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THE METHOD OF "EQUIVALENT LINEARIZATION" OF CONFLICT–CONTROLLED SYSTEMS. ATTITUDE CONTROL OF ASYMMETRIC SOLID

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Abstract: The method "equivalent linearization" of nonlinear conflict–controlled systems and a nonlinear game problem of reorientation of an asymmetric rigid body under uncontrollable disturbances and uncertain parameters are considered. Direct estimates of disturbance domain that is admissible for reorientation and depend on the given constraints imposed on the controls and the initial position of the body are found.

Keywords: attitude control, equivalent linearization of conflict–controlled systems.

1. INTRODUCTION

The approach (Vorotnikov, 1994a, 1997, 1998) to the problem of transferring a non-linear dynamical system, subject to perturbations, to the null equilibrium position in finite time by means of bounded control is considered. Only the levels of uncontrollable perturbations are known, and are not assumed to be small. This approach allows one to conduct *"an equivalent linearization"* of the initial nonlinear problem and obtain its solution on the basis of the *linear* game theory (Krasovskii, 1970). Controls are nonlinear piecewise continuous functions of phase variables and also involve parameters which are refined for each particular initial position of the system. This is performed by iteratively checking the prescribed levels of controls over the set of possible states of auxiliary linear conflict-controlled systems. Conceptually, the approach is close to *decomposition* methods (Chernousko, 1990; Siljak, 1990) as well as to methods of *feedback linearization* of nonlinear controlled systems (Isidori, 1985; Nijmeijer and van der Schaft, 1990; Vorotnikov, 1991).

Within the framework of the approach proposed two methods for solving of *nonlinear* reorientation problem of an asymmetric solid body subjected to uncontrollable interferences was developed in Vorotnikov (1994b, 1995, 1996, 1997, 1998). The first one (Vorotnikov, 1994b, 1996,1998) uses the direct estimations of levels of "auxiliary interferences" in the auxiliary linear systems. The second method (Vorotnikov, 1995, 1998) is based on the principle of "assignment and subsequent confirmation" (ASC principle) of levels of such interferences, which was introduced in Vorotnikov (1995).

The structure of controls is simpler if the second variant is used. In particular, they involve no components compensating gyroscopic moments of a body. This is achieved at the sacrifice of the simplicity of the structure of "auxiliary interferences" in emerging linear systems.

The first method also allows one to consider the game problem of the passage of a body through a prescribed angular position in a three-dimensional inertial space (Vorotnikov, 1998, 1999a). Under the same restriction imposed on control, the reorientation time in the case is substantially shorter than that in reorientation in the equilibrium state.

These methods were further developed in Vorotnikov (1999b). Direct estimations of the levels of interferences which are admissible in reorientation and vary with restrictions imposed on controls have been found. Such estimates are useful at the stage of evaluating the possibility of using the controls to ensure the required reorientation. If these estimates hold with a "reserve", then the guaranteed reorientation time can further be found (the second stage of solving) with iteration algorithms (Vorotnikov, 1994b, 1995, 1996, 1997, 1998).

The estimates for admissible levels of interferences are obtained at component-wise restrictions imposed on the vector of controls, which determine a rectangular domain of admissible controls. Such restrictions correspond to three pairs of *fixed* (with respect to the axes linked to the body) jets. In distinction to these results in Vorotnikov (1999c) the domain of admissible controls is bounded by an ellipsoid and in Vorotnikov (2001) by an sphere.

At last, the method which is based on the ASC-principle have been modified in Vorotnikov (2000) for the case when the solid have uncertain parameters (the principal central moments of inertia).

The approach to nonlinear game problem of reorientation seems to be constructive because the strict solving of nonlinear game problems presents considerable difficulties. The control obtained are robust. They ensure correct reorientation of a body (by one spatial turn) in *a finite* time. In connection with mention should be made of the problem of the motion control of the airplane subjected to interferences: windshear and others (Miele, *et al*., 1986; Botkin, *et al.,* 1989; Leitmann and Pandey, 1991; Bulirsch, *et al*., 1991).

In this paper this approach is further developed in following directions:

1) the method which is based on the ASC-principle is simplified and a estimate for reorientation time is obtained;

2) the case of a relay controls (*bang-bang* controls) is considered;

3) a more common case of solid parameters uncertainty is discussed.

2. STATEMENT OF THE PROBLEM

Consider Euler dynamic equations

$$A_1\dot{x}_1 = (A_2 - A_3)x_2x_3 + u_1 + v_1,$$
$$A_2\dot{x}_2 = (A_3 - A_1)x_1x_3 + u_2 + v_2, \qquad (1)$$
$$A_3\dot{x}_3 = (A_1 - A_2)x_1x_2 + u_3 + v_3,$$

which describe the angular notion of a solid body with respect to its center of mass. Here, x_i are the projection of angular velocity on major central axes of inertia of the body, u_i are the projection of controlling moments on the same axes**,** and A_i are the principal central moments of inertia. Moments v_i characterize the external forces and uncontrolled disturbances. Here and below, $i = \overline{1,3}$; and summation in i from 1 to 3 is assumed. Let us denote by $\mathbf{x}, \mathbf{u}$ and $\mathbf{v}$ the vectors that consist, respectively, of x_i , u_i and v_i .

In addition to (1), let us consider the kinematic equations that determine the orientation of the body in Rodriges-Hamilton variables

$$2\dot{\lambda}_0 = -\sum(x_i\lambda_i),$$
$$2\dot{\lambda}_1 = x_1\lambda_0 + x_3\lambda_2 - x_2\lambda_3,$$
$$2\dot{\lambda}_2 = x_2\lambda_0 + x_1\lambda_3 - x_3\lambda_1, \qquad (2)$$
$$2\dot{\lambda}_3 = x_3\lambda_0 + x_2\lambda_1 - x_1\lambda_2.$$

Variables λ_0 , λ_i that constitute the vector $\boldsymbol{\lambda}$ obey the equation

$$\lambda_0^2 + \sum\lambda_i^2 = 1 . \qquad (3)$$

Controls $\mathbf{u} \in K$ are selected within the class K of piecewise continuous functions $\mathbf{u} = \mathbf{u}(\mathbf{x}, \boldsymbol{\lambda}; \mathbf{x}^0, \boldsymbol{\lambda}^0)$ ($\mathbf{x}^0, \boldsymbol{\lambda}^0$ – initial states) with constraints

$$|u_i| \le \alpha_i = \text{const} > 0. \qquad (4)$$

Disturbances $\mathbf{v} \in K_1$ can be realized as arbitrary piecewise continuous functions $\mathbf{v} = \mathbf{v}[t]$ under the constraints

$$|v_i| \le \beta_i = \text{const} > 0. \qquad (5)$$

Problem 1. *Find the controls* $\mathbf{u} \in K$ *that for arbitrary* $\mathbf{v} \in K_1$ *transfer the body in finite time from initial state* $\boldsymbol{\lambda}(t_0) = \boldsymbol{\lambda}^0$ *to a prescribed* $\boldsymbol{\lambda}(t_1) = \boldsymbol{\lambda}^1$ *one. Both of these are rest states:*

$$\mathbf{x}(t_0) = \mathbf{x}^0 = \mathbf{x}(t_1) = \mathbf{x}^1 = \mathbf{0} .$$

The time moment $t_1 > t_0$ is not fixed. Without losing the generality, we assume $\boldsymbol{\lambda}^1 = (1,0,0,0)$

3. THE FIRST METHOD FOR SOLVING PROBLEM 1

Controls

$$u_1 = 2A_1\lambda_0^{-1}[(\lambda_0^2 + \lambda_1^2)u_1^* +$$
$$(\lambda_1\lambda_2 + \lambda_0\lambda_3)u_2^* + (\lambda_1\lambda_3 - \lambda_0\lambda_2)u_3^*] + \qquad (6)$$
$$+\frac{1}{2}A_1\lambda_1\lambda_0^{-1}\sum x_i^2 - M_1,$$
$$M_1 = (A_2 - A_3)x_2x_3 \quad (123)$$

are implied in Vorotnikov (1994b, 1996, 1998) for solving Problem 1. (Only one is explicitly written; the rest are obtained by a cyclic permutation of indices 1→ 2→ 3.)

As the result, a linear conflict-controlled system

$$\ddot{\lambda}_i = u_i^* + v_i^* \qquad (7)$$

can be constructed where

$$v_1^* = \frac{1}{2}\left(\lambda_0 v_1 A_1^{-1} + \lambda_2 v_3 A_3^{-1} - \lambda_3 v_2 A_2^{-1}\right) \qquad (8)$$
$$(123).$$

Let us interpret u_i^* and v_i^* as auxiliary controls u_i^* and disturbances v_i^* , respectively.

We propose using the game theory solutions of the linear system (7) as a basis for constructing a solution of original nonlinear Problem 1. Parameters of the form, i.e., auxiliary control u_i^*, are determined from solutions of corresponding game theory problems.

Using the inequalities Cauchy-Schwarz, under (8) we obtain following expressions

$$|\, v_1^*[t]\,| \le \beta_1^* = \frac{1}{2}\left[\sum \left(\beta_i A_i^{-1}\right)^2\right]^{1/2} \ (123). \qquad (9)$$

Let us solve the problem of fastest possible transfer of the system (7) to position

$$\lambda_i = \dot{\lambda}_i = 0 \qquad (10)$$

by means of auxiliary controls u_i^*, and under arbitrary admissible v_i^*.

We will interpret this problem as *differential game*, in which a player controls u_i^* and tries to decrease the transition time τ_i, while the "opponent" controls v_i^* and attempts to increase τ_i

For the problem to be solvable, the admissible levels of u_i^* must exceed the levels of v_i^*. We introduce the corresponding constraints as

$$|\, u_i^*\,| \le \alpha_i^*, \ |\, v_i^*\,| \le \beta_i^* = \rho_i \alpha_i^*, \quad 0 < \rho_i < 1\,. \qquad (11)$$

The procedure by which the level α_i^* are assigned is considered below. Here, we assume the levels fixed, so that conditions (11) are net.

The game theory problem for (7) under constraints (11) reduces (Krasovskii, 1970) to the fastest response problem for systems

$$\ddot{\lambda}_i = (1-\rho_i)\, u_i^*, \qquad |\, u_i^*\,| \le \alpha_i^*. \qquad (12)$$

The boundary conditions are the same as for (7). System (12) result from (7) under condition $v_i^* = -\rho_i u_i^*$, which are the "worst" v_i^* - optimal control strategy of opponent.

The solution of fastest response problem for systems (12) is well-known. Thereby the value

$$\tau = \max(\tau_i), \quad \tau_i = 2\left[\,|\,\lambda_i^0\,|\left(\alpha_i^* - \beta_i^*\right)^{-1}\right]^{1/2},$$

determines the guaranteed time τ, after which the position (10) can be reached. If $v_i^* \ne -\rho_i u_i^*$, the transition time to (10) does not exceed τ and a *sliding* regimes of motions along the switching curves for u_i^* take place.

Algorithm 1 for solving Problem 1.

1) Selecting the form (6) for controls u_i.

2) Estimation of v_i^*; see (9).

3) The preliminary choice of α_i^*. This predetermines the values τ_i.

4) Verification of the constraints (4) on u_i, along the trajectories of linear systems (7). In this case we having in mind the relations

$$x_1 = 2\lambda_0^{-1}[\left(\lambda_0^2 + \lambda_1^2\right)\dot{\lambda}_1 + \left(\lambda_1\lambda_2 + \lambda_0\lambda_3\right)\dot{\lambda}_2 + \\ + \left(\lambda_1\lambda_3 - \lambda_0\lambda_2\right)\dot{\lambda}_3] \qquad (123). \qquad (13)$$

If inequalities (4) do not hold or, conversely, there is a wide margin in their validity, we should continue the search fox appropriate τ. Otherwise, it is the choice of τ that determines the guaranteed response time. As result, we obtain an iterative algorithm for Problem 1.

Let us find the restriction which must be imposed on α_i, β_i and λ_0^0, to ensure the possibility of solving Problem 1 by means of controls (6). Introduce the notations

$$\Gamma = \min(\Gamma_i), \quad \Gamma_i = \left[1 + \left(\lambda_0^0\right)^{-2}\left(\lambda_i^0\right)^2\right]^{-1/2} \alpha_i A_i^{-1}.$$

Theorem 1 (Vorotnikov, 1999b). *Let the admissible levels* β_i *of disturbances* v_i *be evaluated by the inequality*

$$\left[\sum \left(\beta_i A_i^{-1}\right)^2\right]^{1/2} < \frac{\sqrt{3}}{3}\Gamma. \qquad (14)$$

Then Problem 1 *can be solved by means of controls* (6) *satisfying restriction* (4) *given.*

4. THE SECOND METHOD FOR SOLVING PROBLEM 1(WITH THE USE OF THE ASC PRINCIPLE)

The above-proposed method have been modified in Vorotnikov (1995, 1997, 1998) toward obtaining controls that are simpler then (8) and, consequently, more convenient in practice. Namely, controls (Vorotnikov, 1995, 1997, 1998)

$$u_1 = 2A_1\lambda_0^{-1}[\left(\lambda_0^2 + \lambda_1^2\right)u_1^* + \left(\lambda_1\lambda_2 + \lambda_0\lambda_3\right)u_2^* + \\ + \left(\lambda_1\lambda_3 - \lambda_0\lambda_2\right)u_3^*] \qquad (123) \qquad (15)$$

are employed for solving Problem 1 as well. In this case, auxiliary linear system (7), in which v_i^* have the form

$$v_1^* = \varphi_1 = \frac{1}{2}[\lambda_0(v_1 + M_1)A_1^{-1} +$$
$$+ \lambda_2(v_3 + M_3)A_3^{-1} - \lambda_3(v_2 + M_2)A_2^{-1}] - \qquad (16)$$
$$-\frac{1}{4}\lambda_1 \sum x_i^2 \quad (123),$$
is extracted from closed system (1)-(3) and (15).

Direct estimations of the form (16) present difficulties. However, it is possible to obtain the solution to Problem 1 by the following algorithm.

Algorithm 2 *for solving Problem* 1.
1) Selecting the form (15) for controls u_i.
2) Preliminary choice of guaranteed reorientation time τ and assignment of levels β_i^*. This predetermines the values α_i^*, ρ_i. With control time equalized in each λ_i so that $\tau_i = \tau$ we obtain $\alpha_i^* = \beta_i^* + 4 | \lambda_i^0 | \tau^{-2}$.
3) Checking the validity of inequalities $| v_i^* | \le \beta_i^*$ in the state set of systems (7).
4) Verification of the constraints (4) on u_i along the trajectories of linear systems (7).

If inequalities (4) do not hold or, conversely, there is a wide margin in their validity, we should continue the search for appropriate τ and β_i^*. Otherwise, it is the choice of τ that determines the guaranteed response time.

Theorem 2 (Vorotnikov, 1999b). *Let the admissible levels* β_i *of disturbances* v_i *be evaluated by the inequality* (14). *Then Problem* 1 *can be solved by means of controls* (15) *satisfying restriction* (4) *given.*

5. THE CASE, WHICH DOMAIN OF ADMISSIBLE CONTROLS IS BOUNDED BY AN ELLIPSOID

In this subsection, in distinction (4),(5) , we suppose

$$\|\mathbf{u}\|^* \le \alpha, \ \|\mathbf{v}\|^* \le \beta \ \ (\alpha, \beta = \text{const} > 0), \qquad (17)$$

where

$$\|\mathbf{u}\|^* = \left[\sum \left(u_i A_i^{-1}\right)^2\right]^{1/2}, \ \ \|\mathbf{v}\|^* = \left[\sum \left(u_i A_i^{-1}\right)^2\right]^{1/2}.$$

Theorem 3 (Vorotnikov, 1999c). *Let the admissible domain of disturbances* v_i *be evaluated by the inequality*

$$\beta < \frac{\sqrt{3}}{3} | \lambda_0^0 | \alpha. \qquad (18)$$

Then Problem 1 *can be solved by means of controls* (6) *or* (15) *satisfying restriction* $\|\mathbf{u}\|^* \le \alpha$ *given.*

Let us note that it is possible restrictions of the type (17) to replace a restrictions in the form (Vorotnikov, 2001)

$$\|\mathbf{u}\| \le a, \ \|\mathbf{v}\| \le b \ \ (a, b = \text{const} > 0), \qquad (19)$$

where

$$\|\mathbf{u}\| = \left[\sum u_i^2\right]^{1/2}, \quad \|\mathbf{v}\| = \left[\sum v_i^2\right]^{1/2}.$$

In this case we have

$$b < \frac{\sqrt{3}}{3} \Theta | \lambda_0^0 | a,$$
$$\Theta = A_*\left(A^*\right)^{-1}, \ \ A^* = \max(A_i), \quad A_* = \max(A_i).$$

6. DISCUSSION OF THE PROPOSED METHODS

1. The construction (6) and (15) is just of possible constructions

$$u_j = \frac{1}{\lambda_j} f_i^{(j)}\left(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{u}^*\right), \ j = \overline{0,3}, \qquad (20)$$

which enable one to select auxiliary linear control systems of the type (7) from the closed system (1)-(3), (20) for a specific choice of $f_i^{(j)}$. The set of indices in (7) depends on the index of λ in the denominator in (20). The index $j = 0$ corresponds to $i = \overline{1,3}$, $\ j = 1 \to i = 0,2,3$, e.t.c. A similar "arsenal" of techniques based on (20) enables one, in principle, to solve the reorientation problems for any boundary conditions. Taking this into account, for the case $\boldsymbol{\lambda}^1 = (1,0,0,0)$ under consideration we can, without loss of generality, assume that $\left|\lambda_0^0\right| \ge 1/2$.

2. Structure (6) and (15) of controls contains the factor λ_0^{-1}, that within a formal approach gives rise to a "singularity". However, in the course of control, $|\lambda_0| \in [| \lambda_0^0 |, 1]$. As a consequence, the singularity mentioned above does not arise.

7. A MODIFICATION OF THE SECOND METHOD OF SOLVING FOR PROBLEM 1

Conditions (14) and (18) guarantees the solution of Problem 1 by using the controls (6) and (15) for sufficiently large (although, finite) value of τ. Assume, for example, the condition (18) holds with a reserve, i.e.,

$$\beta = \frac{\sqrt{3}}{3} | \lambda_0^0 | \alpha - \Delta, \qquad (21)$$

where $\Delta > 0$ is certain number.

In this case the method of solving for Problem 1, which is based on the ASC-principle, can be simplified.

Algorithm 3 for solving Problem 1.
1) Selecting the from (15) for controls u_i.
2) The choice α_i^* in the form

$$\alpha_i^* = \alpha^* = \frac{\sqrt{3}}{6} |\lambda_0^0| \alpha ,$$

which guarantee, that restriction $\|\mathbf{u}\|^* \le \alpha$ will be fulfilled.
3) Preliminary choice of guaranteed reorientation time $\tau = \tau_i$, which predetermines the values β_i^*.
4) Checking the validity of inequalities $\left|v_i^*\right| \le \beta_i^*$ in the state set of systems (7) for v_i^* of the form (16).

If inequalities $\left|v_i^*\right| \le \beta_i^*$ do not hold or, conversely, there is a wide margin in their validity, we should continue the search for appropriate τ. Otherwise, it is the choice of τ that determines the guaranteed response time.

It is possible to obtain a estimation for value of τ.

Theorem 4. *The estimation of the τ have the form*

$$\tau \le \tau^* = 2\lambda^* \sqrt{2\Delta^{-1}} ,$$

where

$$\lambda^* = \left\{ \max\left|\lambda_i^0\right| + 2L\left[\left(\lambda_0^0\right)^{-2} - 1 \right] \right\}^{1/2} ,$$

$$L = (1 + \sum r_i^2)^{1/2}, \quad r_1 = (A_1 - A_3)A_2^{-1} \quad (123).$$

8. BANG-BANG CONTROLS

In distinction to subsections 3-7, let us obtain the solving for Problem 1 by means of a relay controls u_i (*bang-bang* controls).

In this case controls

$$u_i = 2A_i u_i^* \qquad (22)$$

are considered. A linear conflict-controlled system of the type (7) can be constructed, if

$$v_1^* = \varphi_1 + \tfrac{1}{2}[(\lambda_0 - 1)\, u_1 A_1^{-1} + \lambda_2 u_3 A_3^{-1} - \lambda_3 u_2 A_2^{-1}] \qquad (123),$$

and ASC-principle is used. Let restrictions on u_i, and v_i, have the form (4), (5).

$$\Gamma^* = \min\left(\Gamma_i^*\right), \quad \Gamma_i^* = \alpha_i A_i^{-1}.$$

Theorem 5. *Let the admissible domain of disturbances v_i be evaluated by the inequality*

$$\left[\sum \left(\beta_i A_i^{-1}\right)^2 \right]^{1/2} < (1 - \sqrt{3}\lambda_*)\Gamma^* ,$$

where

$$\lambda_* = \frac{1}{2}\sqrt{2\left(1 - \left|\lambda_0^0\right|\right) - \min\left(\lambda_i^0\right)^2} .$$

Then Problem 1 *can be solved by means of controls* (22) *satisfying restriction* (4) *given.*

The construction (6) and (15) is just of possible constructions

$$u_i = 2A_j u_j^*, \quad i,j = \overline{1,3}$$

which enable one, without loss of generality, assume that $\left|\lambda_0^0\right| \ge 1/2$ in the case $\boldsymbol{\lambda}^1 = (1,0,0,0)$ under consideration.

In more details these results see in Vorotnikov (submitted for publication).

9. REORIENTATION UNDER UNCERTAIN PARAMETERS OF SOLID

Let the principal central moments of inertia A_i are *constants*, but vary within the ranges

$$A_i^- \le A_i \le A_i^+ \qquad (23)$$

where A_i^-, A_i^+ a given number. The major central axes of inertia are *the same* in any cases (23).

Suppose following constraints under u_i and v_i

$$\|\mathbf{u}\|^{**} \le \alpha, \ \|\mathbf{v}\|^{**} \le \beta \quad (\alpha, \beta = \text{const} > 0),$$

where

$$\|\mathbf{u}\|^{**} = \left[\sum \left(u_i \left(A^-\right)^{-1} \right)^2 \right]^{1/2} ,$$

$$\|\mathbf{v}\|^{**} = \left[\sum \left(v_i \left(A^-\right)^{-1} \right)^2 \right]^{1/2} ,$$

Under (23) we have $A_i = A_i^- + \delta_i A_i^-$, where $\delta_i \in \left[0, \delta_i^+\right]$, $\delta_i^+ = \left(A_i^+\right)\left(A_i^-\right)^{-1} - 1$. Introduce the notation $\delta^* = \max \delta_i \left(1 + \delta_i\right)^{-1}$ and consider the controls

$$u_1 = 2A_1^- \lambda_0^{-1}[\left(\lambda_0^2 + \lambda_1^2\right)u_1^* + (\lambda_1\lambda_2 + \lambda_0\lambda_3)u_2^* + (\lambda_1\lambda_3 - \lambda_0\lambda_2)u_3^*] \qquad (24)$$

(123).

Theorem 6 (Vorotnikov, 2000). *Let the admissible domain of disturbances* v_i *be evaluated by the inequality*

$$\beta < \left(\frac{\sqrt{3}}{3} | \lambda_0^0 | - \delta^* \right) \alpha. \qquad (25)$$

Then Problem 1 *can be solved by means of controls* (24) *satisfying restriction* $\|\mathbf{u}\|^{**} \leq \alpha$ *given*.

Inequality (25) is applicable if $\sqrt{3}\left|\lambda_0^0\right| - 3\delta^* > 0$. Without any loss of generality, we can assume that $\left|\lambda_0^0\right| \geq 1/2$; hence, the values of δ_i vary within the range $0 \leq \delta_i < \sqrt{3}\left(6 - \sqrt{3}\right)^{-1} = 0{,}4051$.

A more common case of parameters uncertainty (when not only the moments of inertia but the principal central axes of inertia) are inaccurately specified, see in Vorotnikov and Rumyantsev (2001), Vorotnikov (2002).

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