Copyright © 2002 IFAC
15th Triennial World Congress, Barcelona, Spain

COMPOSITE NONLINEAR FEEDBACK CONTROL: THEORY AND AN APPLICATION

Ben M. Chen *,[1] **Tong H. Lee** * **Kemao Peng** *
V. Venkataramanan *

* *Department of Electrical & Computer Engineering, The National University of Singapore, Singapore 117576*

Abstract: The composite nonlinear feedback control technique is developed for a class of linear systems with actuator saturation, which consists of a linear and a nonlinear feedback parts without any switching element. The linear part is to yield a quick response in face of the actuator limits for the desired input levels. The nonlinear part is to reduce the overshoot caused by the linear part as the system output approaches the target. It is shown that the technique is capable of beating the time-optimal control in asymptotic tracking situations and can be applied to design servo systems that deal with "point-and-shoot" fast targeting. *Copyright ©2002 IFAC*

Keywords: Nonlinear Control, Saturation Control, Control Algorithm, Servo Systems, Actuators.

1. INTRODUCTION

Every physical system has nonlinearities and very little can be done to overcome them. Many practical systems are sufficiently nonlinear so that important features of their performance may be completely overlooked if they are analyzed and designed through linear techniques. In hard disk drive (HDD) servo systems, major nonlinearities are friction, high frequency mechanical resonance and actuator saturation. Among these, the actuator saturation could be the most significant nonlinearity in designing an HDD servo system, which deteriorates the system performance seriously.

Traditionally the time optimal control (TOC) is taken to deal with "point-and-shoot" fast-targeting for systems with actuator saturation, which uses maximal acceleration for a predetermined time period. Unfortunately, the TOC is not robust to the system uncertainties and measurement noises and is hardly used in any real situation. Workman (1987) proposed a modification of the TOC, the proximate time-optimal servomechanism (PTOS), to overcome such a drawback. The PTOS essentially uses maximal acceleration. When the error is small, it switches to a linear control law. It is fairly robust to system uncertainties and noises with a discounted tracking time. The TOC is indeed time-optimal for a point-to-point target tracking. However, in most practical situations, it is more appropriate to consider asymptotic tracking instead, *i.e.*, to track the system within a certain neighborhood of the target reference. It will be shown later that the TOC is not time-optimal at all in the asymptotic tracking situation. This is the motivation to search for a better technique. Inspired by a recent work of Lin *et al.*(1998), which was to improve the tracking performance under state feedback laws for a class of second order systems subject to actuator saturation, a nonlinear control technique has been developed in this paper, the so-called composite nonlinear feedback (CNF) control technique, to a more general class of systems with measurement feedback. The technique can be utilized to design

[1] Corresponding author: Ben M. Chen, email: bmchen@nus.edu.sg; Tel: (+65)-874-2289; Fax: (+65)-779-1103.

servo systems that deal with asymptotic target tracking or "point-and-shoot" fast targeting and will be applied to design a servo system for an actual hard disk drive in this paper.

This paper is organized as follows. In Section 2, CNF control technique will be developed. In Section 3, an example will show that the CNF could yield a better performance than the TOC. The application of the CNF technique to an actual HDD servo system will be presented in Section 4. Finally, some concluding remarks and open problems are drawn in Section 5.

2. CNF CONTROL TECHNIQUE

Consider a linear system with an amplitude constrained actuator, characterized by

$$\Sigma : \begin{cases} \dot{x} = A\ x + B\ \mathrm{sat}(u), \ \ x(0) = x_0 \\ y = C_1\ x \\ h = C_2\ x \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^l$, $y \in \mathbb{R}^p$ and $h \in \mathbb{R}$ are respectively the state, control input, measurement output and controlled output of the Σ. A, B, C_1 and C_2 are appropriate dimensional constant matrices, and sat: $\mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\mathrm{sat}(u) = \mathrm{sgn}(u) \min\{\ u_{\max},\ |\ u\ |\ \}, \tag{2}$$

with $u_{\max}$ being the saturation level of the input. The assumptions on the system matrices are required: 1) (A, B) is stabilizable, 2) (A, C_1) is detectable, and 3) (A, B, C_2) is invertible and has no invariant zeros at $s = 0$. The objective is to design a CNF control law that causes the output to track a desired amplitude step input rapidly without experiencing large overshoot and without the adverse actuator saturation effects. This will be done through the design of a linear feedback law with a small closed-loop damping ratio and a nonlinear feedback law through an appropriate Lyapunov function to cause the closed-loop system to be highly damped as system output approaches the command input to reduce the overshoot. The CNF control law will be developed in three distinct cases 1) the state feedback case, 2) the full order measurement feedback case, and 3) the reduced order measurement feedback case.

2.1 *State Feedback Case*

A CNF control technique is developed for the case when all the states of the plant Σ are measurable, i.e., $y = x$. It will be done in three steps. In the first step, a linear feedback control law will be designed and in the second step, the design of nonlinear feedback control will be carried out. Lastly, the linear and nonlinear feedback laws will be combined to give a CNF control law. It is noted that the procedure for this case follows closely to that reported in (Lin *et al.*, 1998), although the result in the section is applicable to a much larger class of systems. Due to the space limitation all proofs will appear elsewhere.

STEP 1: Design a linear feedback law,

$$u_L = Fx + G\,r, \tag{3}$$

where F is chosen such that 1) $A + BF$ is an asymptotically stable matrix, and 2) the closed-loop system $C_2(sI - A - BF)^{-1}B$ has a small damping ratio. Such an F can be designed using any appropriate method (Chen, 2000; Liu, *et al.*, 2001). $G = -\left[C_2(A + BF)^{-1}B\right]^{-1}$ is a scalar, which is well-defined because $A + BF$ is stable, and the triple (A, B, C_2) is invertible and has no invariant zeros at $s = 0$. r is a step reference input.

Lemma 2.1. Given a positive definite matrix $W \in \mathbb{R}^{n\times n}$, let $P > 0$ be the solution of the following Lyapunov equation

$$(A + BF)'P + P(A + BF) = -W. \tag{4}$$

Such a P exists since $A + BF$ is asymptotically stable. For any $\delta \in (0, 1)$, let $c_\delta > 0$ be the largest positive scalar satisfying the following condition:

$$|Fx| \le u_{\max}(1-\delta)\ \ \forall x \in \boldsymbol{X}_\delta := \{x : x'Px \le c_\delta\}. \tag{5}$$

Then, the control law (3) is capable of driving the controlled output, h, to track asymptotically a step command input r, provided that the initial state, x_0 and r satisfy:

$$x_0 - x_e \in \boldsymbol{X}_\delta,\ \ |H\,r| \le \delta u_{\max}. \tag{6}$$

where $H := [1 - F(A + BF)^{-1}B]G$, $x_e := G_e\,r := -(A + BF)^{-1}BG\,r$.

Remark 2.1. For the case $x_0 = 0$, any step input of amplitude r can be asymptotically tracked if $|\ r\ | \le \left[c_\delta(G_e'PG_e)^{-1}\right]^{1/2}$ and $|\ Hr\ | \le \delta\,u_{\max}$. Clearly, the trackable amplitudes of step inputs by the linear feedback control law can be increased by increasing δ and/or decreasing $G_e'PG_e$ through the choice of W. However, the change in gain F will of course affect the damping ratio of the closed-loop system and hence its rising time.

STEP 2: The nonlinear feedback part u_N:

$$u_N = \rho(r, h)B'P(x - x_e) \tag{7}$$

where $\rho(r, h)$ is any non-positive function locally Lipschitz in h, which is used to change the system closed-loop damping ratio as the output approaches the step command input.

STEP 3: Form a CNF control law,

$$u = u_L + u_N = Fx + Gr + \rho(r,h)B'P(x - x_e). \quad (8)$$

The following theorem is on the CNF control law for the state feedback case.

Theorem 2.1. Consider the system (1). Then, for any non-positive function $\rho(r,h)$, locally Lipschitz in h, the CNF control law (8) will drive the controlled output h to asymptotically track the step command input of amplitude r from an initial state x_0, provided that x_0 and r satisfy (6).

Remark 2.2. Theorem 2.1 shows that the additional nonlinear feedback part u_N (7) does not affect the ability of the closed-loop system to track the command input. Any command input that can be asymptotically tracked by the linear feedback law (3), can also be asymptotically tracked by the CNF control law (8). However, this additional term u_N can be used to improve the performance of the overall closed-loop system. This is the key property of the CNF control technique.

2.2 *Full Order Measurement Feedback Case*

The assumption that all the states of Σ are measurable is in general not practical. Thus, the following CNF control law for the measurement feedback case is developed.

$$\begin{cases} \dot{x}_v = (A + KC_1)x_v - Ky + B\,\mathrm{sat}(u), \\ u = Fx_v + Gr + \rho(r,\hat{h})B'P(x_v - x_e). \end{cases} \quad (9)$$

where $\rho(r,\hat{h})$ is a non-positive scalar function, locally Lipschitz in $\hat{h} = C_2 x_v$, and is to improve the performance of the closed-loop system. It turns out that for the measurement feedback case, the choice of $\rho(r,\hat{h})$, is not totally free and subject to a certain constraint. For any $\delta \in (0,1)$, let c_δ be the largest positive scalar such that for all

$$\begin{pmatrix} x \\ x_v \end{pmatrix} \in \boldsymbol{X}_{\mathrm{F}\delta} := \left\{ \begin{pmatrix} x \\ x_v \end{pmatrix} : \begin{pmatrix} x \\ x_v \end{pmatrix}' \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \le c_\delta \right\}$$
$$\Rightarrow \left| [\,F \;\; F\,] \begin{pmatrix} x \\ x_v \end{pmatrix} \right| \le u_{\max}(1-\delta). \quad (10)$$

Where P and Q are the solutions of the appropriate Lyapunov equations. The following theorem is on the CNF control law for the measurement feedback case.

Theorem 2.2. Consider the system (1). Then, there exists a scalar $\rho^* > 0$ such that for any non-positive function $\rho(r,\hat{h})$, locally Lipschitz in $\hat{h}$ and $|\rho(r,\hat{h})| \le \rho^*$, the CNF control law (9) will drive the system controlled output h to asymptotically track the step command input of amplitude r from an initial state x_0, provided that x_0, x_{v0} and r satisfy:

$$|Hr| \le \delta\, u_{\max}, \quad \begin{pmatrix} x_0 - x_e \\ x_{v0} - x_0 \end{pmatrix} \in \boldsymbol{X}_{\mathrm{F}\delta} \quad (11)$$

2.3 *Reduced Order Measurement Feedback Case*

For the system (1), it is clear that there are p states of the system are measurable, if C_1 is of maximal rank. Thus, in general, it is not necessary to estimate these measurable states in the measurement feedback case. A dynamic control law, which has a dynamical order less than that of the plant, can be constructed under the CNF control framework. For simplicity of presentation, C_1 is assumed already in the form $C_1 = [\,I_p \;\; 0\,]$. Then, the system (1) can be rewritten as,

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \mathrm{sat}(u), \\ y = [\,I_p \quad 0\,] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ h = C_2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases} \quad (12)$$

$x_0 = (\,x_{10} \;\; x_{20}\,)'$. where the original state x is partitioned into two parts, x_1 and x_2 with $y \equiv x_1$. Thus, it will only need to estimate x_2 in the reduced order measurement feedback design. Next, let F be chosen such that 1) $A + BF$ is asymptotically stable, and 2) $C_2(sI - A - BF)^{-1}B$ has desired properties, and let K_{R} be chosen such that $A_{22} + K_{\mathrm{R}}A_{12}$ is asymptotically stable. It is noted that (A_{22}, A_{12}) is detectable if and only if (A, C_1) is detectable (Chen, 1991). Thus, there exists a stabilizing K_{R}. Then partition F in conformity with x_1 and x_2, $F = [\,F_1 \;\; F_2\,]$. The reduced order CNF control law is given by

$$\begin{cases} \dot{x}_v = A_{\mathrm{R}}x_v + B_{vy}y + B_{vu}\,\mathrm{sat}(u) \\ u = F(x_r - x_e) + Hr + \rho(r,\hat{h})B'P(x_r - x_e) \end{cases} \quad (13)$$

where $B_{vy} = A_{21} + K_{\mathrm{R}}A_{11} - A_{\mathrm{R}}K_{\mathrm{R}}, A_{\mathrm{R}} = A_{22} + K_{\mathrm{R}}A_{12}, B_{vu} = B_2 + K_{\mathrm{R}}B_1, x_r = (\,y \;\; x_v - K_{\mathrm{R}}y\,)'$ and $\rho(r,\hat{h})$ is non-positive scalar function locally Lipschitz in $\hat{h} = C_2 x_r$. For any $\delta \in (0,1)$, let $c_{\mathrm{R}\delta}$ be the largest positive scalar such that for all

$$\begin{pmatrix} x \\ x_v \end{pmatrix} \in \boldsymbol{X}_{\mathrm{R}\delta} := \left\{ \begin{pmatrix} x \\ x_v \end{pmatrix} : \begin{pmatrix} x \\ x_v \end{pmatrix}' \begin{bmatrix} P & 0 \\ 0 & Q_{\mathrm{R}} \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \le c_{\mathrm{R}\delta} \right\}$$
$$\Rightarrow \left| [\,F \;\; F_2\,] \begin{pmatrix} x \\ x_v \end{pmatrix} \right| \le u_{\max}(1-\delta). \quad (14)$$

Where P and Q_{R} are the solutions of the appropriate Lyapunov equations. The following theorem is on the reduced order CNF control law for the measurement feedback case.

Theorem 2.3. Consider the system (1). Then, there exists a scalar $\rho^* > 0$ such that for any non-positive function $\rho(r, \hat{h})$, locally Lipschitz in $\hat{h}$ and $|\rho(r, \hat{h})| \le \rho^*$, the reduced order CNF control law (13) will drive the controlled output h to asymptotically track the step command input of amplitude r from an initial state x_0, provided that x_0, x_{v0} and r satisfy

$$\begin{pmatrix} x_0 - x_e \\ x_{v0} - x_{20} - K_{\mathrm{R}} x_{10} \end{pmatrix} \in \boldsymbol{X}_{\mathrm{R}\delta}, \ |Hr| \le \delta\, u_{\max}. \quad (15)$$

3. BEATING THE TOC

Would a control system be designed to beat the performance of the TOC? Obviously, the answer is no for a precise point-to-point tracking. However, the answer would be yes for an asymptotic tracking situation, which is widely used in almost all practical situations. Consider a double integrator,

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathrm{sat}(u),\ y = [1\ 0]x,\ h = [1\ 0]x,$$
$$\mathrm{sat}(u) = \mathrm{sgn}(u)\ \min\{\, 1, |u|\, \}. \quad (16)$$

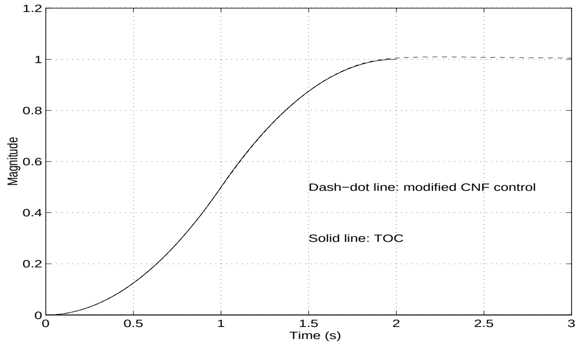
where x, u, y and h are the state, input, the measurement and controlled output respectively and $x(0) = 0$. The minimum time for h to reach precisely the target reference $r = 1$ under the TOC is exactly $2s$. The following CNF control law is designed for the asymptotic tracking situation, where the settling time is defined as the total time for h to enter the $\pm 1\%$ region of r.

$$u = [\,-6.5\ -1\,]\, x + 6.5\, r - (e^{-|1-h|} \quad (17)$$
$$-0.3678)[\,1.4481\ 10.8609\,](x - [\,1\ 0\,]').$$

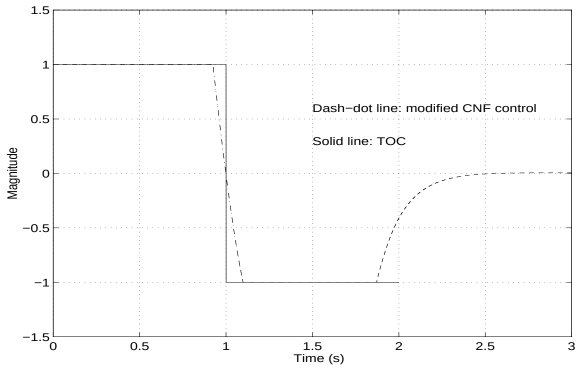
The simulation results of the TOC and the CNF control are shown in Figure 1 and enlarged in Figure 2, which shows that the CNF has a settling time of $1.8453s$ faster than the TOC a settling time of $1.8586s$ when h enters $[0.99, 1.01]$. It can also be shown that, no matter how small the target region is, a suitable control law can be found to beat the TOC in settling time. it is significant enough to address one interesting issue: *there are control laws that can achieve a faster settling time than the TOC in asymptotic tracking situations.* Nonetheless, it might be believed that it would be interesting to carry out some further studies in this subject.

4. AN APPLICATION

The above CNF control technique is applied to design a reduced order CNF control law for the



(a) Controlled output response



(b) Control signal

Fig. 1. Simulation results of the TOC and CNF

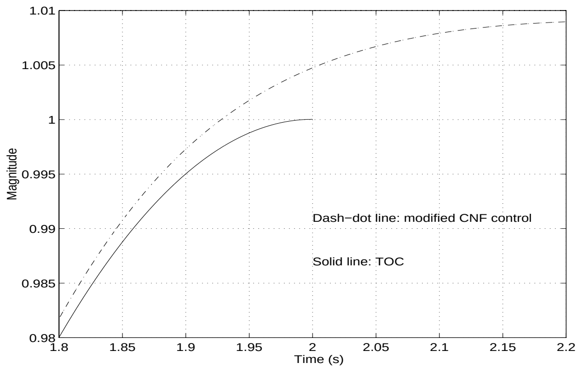


Fig. 2. Enlarged output responses

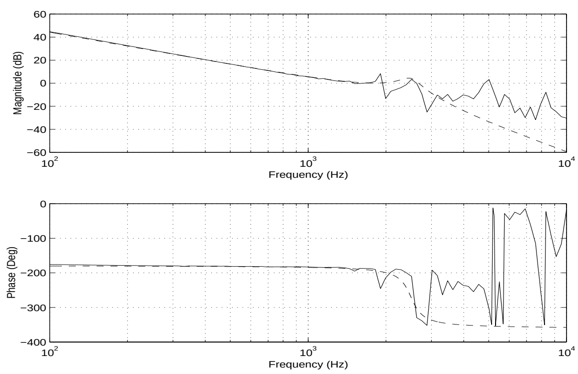


Fig. 3. Frequency response of HDD

HDD servo system. There are two main functions of the head positioning servomechanism in disk drives: track seeking and track following. Track seeking moves the R/W head from the present track to a specified destination track in minimum time using a bounded control effort. Track following maintains the head as close as possible to the destination track center while data is being read from or written to the disk. The actuator, called a voice-coil motor (VCM), carries the carriage and moves the head on a desired track. An actual HDD was modeled through frequency response test. Figure 3 shows the frequency response characteristics of a Maxtor HDD (Model No. 51536U3). The frequency response characteristics was obtained using a Laser Doppler Vibrometer and a HP make dynamic signal analyzer. A fourth order model for the actuator is obtained using the measured data from the actual HDD and the algorithms of Eykhoff (1981) .

$$G_{\rm v}(s) = \frac{a}{s^2}\frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} \qquad (18)$$

where $a = 6.4013 \times 10^7$, $\zeta_n = 0.085$, $\omega_n = 1.57 \times 10^4$ rad/sec and the units of the input and output are respectively in volts and micrometers. During the design only the double integrator model with actuator saturation was taken which is as follows

$$\begin{cases} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ a \end{bmatrix} \mathrm{sat}(u), \\ y &= [1 \quad 0]\, x. \end{cases} \qquad (19)$$

where $u_{\max} = 3V$. For the HDD model (19), the following parameterized feedback gain is derived.

$$F(\varepsilon) = -\frac{1}{a}\left[\frac{4\pi^2 f^2}{\varepsilon^2} \quad \frac{4\pi f\zeta}{\varepsilon}\right]. \qquad (20)$$

The eigenvalues of $A+BF(\varepsilon)$, are placed at $(-\zeta \pm j\sqrt{1-\zeta^2})2\pi f/\varepsilon$. Such a gain is determined with $\zeta = 0.3$, $f = 350$ and $\varepsilon = 1$ roughly corresponding to the normal frequency range of the HDD. The scalar function is chosen as follows,

$$\rho(r, y) = -1.5820\beta(e^{-|1-y/r|} - 0.3678). \quad (21)$$

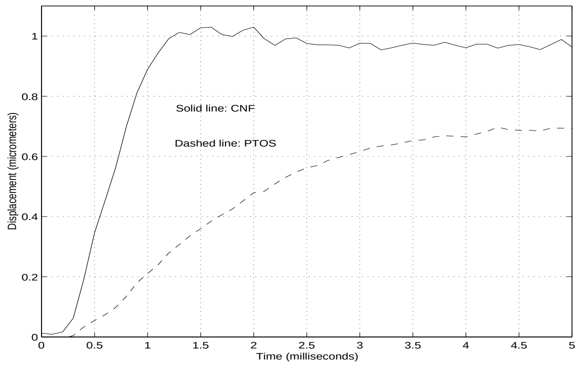
A reduced order CNF control law is designed as only displacement of R/W head is measurable with $K_{\rm R} = 4000$ as follows:

$$\begin{cases} \dot{x}_v = -K_{\rm R}x_v - K_{\rm R}^2 y + a\,\mathrm{sat}(u), \\ u = \kappa_2 x_v + (\kappa_1 + K_{\rm R}\kappa_2)y - \kappa_1 r \\ \quad +\rho(r,y)[\kappa_3 x_v + (K_{\rm R}\kappa_3 - \kappa_1)y + \kappa_1 r]. \end{cases} \qquad (22)$$

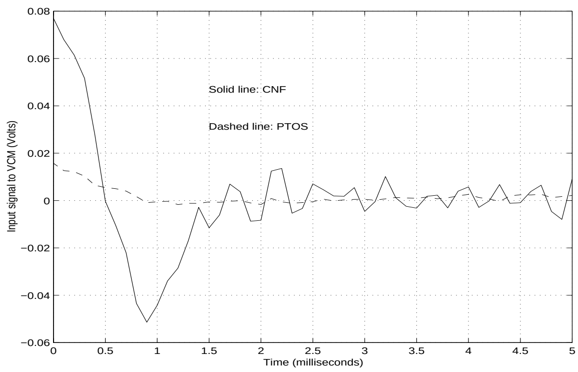
where $\rho(r, y)$ is defined as (21). $\kappa_1 = -0.0755\varepsilon^{-2}$, $\kappa_2 = -2.0613 \times 10^{-5}\varepsilon^{-1}$ and $\kappa_3 = 5.7257 \times 10^{-5}\varepsilon^{-1}$. Note that the parameters ε and β can be adjusted accordingly to the amplitude of the target reference, which are listed in Table 1. The PTOS control law (Workman, 1987) is taken to compare with the CNF control law. The following PTOS control law is found for the HDD model (19).

Table 1. Parameters ε and β

Seek Length (μm)	ε	β
1	1	1
100	2.33	1.59
300	2.76	1.59

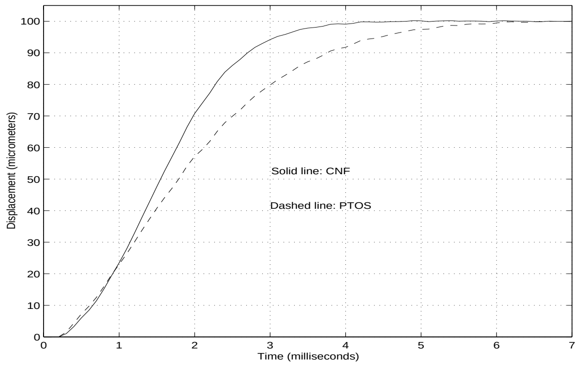


(a) Output response



(b) Control signal

Fig. 4. Experimental results for $1\mu m$ seek length using CNF and PTOS

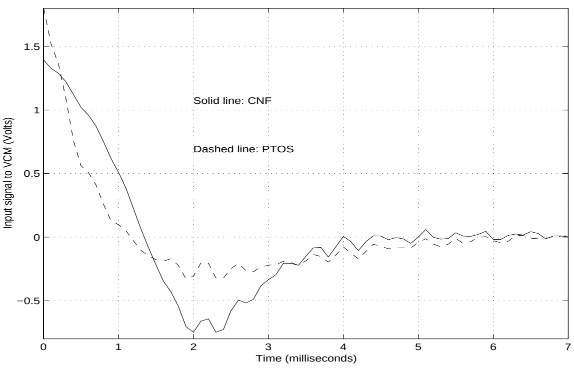


(a) Output response

$$u_p = u_{\max}\,\mathrm{sat}(k_2[f(e) - v]/u_{\max}) \qquad (23)$$

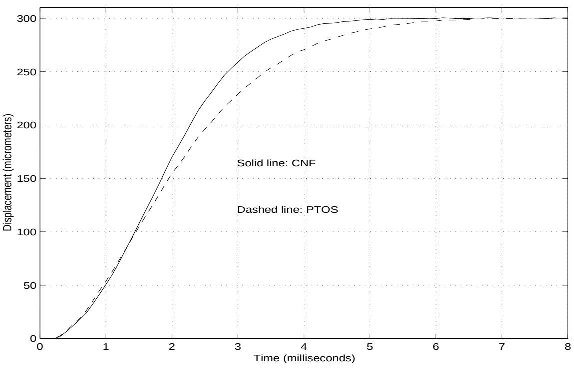
$$f(e) = \begin{cases} (k_1/k_2)e & |e| \le y_l \\ \mathrm{sgn}(e)[\sqrt{2u_{\max}a\alpha|e|} - \frac{u_{\max}}{k_2}] & |e| > y_l \end{cases}$$

where $e = r - y$ and $v = \dot{y}$. The parameters practicable in implementations using PTOS up to a seek length of 300μm were found $a = 6.4013 \times 10^7$, $k_1 = 0.0178$, $k_2 = 2.997 \times 10^{-5}$, $\alpha = 0.62$ and $y_l = 168.32\mu m$. A velocity estimator is designed with a pole -4000 for the PTOS control law.

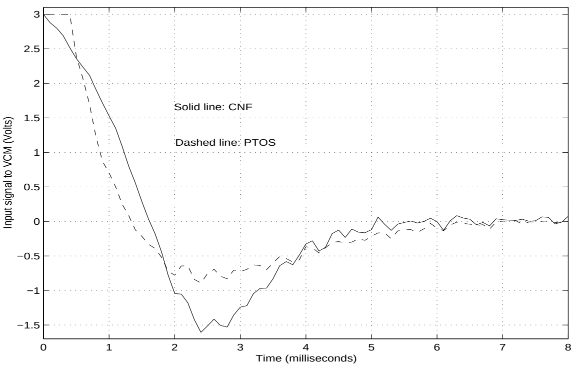


(b) Control signal

Fig. 5. Experimental results for $100\mu m$ seek length using CNF and PTOS



(a) Output response



(b) Control signal

Fig. 6. Experimental results for $300\mu m$ seek length using CNF and PTOS

Table 2. Settling time from experimental results using CNF and PTOS

Seek Length (μm)	Settling Time (ms) PTOS	CNF	Overall Improvement (%)
1	—	1.2	—
100	6.5	4.5	31
300	6.7	5.3	21

The implementations were made on a typical 3.5-inch open hard disk drive with a TMS320 digital signal processor (DSP) with a sampling rate of 10 kHz. The R/W head position was measured using a Laser Doppler Vibrometer (LDV) and the track pitch was assumed to be $1\mu m$. Thus the track density was 25,000 TPI. The implementation results for 1, 100 and 300 μm using CNF and PTOS control laws are respectively shown in Figures 4, 5 and 6. The Table 2 summarizes the settling time from the implementation results which shows that the CNF control improves the performance by more than 30% than the PTOS control. The settling time in the implementations is defined as the time to let the residual error within $0.05, 0.2$ and 0.5μm around the target track for $1, 100$ and 300μm respectively.

5. CONCLUDING REMARKS

The composite nonlinear feedback control technique has been developed for a class of linear systems with actuator saturation. The implementation results show that the new technique has outperformed the conventional PTOS by more than 30% for the HDD servo system. It has been shown by an example that the CNF control is capable of beating the TOC in asymptotic tracking. It would be interesting, although it is pretty hard, to carry out a systematic study on how to derive a TOC law in the asymptotic tracking situations. This will be the subject of the future research.

6. REFERENCES

Chen, B. M. (1991). Theory of Loop Transfer Recovery for Multivarible Linear Systems. PhD thesis. Washington State University. Washington, USA.

Chen, B.M. (2000). *Robust and H_∞ Control.* Springer. London.

Eykhoff, P. (1981). *System Identification - Parameter and State Estimation.* John Wiley. New York.

Lin, Z., M. Pachter and S. Banda (1998). Toward improvement of tracking performance — nonlinear feedback for linear systems. *International Journal of Control* **70**, 1–11.

Liu, K., B. M. Chen and Z. Lin (2001). On the problem of robust and perfect tracking for linear systems with external disturbances. *International Journal of Control* **74**, 158–174.

Workman, M. L. (1987). *Adaptive Proximate Time Optimal Servomechanisms, Ph.D. thesis of Stanford University.* University Microfilms International. Ann Arbor, Michigan, USA.