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A NONLINEAR TRANSFORMATION APPROACH TO GLOBAL ADAPTIVE OUTPUT FEEDBACK CONTROL OF 3RD-ORDER UNCERTAIN NONLINEAR SYSTEMS

Choon-Ki Ahn, Beom-Soo Kim and Myo-Taeg Lim

School of Electrical Engineering, Korea University, Seoul, Korea 136-701
Email: {hironaka, soo, mlim}@elec.korea.ac.kr

Abstract: In this paper, we present a global adaptive output feedback control scheme for a class of 3rd-order uncertain nonlinear systems to which adaptive observer backstepping method may not be applicable directly. In contrast to the existing output feedback form, the allowed extended output feedback structure includes quadratic and multiplicative dependency of unmeasured states. Our novel design technique employs a change of coordinates and adaptive backstepping. With these proposed tools, we can remove linear and quadratic dependence on the unmeasured states in the state equation. Also, the multiplication of the two unmeasured states can be eliminated. From the transformed systems, a state observer can be constructed in a very easy way. The overall scheme achieves globally exponential convergence of the tracking error to zero while maintaining global boundedness of all the signals and states.

Keywords: Output feedback, a change of coordinates, adaptive backstepping, unmeasured states, state observer

1. INTRODUCTION

Adaptive output feedback nonlinear control problems have been given a lot of attention in control community during the recent years. Detailed discussions in such a direction can be found in (Marino and Tomei, 1995; M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, 1995). Under the assumption of full-states measurement, adaptive backstepping scheme can achieve a global stabilization for a class of parametric strict-feedback systems (I. Kanellakopoulos, P.V. Kokotovic, and A. S. Morse, 1991; M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, 1992). Several authors have developed the design methods for a wider class of nonlinear systems under full-state feedback (Yao and Tomizuka, 1997; Yao, 1997; R.A. Freeman, M. Krstic, P. V. Kokotovic, 1998).

In case of only a single output measurement, the existing works show semiglobal results for a class of systems whose nonlinearities depend on the unmeasured variables (H.K.Khalil, 1996; Jankovic, 1997). If the nonlinearities depend on output measurement, current works can achieve global results only for a class of parametric output-feedback systems. With adaptive observer backstepping technique (M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, 1995), an adaptive output-feedback controller that guarantees asymptotic tracking of the reference signal y_r by the output while keeping all the signals bounded can be designed for a class of parametric output-feedback form. As the first step to extend a class of output-feedback nonlinear systems that can be globally stabilized, adaptive controller was constructed for a class of nonlinear systems where the unmeasured states are appearing linearly with regard to nonlinear functions (Freeman and Kokotovic, 1996). Under the assumption that the unmeasured states are generated by pre-stabilized subsystems, Freeman and Kokotovic presented a global stabilization result for a class of extended strict feedback systems. In (Y. Tan, I. Kanellakopoulos, and Z. Jiang, 1998), the assumption of the pre-stabilization of the subsystem represented by Freeman and Kokotovic was removed. More recently, with novel state estimation technique, a global adaptive output feedback controller was designed for a class of uncertain nonlinear systems in (Choon-Ki Ahn, Beom-Soo Kim, and Myo-Taeg Lim, 2001).

In this paper, we consider a class of output feedback nonlinear systems which is a more extended output feedback structure than a class of nonlinear system studied in our previous work (Choon-Ki Ahn, Beom-Soo Kim, and Myo-Taeg Lim, 2001) as one of the recent efforts to extend a class of output-feedback nonlinear systems which can be controlled to guarantee a global stabilization. In contrast to the existing output feedback structure, our extended class of nonlinear systems has quadratic and multiplicative dependency of the unmeasured states. Since a class of introduced systems is not the parametric output-feedback form, the well-known adaptive observer backstepping technique (M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, 1995) can not be applicable directly. The major difficulty is that the system equations depend quadratically and multiplicatively on the unmeasured states. In addition, our extended class allows output-dependent nonlinearities to appear not only additively, but also multiplicatively. However, for this extended structure, we can construct a global adaptive output-feedback tracking controller based on a change of coordinates and adaptive backstepping methodology. By a change of coordinates, the quadratic and multiplicative dependency of the unmeasured states in the state equations can be eliminated from the transformed systems. In addition, with the introduction of ϖ function, we can remove the effect of parametric uncertainties in observer design. Consequently, the proposed novel state estimation technique and the adaptive backstepping scheme achieve a global exponential tracking of the output to the given reference signal while maintaining the global boundedness of all the signals.

The class of nonlinear systems considered in this paper is described in Section 2. In Section 3, state estimation technique based on a change of coordinates is proposed. With this estimation technique, adaptive backstepping scheme is presented in Section 4. The conclusion is given in Section 5.

2. PROBLEM FORMULATION

The class of nonlinear systems to be controlled in this paper is the following three dimensional output-feedback form:

$$\begin{aligned}
\dot{x}_1 &= \phi_1(x_1)x_2 \\
\dot{x}_2 &= \phi_2(x_1)x_3 + \chi_2(x_1) + \theta^T\varphi_2(x_1) + \Phi_2(x_1)x_2 + \psi_2(x_1)x_2^2 \\
\dot{x}_3 &= \phi_3(x_1)u + \chi_3(x_1) + \theta^T\varphi_3(x_1) + \Phi_3(x_1)x_2 + \psi_3(x_1)x_2x_3 \\
y &= x_1
\end{aligned} \tag{1}$$

where $u \in R$, and $y \in R$ are the control input and the output, respectively and x_1 is the measured state while x_2, x_3 represent the unmeasured states. $\varphi_2 \in R^p$, $\varphi_3 \in R^p$ are vectors of known smooth functions. Also, ϕ_1, ϕ_2, ϕ_3, χ_2, χ_3, Φ_2, Φ_3, ψ_2, ψ_3 are known smooth functions. $\theta \in R^p$ is a vector of unknown constant parameters. $\phi_i(x_1) \neq 0$, $1 \leq i \leq 3$, for all $x_1 \in R$. Throughout this paper, we assume that the reference signal y_r and the derivatives of y_r up to the 3rd-order are bounded and piecewise continuous.

The control objective in this paper is to construct an adaptive output feedback nonlinear control law so that the output y tracks a given reference signal y_r while maintaining global boundedness of all the signals.

Remark 1. As seen in (1), it is difficult to design state observer directly since the unmeasured states appear multiplicatively in the output-dependent nonlinear functions. In addition, the major difficulty is that the system equations depend quadratically and multiplicatively on the unmeasured state. However, with the introduction of a change of coordinates, the quadratic and multiplicative dependency of the unmeasured states can be removed from the transformed systems. Therefore, after transformation, we can design state observer very easily from the transformed systems.

Remark 2. In this paper, we consider only the 3rd-order extended output feedback structure. Therefore, it is a further research topic to generalize our result to the n-dimensional extended output feedback form. In addition, it is a topic of another research to determine coordinate-free geometric conditions that are necessary and sufficient for the existence of a global state-space diffeomorphism that transforms uncertain input-affine nonlinear system into the n-dimensional extended output feedback structure.

3. STATE ESTIMATION TECHNIQUE

In this section, we propose state estimation technique based on a change of coordinates. With the introduction of this tool, the quadratic and multiplicative dependency on the unmeasured states can be eliminated from the transformed system. First of all, we introduce the following transformation

$$\xi_2 = x_2 + \theta^T \varpi_2 - l_2(x_1)x_2 - w_2(x_1) \quad (2)$$
$$\xi_3 = x_3 + \theta^T \varpi_3 - l_3(x_1)x_3 - w_3(x_1) \quad (3)$$

where ϖ_2, l_2, l_3, w_2, ϖ_3 and w_3 are smooth design functions of the measured state x_1. $l_2(x_1) \neq 1$ and $l_3(x_1) \neq 1$.

From

$$x_2 = \frac{\xi_2 - \theta^T \varpi_2 + w_2}{1 - l_2} \quad (4)$$
$$x_3 = \frac{\xi_3 - \theta^T \varpi_3 + w_3}{1 - l_3} \quad (5)$$

the time derivative of ξ_2 is given by

$$\begin{aligned}\dot{\xi}_2 =& x_2^2 \left[\psi_2 - \frac{\partial l_2}{\partial x_1}\phi_1 - l_2\psi_2\right] + \theta^T \left[\varphi_2 - l_2\varphi_2 + \dot{\varpi}_2 \right. \\ & \left. - \varpi_3 \frac{1-l_2}{1-l_3}\phi_2 - \frac{\varpi_2}{1-l_2}\left(\Phi_2 - l_2\Phi_2 - \frac{\partial w_2}{\partial x_1}\phi_1\right)\right] \\ & + \frac{1-l_2}{1-l_3}(\xi_3 + w_3)\phi_2 + \left[\frac{\Phi_2 - l_2\Phi_2 - \frac{\partial w_2}{\partial x_1}\phi_1}{1-l_2}\right] \\ & (\xi_2 + w_2) + \chi_2(1 - l_2) \end{aligned} \quad (6)$$

If we choose l_2, ϖ_2, ϖ_3 and w_2 functions to satisfy the following equalities

$$\psi_2 - \frac{\partial l_2}{\partial x_1}\phi_1 - l_2\psi_2 = 0 \quad (7)$$
$$\begin{aligned}&\varphi_2 - l_2\varphi_2 + \dot{\varpi}_2 - \varpi_3\phi_2\frac{1-l_2}{1-l_3} \\ &\quad - \frac{\varpi_2}{1-l_2}\left(\Phi_2 - l_2\Phi_2 - \frac{\partial w_2}{\partial x_1}\phi_1\right) = 0\end{aligned} \quad (8)$$
$$\frac{\Phi_2 - l_2\Phi_2 - \frac{\partial w_2}{\partial x_1}}{1-l_2} = -k_2^2(1-l_2)^2\phi_1^2 \quad (9)$$

where k_2 is a positive design constant, then (6) becomes

$$\begin{aligned}\dot{\xi}_2 =& -k_2^2(1-l_2)^2\phi_1^2(\xi_2 + w_2) + \frac{1-l_2}{1-l_3}\phi_2(\xi_3 + w_3) \\ & + \chi_2(1-l_2)\end{aligned} \quad (10)$$

It should be noted that, by the introduction of l_2 function, we can remove the quadratic dependency of unmeasured state x_2 from ξ system. Also, it is interesting to note that, with the introduction of ϖ function, the effect of parametric uncertainties was eliminated from the transformed system in observer design.

Therefore, we introduce the following observer

$$\begin{aligned}\dot{\hat{\xi}}_2 =& -k_2^2(1-l_2)^2\phi_1^2(\hat{\xi}_2 + w_2) + \frac{1-l_2}{1-l_3}\phi_2(\hat{\xi}_3 + w_3) \\ & + \chi_2(1-l_2)\end{aligned} \quad (11)$$

Denoting the state estimation error by $\tilde{\xi}_2 = \hat{\xi}_2 - \xi_2$ and $\tilde{\xi}_3 = \hat{\xi}_3 - \xi_3$, we can obtain the error equation

$$\dot{\tilde{\xi}}_2 = -k_2^2(1-l_2)^2\phi_1^2\tilde{\xi}_2 + \frac{1-l_2}{1-l_3}\phi_2\tilde{\xi}_3 \quad (12)$$

By the same method, the time derivative of ξ_3 is given by

$$\begin{aligned}\dot{\xi}_3 =& \phi_3[1-l_3]u + \theta^T[\varphi_3 + \dot{\varpi}_3 - l_3\phi_3] + \left[\psi_3 - l_3\psi_3 \right. \\ & \left. - \frac{\partial l_3}{\partial x_1}\phi_1\right] x_2 x_3 + \left[\Phi_3 - \frac{\partial w_3}{\partial x_1}\phi_1 - l_3\Phi_3\right] \\ & \frac{\xi_2 - \theta^T\varpi_2 + w_2}{1-l_2} + \chi_3[1-l_3]\end{aligned} \quad (13)$$

If we select w_3 function, ϖ function and l_3 function to satisfy the following

$$\Phi_3 - \frac{\partial w_3}{\partial x_1}\phi_1 - l_3\Phi_3 = -k_3^2\frac{(1-l_2)^2}{1-l_3}\phi_2 \quad (14)$$
$$\varphi_3 + \dot{\varpi}_3 - l_3\phi_3 + k_3^2\varpi_2\frac{1-l_2}{1-l_3}\phi_2 = 0 \quad (15)$$
$$\psi_3 - l_3\psi_3 - \frac{\partial l_3}{\partial x_1}\phi_1 = 0 \quad (16)$$

where k_3 is a positive design constant, then we have

$$\dot{\xi}_3 = \phi_3[1-l_3]u - k_3^2\frac{1-l_2}{1-l_3}(\xi_2 + w_2)\phi_2 + \chi_3[1-l_3] \quad (17)$$

It is noted that, with the introduction of l_3 function, the mutiplicative dependency of the unmeasured states was eliminated from ξ_3-system (17).

Therefore, we introduce the following observer

$$\dot{\hat{\xi}}_3 = \phi_3[1-l_3]u - k_3^2\frac{1-l_2}{1-l_3}(\hat{\xi}_2 + w_2)\phi_2 + \chi_3[1-l_3] \quad (18)$$

From the definition of the state estimation error, we have

$$\dot{\tilde{\xi}}_3 = -k_3^2\frac{1-l_2}{1-l_3}\tilde{\xi}_2\phi_2 \quad (19)$$

With the error equations (12) and (19), consider the following observer Lyapunov function candidate

$$V_o = \frac{k_3^2}{2}\tilde{\xi}_2^2 + \frac{1}{2}\tilde{\xi}_3^2 \quad (20)$$

The time derivative of V_o is given by

$$\begin{aligned}\dot{V}_o =& k_3^2\tilde{\xi}_2\dot{\tilde{\xi}}_2 + \tilde{\xi}_3\dot{\tilde{\xi}}_3 \\ =& k_3^2\tilde{\xi}_2\left[\frac{1-l_2}{1-l_3}\phi_2\tilde{\xi}_3 - k_2^2(1-l_2)^2\phi_1^2\tilde{\xi}_2\right] \\ & + \tilde{\xi}_3[-k_3^2\frac{1-l_2}{1-l_3}\phi_2\tilde{\xi}_2] \\ =& -k_2^2k_3^2(1-l_2)^2\phi_1^2\tilde{\xi}_2^2 \leq 0\end{aligned} \quad (21)$$

Therefore, we can conclude that the estimation errors $\tilde{\xi}_2$, $\tilde{\xi}_3$ converge to zero.

Remark 3. From (7) and (16), a nonlinear design functions l_2 and l_3 can be constructed by solving the following linear differential equations

$$\frac{\partial l_2}{\partial x_1}\phi_1 + l_2\psi_2 = \psi_2 \quad (22)$$
$$\frac{\partial l_3}{\partial x_1}\phi_1 + l_3\psi_3 = \psi_3 \quad (23)$$

Refer to the basic differential equation text (W.E. Boyce and R. C. Diprima, 1997) for the general solution of these linear differential equations.

Remark 4. From (9) and (14), nonlinear design functions w_2 and w_3 can be explicitly obtained as follows:

$$w_2(y) = \int \frac{1}{\phi_1}[\Phi_2(y)(1-l_2(y)) + k_2^2(1-l_2(y))^3\phi_1^2]dy \quad (24)$$

$$w_3(y) = \int \frac{1}{\phi_1}\left[\Phi_3(y)(1-l_3(y)) + k_3^2\frac{(1-l_2(y))^2}{1-l_3(y)}\phi_2\right]dy \tag{25}$$

Remark 5. From (8), (15) and (9), functions ϖ_2 and ϖ_3 are the outputs of the following filters

$$\dot{\varpi}_2 = -k_2^2(1-l_2)^2\varpi_2\phi_1^2 + \phi_2\frac{1-l_2}{1-l_3}\varpi_3 - \varphi_2(1-l_2) \tag{26}$$

$$\dot{\varpi}_3 = -k_3^2\frac{1-l_2}{1-l_3}\phi_2\varpi_2 - \varphi_3(1-l_3) \tag{27}$$

4. ADAPTIVE CONTROLLER DESIGN

In this section, with the state estimation technique proposed in the previous section, we employ adaptive backstepping method and design a nonlinear controller that guarantees exponential tracking and the boundedness of all the signals

4.1 Step 1

Let $z_1 = y - y_r$, $z_2 = \hat{\xi}_2 - \alpha_1$. With these notations, we can obtain the time derivative of z_1 as

$$\dot{z}_1 = \phi_1\frac{z_2 + \alpha_1 - \theta^T\varpi_2 + w_2 - \tilde{\xi}_2}{1-l_2} - \dot{y}_r \tag{28}$$

If we select α_1 function and the tuning function as

$$\begin{aligned}\alpha_1 =& \frac{1}{\phi_1}\Big[-c_1(1-l_2)z_1 - w_2 + \hat{\theta}^T\varpi_1 + (1-l_2)\dot{y}_r \\ &- \frac{3z_1}{4k_2^2k_3^2(1-l_2)^3}\Big]\end{aligned} \tag{29}$$

$$\tau_1 = -\Gamma z_1\left(\frac{\varpi_2}{1-l_2}\right)\phi_1 \tag{30}$$

where c_1 is a positive constant for design, then it can be shown that the time derivative of the Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{31}$$

satisfies

$$\begin{aligned}\dot{V}_1 =& -c_1z_1^2 + \frac{z_1z_2}{1-l_2}\phi_1 + \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_1) - z_1\phi_1\frac{\tilde{\xi}_2}{1-l_2} \\ &- \frac{3z_1^2}{4k_2^2k_3^2(1-l_2)^4}\end{aligned} \tag{32}$$

where Γ is the positive definite matrix for design, $\tilde{\theta} = \hat{\theta} - \theta$, and z_1 subsystem is

$$\begin{aligned}\dot{z}_1 =& -c_1z_1 + \frac{\phi_1z_2}{1-l_2} + \phi_1\tilde{\theta}^T\left(\frac{\varpi_2}{1-l_2}\right) - \frac{\phi_1\tilde{\xi}_2}{1-l_2} \\ &- \frac{3z_1}{4k_2^2k_3^2(1-l_2)^4}\end{aligned} \tag{33}$$

4.2 Step 2

Let $z_3 = \hat{\xi}_3 - \alpha_2$. From the defined notations and (11), we can obtain the time derivative of z_2 as

$$\begin{aligned}\dot{z}_2 =& -k_2^2(1-l_2)\hat{\xi}_2 + (1-l_2)(z_3+\alpha_2)\phi_2 + w_3(1-l_2)\phi_2 \\ &- k_2^2(1-l_2)w_2 - \dot{\alpha}_1\end{aligned} \tag{34}$$

where

$$\begin{aligned}\dot{\alpha}_1 =& \frac{\partial\alpha_1}{\partial y}\frac{\hat{\xi}_2 + w_2}{1-l_2}\phi_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_1}{\partial\varpi_2}\dot{\varpi}_2 \\ &- \theta^T\left(\frac{\partial\alpha_1}{\partial y}\phi_1\frac{\varpi_2}{1-l_2}\right) - \frac{\partial\alpha_1}{\partial y}\frac{\tilde{\xi}_2}{1-l_2}\phi_1\end{aligned} \tag{35}$$

If we select α_2 function and the tuning function as

$$\begin{aligned}\alpha_2 =& \frac{1}{\phi_2}\Big[-c_2\frac{z_2}{1-l_2} - \frac{z_1\phi_1}{(1-l_2)^2} + k_2^2(\hat{\xi}_2 + w_2) - w_3\phi_2 \\ &- \frac{3z_2}{4k_2^2k_3^2(1-l_2)^5}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \frac{1}{1-l_2}\left(\frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2\right) \\ &+ \frac{1}{1-l_2}\Big[\frac{\partial\alpha_1}{\partial y}\frac{\hat{\xi}_2 + w_2}{1-l_2}\phi_1 + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_1}{\partial\varpi_2}\dot{\varpi}_2 \\ &- \hat{\theta}^T\left(\frac{\partial\alpha_1}{\partial y}\frac{\varpi_2}{1-l_2}\phi_1\right)\Big]\Big]\end{aligned}$$

$$\tau_2 = \tau_1 + \Gamma z_2\frac{\varpi_2}{1-l_2}\left(\frac{\partial\alpha_1}{\partial y}\right)\phi_1 \tag{36}$$

where c_2 is a positive design constant then, it can be shown that the time derivative of the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{37}$$

satisfies

$$\begin{aligned}\dot{V}_2 =& -\sum_{i=1}^{2}c_iz_i^2 + (1-l_2)z_2z_3\phi_2 + \sum_{i=1}^{2}\frac{z_i}{1-l_2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)\tilde{\xi}_2\phi_1 \\ &- \sum_{i=1}^{2}\frac{3}{4k_2^2k_3^2}\frac{z_i^2}{(1-l_2)^4}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 \\ &+ (\dot{\hat{\theta}} - \tau_2)\left[\tilde{\theta}^T\Gamma^{-1} - z_2\left(\frac{\partial\alpha_1}{\partial\hat{\theta}}\right)\right]\end{aligned} \tag{38}$$

where for notational convenience we have introduced $\frac{\partial\alpha_0}{\partial y} \triangleq -1$ and z_2 subsystem is

$$\begin{aligned}\dot{z}_2 =& \phi_2(1-l_2)z_3 - c_2z_2 - \frac{z_1}{1-l_2}\phi_1 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \tilde{\theta}^T\left(\frac{\partial\alpha_1}{\partial y}\frac{\varpi_2}{1-l_2}\phi_1\right) \\ &+ \frac{\partial\alpha_1}{\partial y}\frac{\tilde{\xi}_2}{1-l_2}\phi_1 - \frac{3z_2}{4k_2^2k_3^2(1-l_2)^4}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2\end{aligned} \tag{39}$$

4.3 Step 3

From the above defined notations and (18), we can obtain the time derivative of z_3 as

$$\dot{z}_3 = \phi_3u - k_3^2(1-l_2)\phi_2\hat{\xi}_2 - k_3^2(1-l_2)w_2\phi_2 - \dot{\alpha}_2 \tag{40}$$

where

$$\begin{aligned}\dot{\alpha}_2 =& \frac{\partial\alpha_2}{\partial y}\frac{\hat{\xi}_2 + w_2}{1-l_2}\phi_1 + \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_2}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_2}{\partial\ddot{y}_r}y_r^{(3)} \\ &+ \frac{\partial\alpha_2}{\partial\hat{\xi}_2}\dot{\hat{\xi}}_2 + \frac{\partial\alpha_2}{\partial\varpi_2}\dot{\varpi}_2 + \frac{\partial\alpha_2}{\partial\varpi_3}\dot{\varpi}_3 - \theta^T\left(\frac{\partial\alpha_2}{\partial y}\frac{\varpi_2}{1-l_2}\phi_1\right) \\ &- \frac{\partial\alpha_2}{\partial y}\frac{\tilde{\xi}_2}{1-l_2}\phi_1\end{aligned} \tag{41}$$

If we select the control input u as

$$\begin{aligned}u =& \frac{1}{\phi_3}\Big[-c_3z_3 - (1-l_2)\phi_2z_2 + k_3^2(1-l_2)(\hat{\xi}_2 + w_2)\phi_2 + \frac{\partial\alpha_2}{\partial\hat{\theta}}\tau_3 \\ &- \frac{3z_3}{4k_2^2k_3^2(1-l_2)^4}\left(\frac{\partial\alpha_2}{\partial y}\right)^2 + z_2\left(\frac{\partial\alpha_1}{\partial\hat{\theta}}\right)\Gamma\left(\frac{\partial\alpha_2}{\partial y}\right)\frac{\varpi_2}{1-l_2}\phi_1 \\ &+ \frac{\partial\alpha_2}{\partial y}\frac{\hat{\xi}_2 + w_2}{1-l_2}\phi_1 + \frac{\partial\alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_2}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_2}{\partial\ddot{y}_r}y_r^{(3)} + \frac{\partial\alpha_2}{\partial\hat{\xi}_2}\dot{\hat{\xi}}_2\end{aligned}$$

$$+\frac{\partial\alpha_2}{\partial\varpi_2}\dot{\varpi}_2+\frac{\partial\alpha_2}{\partial\varpi_3}\dot{\varpi}_3-\hat{\theta}^T\left(\frac{\partial\alpha_2}{\partial y}\frac{\varpi_2}{1-l_2}\phi_1\right)\Big] \quad (42)$$

where c_3 is a positive design constant and the tuning function satisfies the following equation

$$\tau_3=\tau_2+\Gamma z_3\frac{\varpi_2}{1-l_2}\left(\frac{\partial\alpha_2}{\partial y}\right)\phi_1 \quad (43)$$

then, it can be shown that the time derivative of the Lyapunov function candidate

$$V_3=V_2+\frac{1}{2}z_3^2 \quad (44)$$

satisfies

$$\begin{aligned}\dot{V}_3=&-\sum_{i=1}^{3}c_iz_i^2+\sum_{i=1}^{3}\frac{z_i}{1-l_2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)\phi_1\tilde{\xi}_2\\&-\sum_{i=1}^{3}\frac{3}{4k_2^2k_3^2}\frac{z_i^2}{(1-l_2)^4}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\\&+(\dot{\hat{\theta}}-\tau_3)\left[\tilde{\theta}^T\Gamma^{-1}-\sum_{i=1}^{2}z_{i+1}\left(\frac{\partial\alpha_i}{\partial\hat{\theta}}\right)\right]\end{aligned} \quad (45)$$

where z_3 subsystem is

$$\begin{aligned}\dot{z}_3=&-(1-l_2)\phi_2z_2-c_3z_3-\frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}}-\tilde{\theta}^T\left(\frac{\partial\alpha_2}{\partial y}\frac{\varpi_2}{1-l_2}\right)\phi_1\\&+\frac{\partial\alpha_2}{\partial y}\frac{\tilde{\xi}_2}{1-l_2}\phi_1-\frac{3z_3}{4k_2^2k_3^2(1-l_2)^4}\left(\frac{\partial\alpha_2}{\partial y}\right)^2+\frac{\partial\alpha_2}{\partial\hat{\theta}}\tau_3\\&+z_2\left(\frac{\partial\alpha_1}{\partial\hat{\theta}}\right)\Gamma\left(\frac{\partial\alpha_2}{\partial y}\right)\frac{\varpi_2}{1-l_2}\phi_1\end{aligned} \quad (46)$$

In this case, selecting the parameter update law as

$$\dot{\hat{\theta}}=\tau_3 \quad (47)$$

yields

$$\begin{aligned}\dot{V}_3=&-\sum_{i=1}^{3}c_iz_i^2+\sum_{i=1}^{3}\frac{z_i}{1-l_2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)\phi_1\tilde{\xi}_2\\&-\sum_{i=1}^{3}\frac{3}{4k_2^2k_3^2}\frac{z_i^2}{(1-l_2)^4}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\end{aligned} \quad (48)$$

We are ready to state and prove the following theorem.

Theorem 1. *Consider the system (1). If we apply the control input (42), the parameter update law (47) and the design procedure in the previous subsections, then*
(1) all signals are globally bounded.
(2)

$$\lim_{t\to\infty}[y(t)-y_r(t)]=0 \quad (49)$$

Proof. Consider the following Lyapunov function candidate

$$V=V_3+V_o=V_3+\frac{k_3^2}{2}\tilde{\xi}_2^2+\frac{1}{2}\tilde{\xi}_3^2 \quad (50)$$

The time derivative of V can be calculated as

$$\begin{aligned}\dot{V}=&-\sum_{i=1}^{3}c_iz_1^2+\sum_{i=1}^{3}\frac{z_i}{1-l_2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)\tilde{\xi}_2\\&-\sum_{i=1}^{3}\frac{(\sqrt{3})^2}{4k_2^2k_3^2}\frac{z_i^2}{(1-l_2)^4}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2-\sum_{i=1}^{3}\frac{k_2^2k_3^2}{(\sqrt{3})^2}(1-l_2)^2\tilde{\xi}_2^2\\=&-\sum_{i=1}^{3}c_iz_1^2-\sum_{i=1}^{3}\left[\frac{\sqrt{3}}{2k_2k_3}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)\frac{z_i}{(1-l_2)^2}\right.\\&\left.-\frac{k_2k_3}{\sqrt{3}}(1-l_2)\tilde{\xi}_2\right]^2\\\le&-\sum_{i=1}^{3}c_iz_1^2\end{aligned} \quad (51)$$

From the above result, we know that z, $\tilde{\xi}$ and $\tilde{\theta}$ are globally uniformly bounded. Since y_r is bounded, from $z_1=y-y_r$, the output y remains globally bounded. Therefore, from (26) and (27), ϖ function is also bounded. With these informations of boundedness, we can conclude that α_1 function is also globally bounded. By $z_2=\hat{\xi}_2-\alpha_1$, the global boundedness of $\hat{\xi}_2$ holds. By the same method, we can obtain the global boundedness of α_2, u, $\hat{\xi}_3$, ξ_2, ξ_3 and the other signals. Thus, the property (1) of Theorem 1 follows from some observations. From (33), (39) and (46), we have $\dot{z}\in L_\infty$. Since $z\in L_\infty$, we can obtain $z,\dot{z}\in L_\infty$. From the definition of V and (51), we see that $z\in L_2$. According to Barbalat lemma (Marino and Tomei, 1995; M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, 1995), we can get the property (2) of Theorem 1. ■

5. CONCLUSION

In this paper, we presented a global adaptive output-feedback control scheme for a class of 3rd-order uncertain nonlinear systems. In contrast to the existing output feedback structure, our extended class of nonlinear systems has quadratic and multiplicative dependency of unmeasured states. As one of the recent efforts to extend a class of output-feedback nonlinear systems which can be controlled to guarantee a global stabilization, we constructed a global adaptive output-feedback controller based on a change of coordinates and adaptive backstepping. With the introduction of a change of coordinates, the quadratic and multiplicative dependency of the unmeasured states can be removed from the transformed systems. Therefore, we could design state observer very easily from the transformed systems. For a class of output-feedback systems to which adaptive observer backstepping technique can not be applicable directly, the proposed schemes achieve globally exponential convergence of the tracking error to zero while maintaining global boundedness of all the signals and states.

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