \Psi\, \bar{\tau}(w_{\mathrm{a}}(t), \mu)\, \mathrm{sgn}(\theta)\ ) \ .$$

In particular it is easy to show that the function $\phi(t, \theta)$ has positive lower and upper bounds independent of θ, namely

$$0 < \mu^{\mathrm{L}}\, \beta \le \phi(t, \theta) \le \mu^{\mathrm{U}}\, (2g - \beta) \ .$$

In view of this we drop the dependence on θ considering $\phi(t)$ as an uncertain time varying input gain and we study the uncertain system

$$\begin{aligned} \dot{e}_1 &= x_2 \\ \dot{x}_2 &= \phi(t)\, \mathrm{sat}_g \left(v_{\mathrm{st}} + \frac{1}{\mu}\Psi G x_2 + \Psi\chi \right) \\ \dot{\chi} &= F\chi + \frac{1}{\mu} F G x_2 \ . \end{aligned} \tag{13}$$

For such a system the following claim will be proved.

Proposition 1. There exists $\lambda^\star > 0$, $k^\star > 0$ and c_1, c_2 such that for all $\lambda \le \lambda^\star$, $k \ge k^\star$ the control law

$$v_{\mathrm{st}} = -k x_2 - \lambda \mathrm{sat}_1 \left(\frac{c_1 e_1 + c_2 x_2}{\lambda} \right)$$

globally asymptotically stabilizes the origin of (13).

The proof (for more details see (Gentili and Marconi, 2001)) is based on the small gain theorem for saturated interconnected systems (see (Teel, 1996)).

4. ERROR FEEDBACK DESIGN

The goal of this section is the design of regulator without explicit knowledge of the ball velocity. In par-

ticular we re-formulate the specific problem of disturbance suppression for the magnetic levitation system (4)-(5) in more general terms. The aim is to obtain a general result in the context of output regulation of linear systems with constant uncertain parameters affecting the inputs channel and with inputs constraint. In this sense the result here presented can be seen as a nontrivial extension of the results proposed in (Isidori and DeSantis, 2001) where uncertain parameters was not allowed. Clearly, by solving the general problem, we obtain also the solution for the motivating example specified in section 2.
In particular given the system

$$\begin{aligned} \dot{x} &= Ax + \mu B v + Q w \\ \dot{w} &= S w \\ e &= C x + P w \end{aligned}$$

with inputs $v \in \mathbb{R}^m$ and outputs $e \in \mathbb{R}^m$, given a set $W \in \mathbb{R}^r$ and a real number $g > 0$, we consider the problem of designing an error feedback regulator of the form

$$\dot{\xi} = \varphi(\xi, e) \qquad v = \gamma(\xi) \qquad |v(\cdot)| \leq g$$

such that for all $x(0) \in \mathbb{R}^n$ and for all constant μ such that $0 < \mu^{\mathrm{L}} \leq \mu \leq \mu^{\mathrm{U}}$,

(i) the closed loop system with $w = 0$ is asymptotically stable;

(ii) for all $w(0) \in W \subset \mathbb{R}^r$ the response of the closed loop system is bounded and $\lim_{t\to\infty} e(t) = 0$.

The first assumption which will be used to solve the problem is the existence of matrices Π and Γ solution of the *Francis* (or *regulator*) *equations*

$$\begin{aligned} \Pi S &= A\,\Pi + B\,\Gamma + Q \\ 0 &= C\,\Pi + P\ . \end{aligned} \tag{14}$$

In particular the function $\Gamma w(t)/\mu$ represents the control law which must be provided asymptotically by the controller in order to offset the exogenous disturbance $Qw(t)$ while keeping the error e identically zero. Hence, in this general context, assumption (7) specializes in requiring that the initial state of the exosystem satisfies

$$w(0) \in \mathcal{W} := \{w \in \mathbb{R}^r : \frac{\|\Gamma w(\cdot)\|_\infty}{\mu^{\mathrm{L}}} < g - \beta\}. \tag{15}$$

for some positive $\beta < g$.
Moreover, in order to reconstruct the value of the exogenous variable, we assume that the pair

$$C_{\mathrm{e}} := \begin{pmatrix} C & 0 \end{pmatrix} \qquad A_{\mathrm{e}} := \begin{pmatrix} A & -\mu B\Gamma \\ 0 & S \end{pmatrix}$$

is observable for any $\mu \in [\mu^{\mathrm{L}}, \mu^{\mathrm{U}}]$. Finally we assume that the pair (A, B) is stabilizable and the system is *null-controllable*, namely the spectrum of A is all contained in the closed left half plane, i.e. $\sigma(A) \subset \bar{\mathcal{C}}^-$.

In particular it is trivial to check that all these assumptions are fulfilled specializing the system (A, B, C) to he magnetic levitation plant presented in section 2.
Now call P_A the positive definite matrix such that

$$P_A A + A^T P_A \leq 0 \tag{16}$$

and consider the *observer-based* controller

$$\begin{aligned} \dot{\xi}_{\tilde{x}} &= A\xi_{\tilde{x}} + \mu^0\, B\, v - \mu^0 B\Gamma\xi_w + G_x(e - C\xi_{\tilde{x}}) \\ \dot{\xi}_w &= S\xi_w + G_w(e - C\xi_{\tilde{x}}) \\ v &= \mathrm{sat}_g(\Gamma\xi_w - \lambda\,\mathrm{sat}_1(\frac{B^T P_A \xi_{\tilde{x}}}{\lambda})) \end{aligned} \tag{17}$$

where G_x and G_w are output injection matrices to be specified, $\mathrm{sat}_g(\cdot)$ is the classical saturation function defined in the previous section and $\lambda \leq \|\delta\|_\infty/2$. The claim is that this controller, suitably tuned, solves the problem.

Proposition 2. There exists a $\gamma^\star > 0$ such that if the output injection matrix $G_{\mathrm{e}} = (G_x, G_w)$ is chosen so that all the eigenvalues of $A_{\mathrm{e}} - G_{\mathrm{e}} C_{\mathrm{e}}$ have real part lower than $-\gamma^\star$, i.e.

$$\mathrm{Re}\,(\sigma_i(A_{\mathrm{e}} - G_{\mathrm{e}} C_{\mathrm{e}})) < -\gamma^\star \qquad \begin{aligned} i &= 1, \ldots, n+r \\ \forall\, \mu &\in [\mu^{\mathrm{L}}, \mu^{\mathrm{U}}]\ , \end{aligned} \tag{18}$$

then the controller (17) solves the problem in question for any $(x(0), w(0)) \in \mathbb{R}^n \times \mathcal{W}$ and for any $(\xi_{\tilde{x}}(0), \xi_w(0)) \in \mathbb{R}^n \times \mathbb{R}^r$.

Before presenting a sketch of the proof of this claim we provide a result which turns out to be instrumental in the following. This result refers to the linear uncertain system

$$\left\{ \begin{aligned} \dot{\tilde{\xi}}_{\tilde{x}} &= (A - G_x C)\tilde{\xi}_{\tilde{x}} - \mu B\Gamma\tilde{\xi}_w + q \\ \dot{\tilde{\xi}}_w &= S\tilde{\xi}_w - G_w C\tilde{\xi}_{\tilde{x}} \end{aligned} \right. \tag{19}$$

with input q and uncertain parameter μ. The result states that it is possible to choose the output injection matrix G_{e} in order to impose an arbitrary input-to-state linear gain, robustly with respect to the uncertain parameter μ.

Proposition 3. Consider system (19) with μ ranging within a compact set. Then for all $\gamma > 0$ there exists a $\gamma^\star > 0$ such that if G_{e} is chosen so that (18) is fulfilled, then (19) is ISS with no restriction on the input q and on the initial state and linear gain γ, namely the following asymptotic estimate holds [2]

$$\|(\tilde{\xi}_{\tilde{x}}(\cdot), \tilde{\xi}_w(\cdot))\|_a \leq \gamma \|y_x(\cdot)\|_a\ .$$

In particular for all $\epsilon > 0$ there exists a $T^\star > 0$ such that

$$\|(\tilde{\xi}_{\tilde{x}}(t), \tilde{\xi}_w(t))\| \leq (\gamma + \epsilon)\|y_x(t)\| \qquad \text{for all } t \geq T^\star\ .$$

[2] Here $\|s(\cdot)\|_a := \lim_{t\to\infty} \sup \|s(t)\|$

Note that for $\mu = \mu^0$ the result is trivial due to the observability assumption of the pair $(C_\mathrm{e}, A_\mathrm{e})$. For uncertain values of μ the result can be easily proved for instance by small gain arguments.

Considering now the change of coordinates

$$x \quad \to \quad \tilde{x} := x - \Pi w \tag{20}$$

and

$$\begin{array}{lcl} \xi_{\tilde{x}} & \to & \tilde{\xi}_{\tilde{x}} = \xi_{\tilde{x}} - \tilde{x} \\ \xi_w & \to & \tilde{\xi}_w = \xi_w - \dfrac{w}{\mu} \ , \end{array} \tag{21}$$

the overall system in the new coordinates reads as (recall equations (14))

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + \mu B v - \mu^0 B\Gamma w \\ \dot{\tilde{\xi}}_{\tilde{x}} &= (A - G_x C)\tilde{\xi}_{\tilde{x}} - B\Gamma\tilde{\xi}_w + (\mu^0 - \mu)Bv + \\ & \qquad + \frac{\mu - \mu^0}{\mu} B\Gamma w \\ \dot{\tilde{\xi}}_w &= S\tilde{\xi}_w - G_w \tilde{\xi}_{\tilde{x}} \ . \end{aligned} \tag{22}$$

The proof of the result is then divided in two parts. In the first it is shown that there exists a choice of $G_\mathrm{e} = (G_x, G_w)$ and a time $T^\star > 0$ such that for the closed loop system the following holds

$$\Gamma\xi_w(t) - \lambda\, \mathrm{sat}_1(\frac{B^T P \xi_{\tilde{x}}}{\lambda}) \leq g \qquad \forall\, t \geq T^\star \tag{23}$$

namely the control law becomes lower than the saturation level in finite time. In the second part, due to proposition 1, it can be shown that the origin of the system (22) for $t \geq T^\star$ is globally asymptotically stable. This, in view of the definitions (20)-(21) and of the equations (14), proves the result (for more details see (Gentili and Marconi, 2001)).

5. SIMULATION RESULTS

In order to check the performances of both the controllers above designed, a certain number of tests have been made, simulating the response of the system subject to various periodic disturbances. The results of these tests are shown in fig. 1 and fig. 2. In particular the disturbance that has been simulated is a sinusoidal signal $\delta(t) = V\sin(\Omega\, t)$ with amplitude $V = 5\ \mathrm{m/s^2}$ and frequency $\Omega = 2\, rad/sec$ occurring at time $t = 10\ sec$. The parameter α is assumed to range within of $\pm\, 20\%$ of the nominal value $\alpha_0 = 6.612 \times 10^{-3}$ $Kg\, m^3/s^2\, A^2$; the internal model of both controllers was connected at time $t = 20$.
The state feedback controller was designed choosing, $k = 800$, $c1 = 1.252 \times 10^3$, $c2 = 900$ and $\lambda = 10^5$; the reference position was assumed $x_1^{\mathrm{des}} = -0.022\ m$ and the initial state of the system was taken as $x_1(0) = -0.03\ m$ and $x_2(0) = -0.1\, m/s$.
The error feedback controller was designed choosing $G_x = 10^3 * [0.072\ 2.043]^T$, $G_w = 10^5 * [-1.08\ 0.95564\ 1.365]^T, \lambda = V/3, B^T P = [100\ 100]$; the reference position was assumed still $x_1^{\mathrm{des}} = -0.022\ m$ and the initial state of the system was taken as $x_1(0) = -3.03\ m$ and $x_2(0) = -2\, m/s$.
Just note that the choice of two different initial states for these simulations was made just to point out that, due to the presence of a non saturated high gain state feedback, the state feedback stabilizer controller also without the internal model is able to reduce the effect of the disturbance but not to totally overcome it; on the other hand the error feedback saturated stabilizer controller without the internal model unit is in no way able to contrast the sinusoidal disturbance.

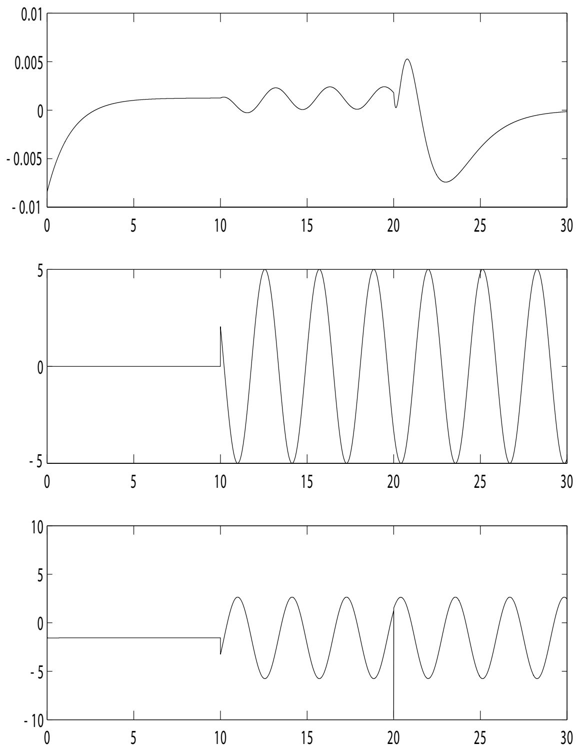

Fig. 1. state feedback controller: tracking error (e_1), disturbance ($\delta(t)$), control input (v)

6. CONCLUSIONS

In this paper the problem of disturbance suppression for a magnetic levitation system has been addressed. In the realistic case the model of the system is uncertain, we have proposed two design procedure yielding an internal model-based regulator that achieves tracking of the reference position while rejecting the disturbance for any initial state of the system and for any initial state of the exosystem in a certain set. In particular we presented a state feedback design procedure able to obtain the *lowest order* controller and an error feedback design procedure able to obtain an observer-based controller; the latter solves also a more general problem in the context of output regulation of linear systems with constant uncertain parameters affecting the inputs channel and with input constraints.

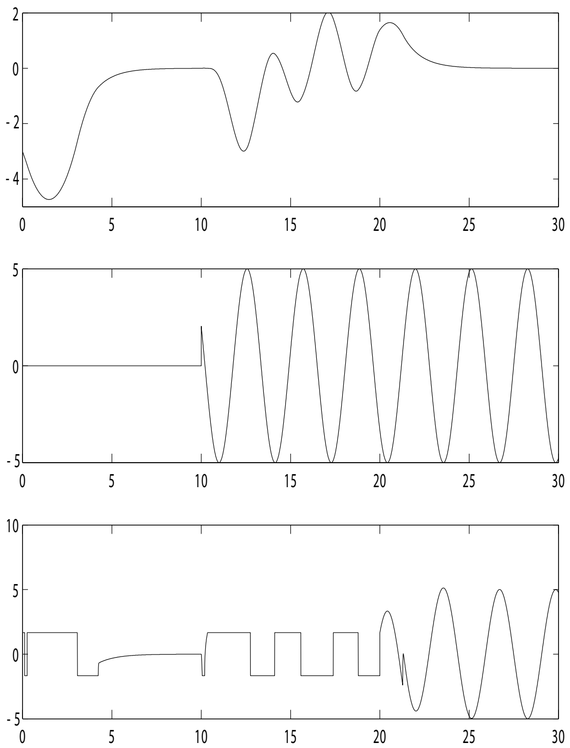

Fig. 2. error feedback controller: tracking error (e_1), disturbance ($\delta(t)$), control input (v)

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