

## GLOBAL OPTIMIZATION FOR HIGHLY DYNAMIC SUPPLY CHAINS

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Abstract: Supply chains in automotive industry are both very complex and highly dynamic. Thus any approach for mastering this complexity and dynamics has to be considered. In this contribution we apply approaches from global optimization. The context is a supply chain scenario related to a truck plant. We demonstrate how in a fixed, but rolling, period of time the daily orders for each partner in the supply net can be optimized with respect to the overall costs. This investigation is to be understood as a starting point for applying methods of modern optimization to supply net management.

Keywords: automotive industry, enterprise modelling, optimization, dynamic systems, oscillation, stability analysis, control theory

### 1. INTRODUCTION

Supply chains in automotive industry are highly dynamic and very complex: Thousands of suppliers, high degree of product complexity, high degree of customization (esp. for trucks), long cycle times in some chains due to long lead times, various time and space constraints (w. r. t. capacity, stock size, part and component size). Simple changes in the demands from the market consumer lead to rather unexpected stock and production policies at the OEM<sup>1</sup> and at

the nodes further down the chain, i. e. the system and component suppliers. This demonstrates the bullwhip effect: forecasts and actual demand differ the more one goes down the chain.

Supply chain management aims at a better synchronization amongst the OEM and its suppliers compared to traditional logistics, especially for production program planning and for scheduling orders and capacities. Only by a genuine co-operation one will be able to reduce costs in the chain, which ultimately will lead to a win-win situation for all partners in a supply community and the customer as well.

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<sup>1</sup> Original Equipment Manufacturer

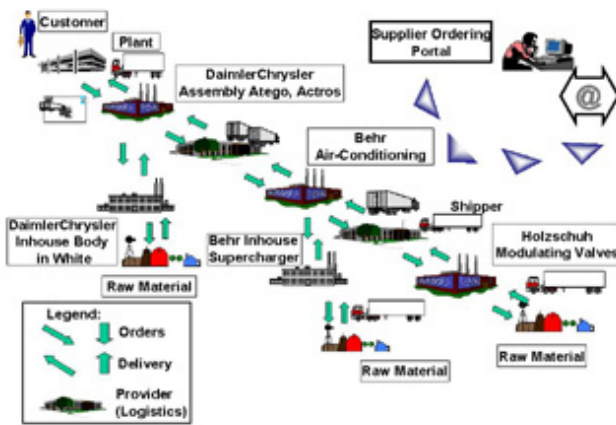


Fig. 1. Supplier ordering portal for a ten stage supply chain

Various methods of how to support this co-operation are currently investigated in research and in practice. This contribution describes modern methods from system theory, specifically discrete time nonlinear optimization. The scenario presented assumes that the market demand is known for a fixed and rolling period of time. This information together with logistic data and parameters from the OEM and the suppliers is handed to some central system, which in turn determines the optimal ordering behavior for each partner in the supply context, where optimality relates to the minimization of total costs.

To demonstrate this approach we take a supply chain with ten partners, which leans upon the structure of a real supply chain of DaimlerChrysler Truck Division, namely a plant in Woerth (Germany) with its body-in-white and its assembly plant, one company assembling the air-conditioning unit, another one delivering valves for this unit, three raw material plants, and two logistic providers. We model the OEM and the suppliers in a discrete time state space model and exhibit the constraint relation amongst logistic parameters. As for the cost function we take in this contribution an additive function summing both the inventory (warehouse) costs and the backlog costs for those trucks which could not be delivered in time to the customer. We show and discuss the simulation results with their optimal order sequences for the first-tier in this chain. Although this model is still simple compared to the full complexity and dynamics in automotive supply, it gives insights about further extensions both in the application scope as well for research. Especially, in more complex domains with more elaborated cost functions we foresee the need for more sophisticated methods of global optimization, as for example interval arithmetic. In the concluding chapter we indicate a pathway to future investigations.

## 2. CONTROL SYSTEM THEORY FOR SUPPLY CHAIN OPTIMIZATION

In modelling and optimizing the supply chain a simple input-output model is not appropriate. Since in supply chain management the reaction to decisions typically

happens with time-lags and since positive and negative feed-back-loops exist, we are forced to model the internal state behavior of the system. Therefore we will use the state space formulation of system theory. The state space modelling has to respect the inherent non-linearity of supply chains, which arises amongst others from limited logistic functions (like capacities, safety stocks, buffer sizes, batch sizes) . Thus the full model will be a nonlinear state space model.

In the past, methods of system and control theory were typically applied to technical systems. In this paper we will show that there is a smart use of system and control theory for economical applications and especially for supply chain management. There is also an additional value for operations research, because in the past there was almost no consideration of dynamical aspects together with non-linearity.

## 3. GLOBAL OPTIMIZATION

Global optimization covers all activities for developing algorithms which calculate the global minimum (or maximum) of functions, especially performance functions. Performance functions which include several variables can have several different minima. If the performance function has the meaning of a cost function it is a must to calculate the absolute minimum. So for the operational logistic it is very advantageous to rely upon algorithms which calculate parameters or instructions for globally minimizing a corresponding cost function. In the scenario of this contribution optimal order policies for each supplier and the OEM will be calculated. With these optimal orders the logistic departments of suppliers and OEM operate the supply chain in a global minimum, with respect to costs.

### 3.1 Scenario

The ordering scenario considered is a supply chain formed by the assembly of DaimlerChrysler commercial vehicles (OEM), the DaimlerChrysler body-in-white unit, the production of the air-conditioning (Behr) including the in-house production of superchargers, and the production and delivery of modulating valves (Holzschuh). Furthermore there are two logistics providers. The supply chain, is "terminated" by three units for raw material.

The information of the orders from the customers are bundled by sales organizations and given to the plants. Then the ordering information "flows" from the OEM to his immediate suppliers and in the same manner from these suppliers to their next suppliers etc. As of today, the "flowing" information is bound to the BOM<sup>2</sup> and each site uses the exploded BOM together with the demand from previous units for its own production planning. The demand is propagated to the chain in

<sup>2</sup> Bill Of Material

forecast horizons defined by VDA<sup>3</sup> standards. Each forecast can be interpreted as a funnel, where the orders further back in time have a higher uncertainty than the ones nearer to execution. By our approach we aim to propagate not only pure demand data but additional control information. So we assume the existence of a central optimization unit (where ever it resides), which collects internal data of the supply partners and afterwards informs each partner about his optimal order policy.

### 3.2 Modelling

As for the aggregation level of the quantities to be modelled we lean upon the SCOR<sup>4</sup> model which standardizes the source, make and delivery processes. We decide to model the system in terms of demands and inventories. These functions are in general described in terms of coupled differential equations. But since the OEM and the suppliers usually have strict run off, the formulation in terms of difference equations is not convenient but even appropriate.

For notational purposes we use the following abbreviations:  $S$  stands for each partner in the supply net (including the OEM itself),  $P$  stands for the producers (OEM, body-in-white, Behr, Holzschuh),  $L$  for the two logistics provider, and  $R$  for the three units for raw material. The state space functions, depending on discrete time  $k$ , are as follows:

daily demand:  $u_S(k)$ ,  $k = 0, \dots$

delivery finished goods:  $y_S(k)$ ,

incoming goods:  $w_S(k)$ ,

orders:  $B_S(k)$ ,

stock of finished goods:  $L_S(k)$ ,

stock of incoming goods:  $R_S(k)$ ,

open customer demand:  $KO_S(k)$ ,

total demand:  $G_S(k)$

The state space functions for the producers obey the following equations:

$$\begin{aligned} KO_P(k) &= G_P(k) - y_P(k-1), \\ G_P(k) &= KO_P(k) + u_P(k), \\ y_P(k) &= \text{Min}(G_P(k), L_{max}, L_P(k-1)) + p(k), \\ L_P(k) &= L_P(k-1) - y_P(k) + p(k), \\ R_P(k) &= R_P(k-1) + w_P(k) - p(k), \\ B_P(k) &= f(L_P(k), KO_P(k)). \end{aligned}$$

Here  $p(k)$  is the production function, which are the daily production rates at each of the producing units. In its most simple form it is:

$$p(k) = \text{Min}(G_P(k) - L_P(k), p_{max}, R_P(k-1) + w_P(k))$$

Because of the minimum equation for production and delivery we have a nonlinear system model. The logistic providers  $L$  and the supplier  $R$  for raw material have only one warehouse and no production, so their dynamic equations have no production function:

$$\begin{aligned} KO_{L,R}(k) &= G_{L,R}(k) - y_{L,R}(k-1), \\ G_{L,R}(k) &= KO_{L,R}(k) + u_{L,R}(k), \\ y_{L,R}(k) &= \text{Min}(G_{L,R}(k), L_{max}, L_{L,R}(k-1)) + p(k), \\ L_{L,R}(k) &= L_{L,R}(k-1) - y_{L,R}(k) + w_{L,R}(k), \\ B_{L,R}(k) &= f(L_{L,R}(k), KO_{L,R}(k)). \end{aligned}$$

The  $B_S(k)$  have to be optimized in the sense that an associated cost function is minimized.

Orders of time step  $k - k'$  from one supplier are linked to the daily demand of time step  $k$  for the next supplier:

$$u_S(k) = B_{S'}(k - k'),$$

where the supplier  $S$  is next to supplier  $S'$ . For the sake of complicity we set  $k' = 1$ . A similar relation holds for the finished goods and the incoming parts:

$$w_S(k) = y_{S'}(k - k').$$

### 3.3 Performance function

The performance function is realized as a cost function with respect to the costs of the producers, the logistics providers and the supplier of raw material. In general the performance function will include variables like time delivery data, transport costs, storages, finished products, materials, orders etc.. We will confine this scenario to stock- and backlog costs. The costs for goods (incoming goods and finished goods) on the stock are assumed to be 0.5 units/day. The backlog costs are costs for punishing the lateness between ordering and delivery. The backlog costs for open customer demand are 1.0 unit/day. Backlog costs and stock costs are conflicting and so they have to be optimized in one performance function. This function looks like

$$\begin{aligned} totalSupplyCosts &= backlogCosts + \\ & 0.5 * warehousingCosts. \\ backlogCosts &= \sum_{x=S} \sum_{k=1}^{20} KO_x(k) \\ warehousingCosts &= \sum_{x=S} \sum_{k=1}^{20} L_x(k) + \\ & \sum_{x=P} \sum_{k=1}^{20} R_x(k) \end{aligned}$$

<sup>3</sup> Verein der Automobilindustrie

<sup>4</sup> Supply Chain Council, Inc.

This performance function is a cost function for the whole supply chain which has to be minimized.

Normally there exist several minima. But we have to find the global minimum of the performance function for getting the highest benefit of cost potential.

In order to compute the optimal control input sequence (the orders) we will use global optimization algorithms (Tibken *et al.*, 1999; Hofer *et al.*, 1998; Heeks *et al.*, 1999). Because we search for optimal order functions with the time as the independent variable we need the research results from dynamics optimization. Due to the piecewise linear structure of our state equations interval arithmetic based algorithms will used very well to find the global minimum (Alefeld, 1994; Moore, 1979).

### 3.4 Simulation results

The last things to be fixed before solving the optimization problem are the data (demand functions, ...) and internal parameters of the suppliers. Instead of using real data we abstract them as follows: The forecast duration is 20 days. At the 1-st day the daily demand increases from 1 to 38. This step function is to be understood as a test function respecting the forecast funnel described in chapter 3.1. The OEM has 190 commercial vehicles as initial conditions in its stock. We furthermore assume that the other suppliers start with 38 units for their initial product inventory. The maximum production capacity is 70 for the OEM and for Behr; the other suppliers have a maximum capacity of 200 units per day. The raw material suppliers are assumed to have an unlimited capacity.

We next present our simulation results for an optimal ordering behavior of ten supply chain members. For a first impression it is sufficient to focus to the ordering and delivery from the OEM, because he is the closest to the customer.

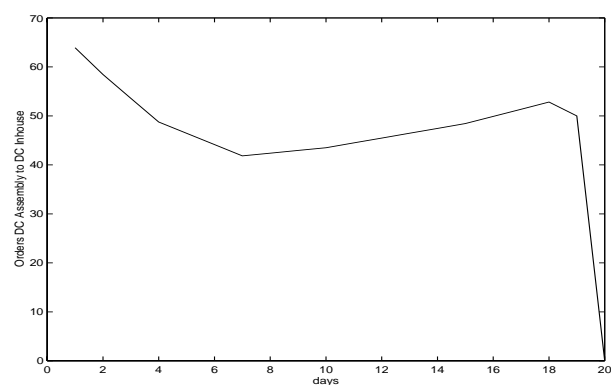


Fig. 2. Orders from Assembly to Body-In-White

In figure two we see that the OEM first decreases his orders to DC Body-in-White because he starts with a sufficient filled product inventory and can satisfy the demand from the market. At day 18 the OEM stops ordering because he knows the "supply final" at time step 20. So he wants to exhaust the product inventory avoiding warehousing costs.

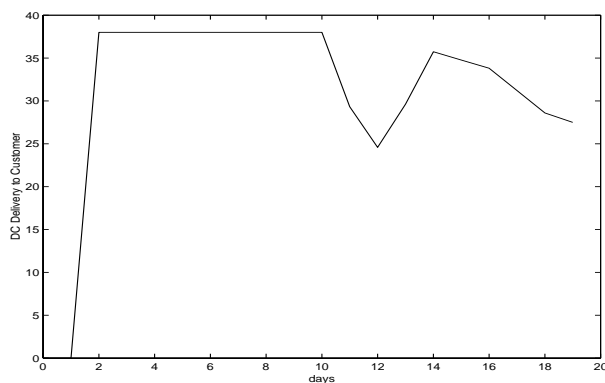


Fig. 3. DC-Delivery to customer

According to figure three up to day 11 the OEM delivers 38 units per day promptly. Because of the compromise between backlog costs and warehousing costs he defers the delivery to customer at day 11. In figure four we present the quantity of commercial vehicles over the 20 days. First the warehouse overshoots but then it drops directly to the goal of zero finished goods. This goal was given implicitly in the cost function.

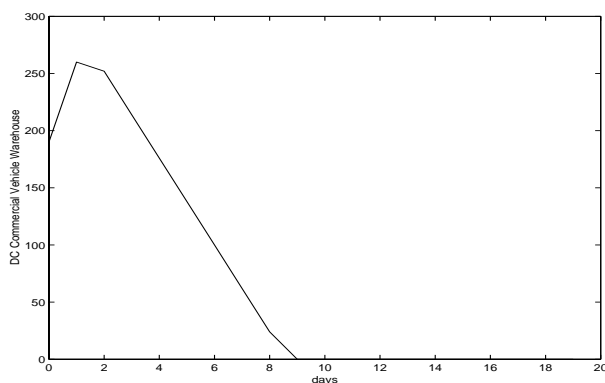


Fig. 4. DC warehouse finished goods

To get closer to reality we of course have to perform the optimization for each time step in order to respect the practise of rolling time planning.

### 3.5 Interval Arithmetic Methods for Global Optimization

The optimal ordering sequence for the suppliers can only be calculated when the corresponding cost function reaches its global minimum. But algorithms like SQP (Sequential Quadratic Programming) have difficulties because functions will only analyze finite points -with danger not reaching a "valley"- whereas interval methods uses the continuum, so a possible local and global minima cannot escape because it is enclosed in an interval. In this section we will give a short overview about global optimization. In the following we assume that  $X$  is a closed compact subset of  $R^n$  and that the function  $f : X \rightarrow R$  is continuous, then, by the term global optimization we mean that the optimization problem

$$f^* = \min_{x \in X} f(x)$$

is solved for the global minima without getting stuck in local suboptimal points. In practice it is usually the goal to compute an  $\epsilon$ -optimal solution, e.g. we look for an  $\bar{f}$  such that the following inequality  $\bar{f} - \epsilon \leq f^* \leq \bar{f}$  holds for the chosen  $\epsilon$ . Thus, we compute  $f^*$  to accuracy  $\epsilon$ . For the set  $X^*$  of optimal points of the function  $f$  we have the following inclusion

$$\begin{aligned} X^* &= \{x \in X \mid f(x) = f^*\} \subset \\ &\quad \{x \in X \mid f(x) \leq \bar{f}\} \\ &= \bar{X} \end{aligned}$$

and the set  $\bar{X}$  often is computed during the global optimization. It should be noted that the set  $X^*$  may consist of infinitely many points. Thus,  $f^*$  is unique but the set of minimizing points is in general not one point. Nevertheless, in practice there is often only one optimal point.

In the literature several different methods for global optimization have been published. The basic idea in all global optimization methods is a branch and bound approach which bounds the values of the function  $f$  from below and branches according to some criteria. In order to obtain guaranteed and reliable results the approach by (Hansen, 1992) seems to be most appropriate. Hansen used interval arithmetic to do the bounding from below and a bisection strategy with respect to the set  $X$  in order to do the branching. Due to the special piecewise linear structure of our system equations with the min function as the only nonlinearity we can guarantee that our bounding step is sharp, i.e. the function  $f$  on an subset of  $X$  can be bounded very accurately with interval arithmetic. Ongoing research will concentrate on other approaches which use this piecewise linearity in order to improve the bounding and the branching. We will report on this in a separate publication.

#### 4. RESULTS AND OUTLOOK

Control theory is usually the domain for technical systems. We have shown in this paper that modern methods of control theory, especially optimal control and global optimization can be utilized in a non-technical domain, namely the process world of supply chain management. We especially demonstrated on an automotive inspired scenario how the global minimization of costs for a supply chain can be reached by calculating the optimal order sequence of each partner in the supply chain, assuming that the demand from the market is known for a given period of time. Several additional investigations are to be undertaken before testing it in the field. These additions refer both to algorithmic and organizational aspects.

As from the algorithmic side further investigations need to consider

- sensitivity of the results upon variations of the demand functions within the funnel defined by the standardized forecast windows,

- sensitivity of the results on variations in the cost function,
- global optimization techniques for "rough" performance functions, like indicated in the main part, interval arithmetic,
- the aspect of rolling planning,
- production rates in accordance with results of MRP II calculations,
- scalability of the method with the topology and the size of a supply network.

However, improved supply chain management is not simply a question of algorithms but heavily needs support from business organization and must reside on an open-minded business culture. Here are questions like

- Where resides the optimization unit? Is it separate from the supplier (a supplier portal for instance)? Is it one of the partners in the supply chain?
- Does every participant trust this optimization unit? Will every supplier deliver those information which are needed as input for the optimization? Will every supplier in the chain follow those policies advised by the optimization unit? Do the optimization results drastically change, if one of the suppliers deviates even slightly from the order sequence calculated by the global optimization?
- How to distribute the cost savings gained by this approach amongst the participants?

Even if the approach considered here cannot be immediately implemented in a real life application, it can be used as a benchmark for other approaches, for instance distributed algorithms like agent-based algorithms.

Quite another - and more future-oriented - research thread is to extend the approach from pure control to control loops. In closed control loops one can refrain from the assumption of exact demand functions - and this is more the reality in automotive supply. The idea is of course to interpret sudden changes in demand as disturbances. These disturbances will be taken as requirements for designing a control loop. Furthermore we envisage to use methods of stochastic system theory when signals are not predictable and available for optimization algorithms.

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