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NON-LINEAR OBSERVER FOR BIOPROCESSES

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Abstract
In this paper a new non-linear observer is designed for on-line estimation of the broth composition in the bioreactors. The design is based on asymptotic observer. An additional correction is done to asymptotic estimation, taking into account the structure of the reaction rate equations. Two examples of biotechnological processes are used to show the performance of the proposed observer.

Keywords: bioreactors, non-linear observers

1. INTRODUCTION

Because of the lack of reliable and cheap sensors, the observers for state variables estimation are used in the on-line measurement of broth composition in bioreactors. Some studies have been reported in the literature in which different approaches had been considered (Stephanopoulos and San, 1984; Staniškis and Simutis, 1986; Farza *et al.*, 1998; Lombardi *et al.*, 1999; Gudi *et al.*, 1997; Keesman, 2000).

Asymptotic observer (Bastin and Dochain, 1990) is a special algorithm, because it is not depending on the reaction rates. A consequence is that the convergence speed of the estimation is completely determined by the experimental condition, through the value of the dilution rate. This feature restricts the application of this approach to fed-batch processes, due to dilution rate decreases as the volume rises. Besides, low inlet flow is used for these processes, because the volume is limited to the vessel of the bioreactor.

In practice, the information about the reaction kinetics is usually available, thus a priori knowledge can be used to improve the behaviour of the asymptotic observers. In this paper, a new non-linear observer for bioreactor processes is presented, which takes into account the reaction rate equations.

The paper is structured as follows. The section 2 introduce the asymptotic observers and basic notations. In the section 3 the new observer is described, and some aspects about the convergence are pointed out. Two case studies are presented in the section 4. Finally, section 5 states the conclusions derived from this work.

2. ASYMPTOTIC OBSERVER

In (Bastin and Dochain, 1990), a universal model for bioreactor dynamics in terms of reaction rates is expressed by:

$$\dot{\xi} = K\Phi(\xi) - D\xi + D\xi_i \qquad (1)$$

where:

- ξ: component concentrations in the bioreactor (state variables).
- $K_{N\times M}$: yield coefficient matrix. The value k_{ij} is the yield coefficient of the component i in the reaction j. If the component is reactant, then the coefficient is negative and

if the component is a product the coefficient is positive. N is the number of components and M is the number of reactions.
- $\Phi(\xi)_{M\times 1}$: Vector of the reaction rates. The component Φ_j is the rate of the reaction j.
- ξ_i: inlet component concentrations.
- $D = F/V$, dilution rate, F inlet flow rate, V volume.

The asymptotic observer design is based on the sequel property of the model 1:

If $p = rank(K)$, then equation 1 can be decoupled in two subsystems:

$$\begin{aligned}\dot{\xi}_a &= K_a\Phi(\xi_a,\xi_b) - D\cdot\xi_a + D\xi_{i_a}\\ \dot{\xi}_b &= K_b\Phi(\xi_a,\xi_b) - D\cdot\xi_b + D\xi_{i_b}\end{aligned} \tag{2}$$

where $K_{a_{(p\times M)}}$ is a full rank submatrix of K; K_b holds the remaining coefficients of K, and the vector variables are induced by the decomposition. Then, a transformation can be defined by

$$Z = A_0\xi_a + \xi_b \tag{3}$$

where $A_{0_{(N-p)\times p}}$ is the solution of the matrix equation:

$$A_0K_a + K_b = 0 \quad\Rightarrow\quad A_0 = -K_bK_a^{-1} \tag{4}$$

and the model 1 is equivalent to

$$\begin{aligned}\dot{\xi}_a &= K_a\Phi(\xi_a,\xi_b) - D\xi_a + D\xi_{ia}\\ \dot{Z} &= -DZ + A_0(D\xi_{ia}) + (D\xi_{ib})\end{aligned} \tag{5}$$

It can be noted that the dynamics of Z are independent of the reaction rates Φ. This allows the construction of the asymptotic observer.

If the number of measured variables ξ_1 is enough $q = dim(\xi_1) \geq p$, Z can be obtained as a linear combination of the measured and unmeasured variables:

$$Z = A_1\xi_1 + A_2\xi_2 \tag{6}$$

Then, the unmeasured variables ξ_2 can be observed asymptotically by

$$\begin{aligned}\dot{\hat{Z}} &= -D\hat{Z} + A_0(D\xi_{ia}) + (D\xi_{ib})\\ \hat{\xi}_{2_A} &= {A_2}^{-1}(\hat{Z} - A_1\xi_1)\end{aligned} \tag{7}$$

The asymptotic observer can be used if A_2 is invertible. It can be proved that the error dynamics are driven by

$$\dot{e}_{_A} = -De_{_A} \tag{8}$$

Equation 8 shows that the observer converges asymptotically if $D > 0$.

3. NEW NON-LINEAR OBSERVER FOR THE BIOPROCESSES

In the asymptotic observer, no information about the reaction rates is used. This guarantees convergence, but the speed depends on D, that is small usually. The information about the reaction rates can be used if an additional term is added to the equation of the asymptotic estimation. Suppose that $\overline{\xi}_2$ is the subset of the components of ξ_2 that appears explicitly in the reaction rate equations, then a further correction can be done to the asymptotic estimations of $\overline{\xi}_2$ as follows.

$$\hat{\overline{\xi}}_2 = \hat{\overline{\xi}}_{2_A} + \gamma(\xi_1 - \hat{\xi}_1) \tag{9}$$

where $\hat{\overline{\xi}}_{2_A}$ is the asymptotic estimation by 7; γ is a design matrix; ξ_1 is the measured variable and $\hat{\xi}_1$ is an estimation of the measured variable by the following equation

$$\dot{\hat{\xi}}_1 = K_1\Phi(\xi_1,\hat{\overline{\xi}}_2) - D\hat{\xi}_1 + D\xi_{i_1} \tag{10}$$

where K_1 holds rows of K corresponding with ξ_1.

3.1 *Convergence analysis*

The estimation error is

$$\begin{aligned}e = \overline{\xi}_2 - \hat{\overline{\xi}}_2 &= \overline{\xi}_2 - \hat{\overline{\xi}}_{2_A} - \gamma(\xi_1 - \hat{\xi}_1)\\ &= e_{_A} - \gamma(\xi_1 - \hat{\xi}_1)\end{aligned} \tag{11}$$

where $e_{_A}$ is the error of the asymptotic observer.

If equation 11 is derived

$$\dot{e} = \dot{e}_{_A} - \gamma(\dot{\xi}_1 - \dot{\hat{\xi}}_1) \tag{12}$$

Using 8, 10 and 12

$$\begin{aligned}\dot{e} = -De_{_A} - \gamma D(\xi_1 - \hat{\xi}_1) -\\ \gamma K_1\left(\Phi(\xi_1,\overline{\xi}_2) - \Phi(\xi_1,\hat{\overline{\xi}}_2)\right)\end{aligned} \tag{13}$$

$$\dot{e} = -De - \gamma K_1\left(\Phi(\xi_1,\overline{\xi}_2) - \Phi(\xi_1,\hat{\overline{\xi}}_2)\right) \tag{14}$$

Assuming that the functions in $\Phi(\xi_1,\overline{\xi}_2)$ are continuously differentiable with respect to $\overline{\xi}_2$, by the mean value theorem,

$$\Phi(\xi_1,\overline{\xi}_2) - \Phi(\xi_1,\hat{\overline{\xi}}_2) = \Psi(\xi_1,{\overline{\xi}_2}^*)\left(\overline{\xi}_2 - \hat{\overline{\xi}}_2\right) \tag{15}$$

for some ${\overline{\xi}_2}^*$ which lie between the segment with end points $\overline{\xi}_2$ and $\hat{\overline{\xi}}_2$, where

$$\Psi(\xi_1, \overline{\xi}_2{}^*) = \left.\frac{d\Phi(\xi_1, \overline{\xi}_2)}{d\overline{\xi}_2}\right|_{(\xi_1, \overline{\xi}_2{}^*)} \tag{16}$$

Then

$$\dot{e} = -\left(DI + \gamma K_1 \Psi(\xi_1, \overline{\xi}_2{}^*)\right) e \tag{17}$$

In the case of the asymptotic observer, the convergence rate is only determined by the dilution rate, equation 8. However, in 17 there is an extra term that allows a higher convergence rate if γ is properly designed. This term is related to the reaction kinetics as will be shown in the case studies.

3.1.1. *γ selection*

It's clear that

$$\lim_{\hat{\overline{\xi}}_2 \to \overline{\xi}_2} \Psi(\xi_1, \overline{\xi}_2{}^*) = \left.\frac{d\Phi(\xi_1, \overline{\xi}_2)}{d\overline{\xi}_2}\right|_{(\xi_1, \hat{\overline{\xi}}_2)} \tag{18}$$

So if $\frac{d\Phi(\xi_1, \overline{\xi}_2)}{d\overline{\xi}_2}$ is nonsingular, then the design matrix can be selected as

$$\gamma = \Gamma \left(\frac{d\Phi(\xi_1, \overline{\xi}_2)}{d\overline{\xi}_2}\right)^{-1}\Bigg|_{(\xi_1, \hat{\overline{\xi}}_2)} \mathcal{K} \tag{19}$$

If $\mathcal{K}$ satisfies: $\mathcal{K}K_1 = I$; then using equation 19 in equation 17

$$\begin{aligned} &DI + \gamma K_1 \Psi(\xi_1, \overline{\xi}_2{}^*) \\ &= DI + \Gamma \left(\frac{d\Phi(\xi_1, \overline{\xi}_2)}{d\overline{\xi}_2}\right)^{-1}\Bigg|_{(\xi_1, \hat{\overline{\xi}}_2)} \Psi(\xi_1, \overline{\xi}_2{}^*) \end{aligned} \tag{20}$$

taking limits like in 18

$$DI + \gamma K_1 \Psi(\xi_1, \overline{\xi}_2{}^*) = DI + \Gamma \tag{21}$$

Therefore it's proved that if Γ is selected in suitable form, then the observer has an attractor region in the vicinity of $\hat{\overline{\xi}}_2 = \overline{\xi}_2$ for this γ.

4. CASE STUDIES

Two computer simulation examples of biotecnological processes are presented in this section to illustrate the performance of the observer. The proposed and asymptotic observers are compared in these applications. Low values of the dilution rate are considered in order to check the properties of both algorithms under critical condition for the convergence for the asymptotic observer.

4.1 *Case I. Simple microbial growth process.*

This is the simplest biological process, involving growth of micro-organisms on a simple limiting substrate. The dynamic model of the concentrations of the components is:

$$\begin{bmatrix} \dot{x} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} 1 \\ -k_1 \end{bmatrix} \frac{\mu_{max} s x}{\kappa + s} - D \begin{bmatrix} x \\ s \end{bmatrix} + D \begin{bmatrix} 0 \\ s_i \end{bmatrix} \tag{22}$$

where x is the concentration of the micro-organisms and s is the substrate concentration. The reaction kinetic is assumed to be described by Monod law; μ_{max} is the maximum growth rate and κ is the Michaelis-Menten constant.

If the biomass concentration is measured, then an asymptotic observer for substrate estimation can be used:

$$\begin{aligned} \dot{z} &= -Dz + Ds_i \\ \hat{s}_a &= z - k_1 x \end{aligned} \tag{23}$$

The difference $\Phi(\xi_1, \xi_2) - \Phi(\xi_1, \hat{\xi}_2)$ in this case is

$$\Phi(x, s) - \Phi(x, \hat{s}) = \left(\frac{\mu_{max} x \kappa}{(\kappa + s)(\kappa + \hat{s})}\right)(s - \hat{s}) \tag{24}$$

so, the dynamic equation of the estimation error is as follows

$$\dot{e} = -\left(D + \gamma \frac{\mu_{max} x \kappa}{(\kappa + s)(\kappa + \hat{s})}\right) e \tag{25}$$

For convergence

$$\left(D + \gamma \frac{\mu_{max} x \kappa}{(\kappa + s)(\kappa + \hat{s})}\right) > 0 \tag{26}$$

Therefore, a sufficient condition is $\gamma \geq 0$.

The estimation error for different values of the parameter γ are presented in the figure 1. Higher convergence rates are attained as γ rises. If $\gamma = 0$ the behaviour of the new observer is like that of the asymptotic observer.

For the comparative study the following conditions were considered. The inlet flow was a ramp function with slope 0.001 $g/l^{-1}h^{-1}$, and the duration of the experiment was 100 h. Therefore, the maximal dilution rate was 0.007 h^{-1}. The inlet flow substrate concentration was 80 gl^{-1}. Model parameters: $\mu_{max} = 0.22$; $\kappa = 0.14$; $k_1 = 1.43$. The behaviours of the observers are compared in the figures 2 and 3. It can be seen that the new observer converge more quickly than asymptotic one if $\gamma > 0$.

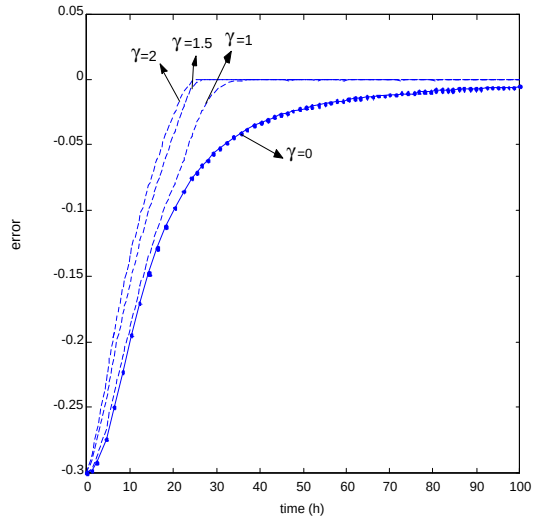

Figure 1. Estimation errors for different values of γ; dotted line: asymptotic observer; dashed line: new observer.

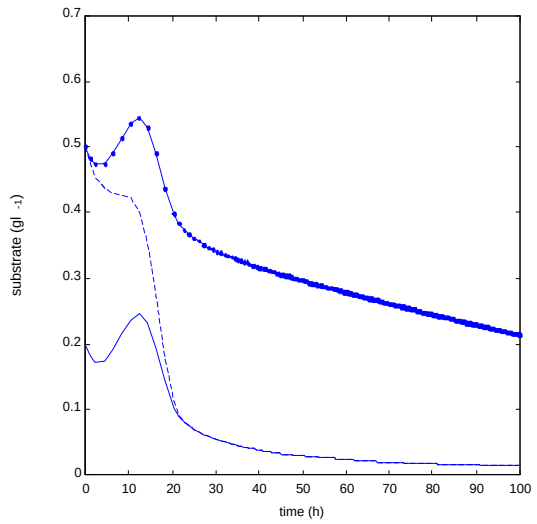

Figure 2. Estimation of the substrate concentration. Solid line: actual substrate concentration; dotted line: asymptotic observer; dashed line: new observer.

4.2 *Case II. Anaerobic digestion process*

Anaerobic digestion is a process for the biological treatment for organic wastes. Real process consists of nine components and four reactions. Under some assumptions the model is reduced to four components and two reactions. For details see (Bastin and Dochain, 1990). The reduced model is

$$\begin{bmatrix} \dot{s_1} \\ \dot{x_1} \\ \dot{s_2} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ 1 & 0 \\ k_3 & -k_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\mu max_1 s_1 x_1}{\kappa_1 + s_1} \\ \frac{\mu max_2 s_2 x_2}{\kappa_2 + s_2} \end{bmatrix} - D \begin{bmatrix} s_1 \\ x_1 \\ s_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} s_{1i} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

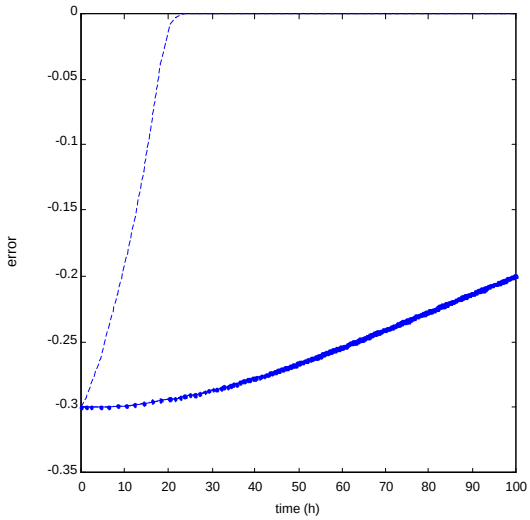

Figure 3. Estimation error; dotted line: asymptotic observer; dashed line: new observer.

where: x_1: acidogenic bacteria concentration; x_2: acetoclastic methanogenic bacteria concentration; s_1: glucose concentration; s_2: acetate concentration. Furthermore, Monod law is considered for both reactions.

Suppose that there are measurements available for x_1 and s_2. This is not a realistic assumption for x_1, but this situation is considered here in order to show the capability of the new observer on the estimation of substrate and biomass concentrations. The states s_1 and x_2 can be asymptotically estimated by means of the following algorithm

$$\begin{bmatrix} \dot{z_1} \\ \dot{z_2} \end{bmatrix} = -D \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + D \begin{bmatrix} s_{1i} \\ 0 \end{bmatrix} \tag{28}$$

$$\begin{aligned} \hat{s}_1 &= z_1 - k_1 x_1 \\ \hat{x}_2 &= \frac{z_2 + k_3 x_1 - s_2}{k_2} \end{aligned} \tag{29}$$

Since both x_1 and s_2 appears in the reaction rate equations ($\overline{\xi}_2 = \xi_2$), it's possible to make a further correction for their asymptotic estimation. The tuning parameter γ is selected according to the section 3.1.1.

For this application

$$K_1 = \begin{bmatrix} 1 & o \\ k_3 & -k_2 \end{bmatrix} \Rightarrow \mathcal{K} = K_1^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{k_3}{k_2} & -\frac{1}{k_2} \end{bmatrix} \tag{30}$$

and

$$\left(\frac{d\Phi}{d\xi_2} \right)^{-1} = \begin{bmatrix} \frac{(\kappa_1 + s_1)^2}{\mu max_1 x_1 \kappa_1} & 0 \\ 0 & \frac{\kappa_2 + s_2}{\mu max_2 s_2} \end{bmatrix} \tag{31}$$

Then

$$\gamma = \Gamma \left[\begin{array}{cc} \frac{(\kappa_1+s_1)^2}{\mu_{max_1} x_1 \kappa_1} & 0 \\ \frac{k_3}{k_2}\frac{\kappa_2+s_2}{\mu_{max_2} s_2} & -\frac{1}{k_2}\frac{\kappa_2+s_2}{\mu_{max_2} s_2} \end{array} \right] \Bigg|_{(x_1, s_2, \hat{s}_1)} \quad (32)$$

In this case, as in *case I*, is possible to obtain $\Psi(\cdot)$ by direct factorization of the difference

$$\Phi(\xi_1, \xi_2) - \Phi(\xi_1, \hat{\xi}_2) = \left[\begin{array}{cc} \frac{\mu_{max_1} x_1 \kappa_1}{(\kappa_1+s_1)(\kappa_1+\hat{s}_1)} & 0 \\ 0 & \frac{\mu_{max_2} s_2}{\kappa_2+s_2} \end{array} \right] \left[\begin{array}{c} s_1 - \hat{s}_1 \\ x_2 - \hat{x}_2 \end{array} \right] \quad (33)$$

It's easy to prove that the dynamic equations of the estimation errors are

$$\left[\begin{array}{c} \dot{e}_{s_1} \\ \dot{e}_{x_2} \end{array} \right] = \left[\begin{array}{cc} -\left(D + \Gamma_1 \frac{(\kappa_1+s_1)^2}{(\kappa_1+s_1)(\kappa_1+\hat{s}_1)} \right) & 0 \\ 0 & -(D+\Gamma_2) \end{array} \right] \left[\begin{array}{c} e_{s_1} \\ e_{x_2} \end{array} \right] \quad (34)$$

where

$$e_{s_1} = s_1 - \hat{s}_1 \quad (35)$$

$$e_{x_2} = x_2 - \hat{x}_2 \quad (36)$$

and Γ_1, Γ_2 are the diagonal coefficients in Γ. Therefore, a sufficient conditions for convergence are: $\Gamma_1 \geq 0$ and $\Gamma_2 \geq 0$.

For the comparative study with asymptotic observer the following conditions were considered. The inlet flow was a ramp function with slope 0.003 $g/l^{-1}h^{-1}$, and the duration of the experiment was 50 h. Therefore, the maximal dilution rate was 0.04 h^{-1}. The inlet flow glucose concentration was 5 gl^{-1}. Model parameters: $\mu_{max_1} = 0.24$; $\kappa_1 = 0.15$; $\mu_{max_2} = 0.5$; $\kappa_2 = 0.3$; $k_1 = 3.2$; $k_2 = 28.3$; $k_3 = 5.7$. The behaviours of the observers are compared in the figures 4 and 5 for different values of Γ_1 y Γ_2.

High values of Γ_1 and Γ_2 lead to faster convergence. Transient oscillations appears in the x_2 estimation for large values in Γ_2.

Some perturbations had been considered on the model in order to show the robustness of the proposed observer: white noise with $Var(\omega) = 0.001$ was added to the state equation. Because of the modeling errors, the asymptotic observer doesn't work properly for the estimation of glucose. For low values on Γ_1 the performance of new observer is like that for asymptotic one. The results are better as Γ_1 rise, figures 6 and 7.

On the other hand, high values in Γ increase the estimation error variance when noises are presented, figures 8 and 9. Therefore, both modeling errors and noises must be considered for Γ selection.

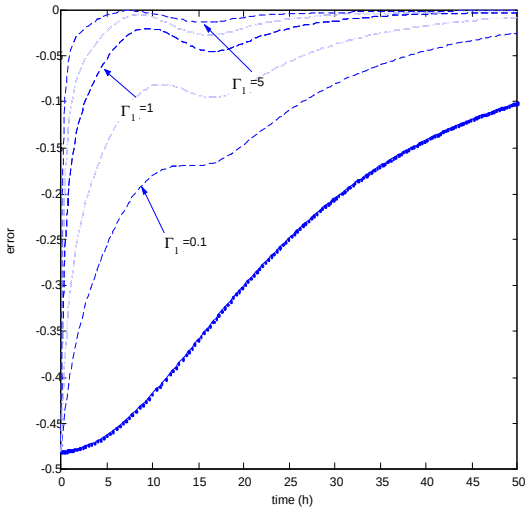

Figure 4. The estimation errors of the glucose concentration for different values of Γ_1; dotted line: asymptotic observer; dashed line: new observer.

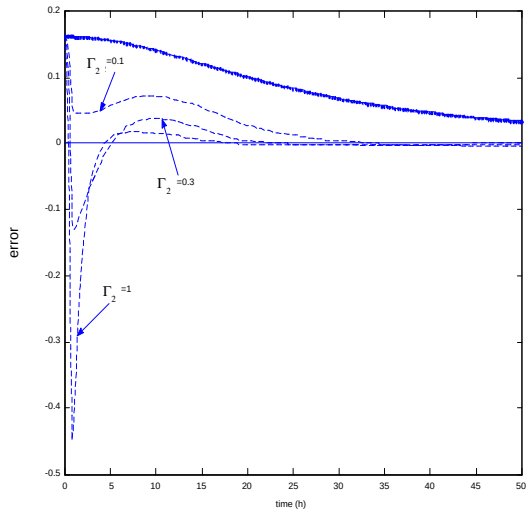

Figure 5. The estimation errors of the acetoclastic methanogenic bacteria concentration for different values of Γ_2; dotted line: asymptotic observer; dashed line: new observer.

5. CONCLUSIONS

In this paper, new non-linear observer is proposed for the on-line measurement in the bioreactors. The asymptotic approach is used in the design of the new non-linear observer. An additional correction is done by taking into account the error between actual and estimated outputs by the model.

Two simulation experiments were carried out for validation. The best performance of the proposed observer is noted from comparison with the asymptotic one. The proposed observer is shown to be robust with regard to modeling errors and noise.

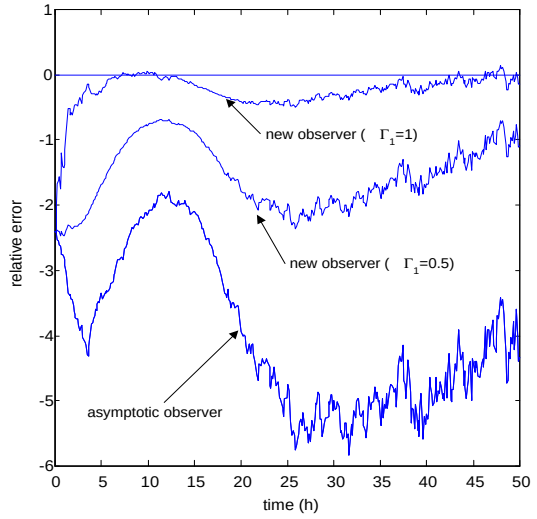

Figure 6. The estimation errors of the glucose concentration by asymptotic observer and new observer dealing with modeling errors.

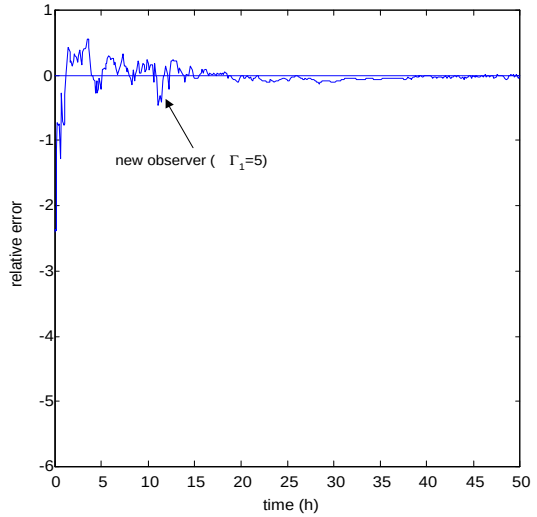

Figure 7. The estimation error of the glucose concentration by new observer dealing with modeling errors.

6. REFERENCES

Bastin, G. and D. Dochain (1990). *On-line estimation and Adaptive Control of Bioreactors.*. Elsevier Science Plublishers.

Farza, M., K. Busawon and H. Hammouri (1998). Simple nonlinear observers for on-line estimation of kinetic rates in bioreactors.. *Automática*.

Gudi, Ravindra D., Sirish L. Shah, Murray R. Gray and P. K. Yegneswaran (1997). Adaptive multirate estimation and control of nutrient levels in a fed-atch fermentation using off-line and on-line measurements.. *The Canadian Journal of Chemical Engineering.* **75**, 562–573.

Keesman, Karel J. (2000). State and parameter estimation in biotechnical batch reactor.. *Proceedings on IFAC System Identification, Santa Barbara, California, USA*.

Lombardi, M., K. Fiaty and P. Laurent (1999). Implementation of observer for on-line estimation of concentration in continuous-stirred membrane bioreactor: Application to the fermentation of lactose. *Chemical Engineering Science* **54**, 2689–2696.

Staniškis, J. and R. Simutis (1986). A measuring system for biotechnical processes based on discrete methods of estimation. *Biotechnology and Bioengineering* **XXVIII**, 362–371.

Stephanopoulos, Gregory and Ka-Yiu San (1984). Studies on on-line bioreactor identification. i. theory.ii numerical and experimental results.. *Biotechnology and Bioengineering* **XXVI**, 1176–1188.

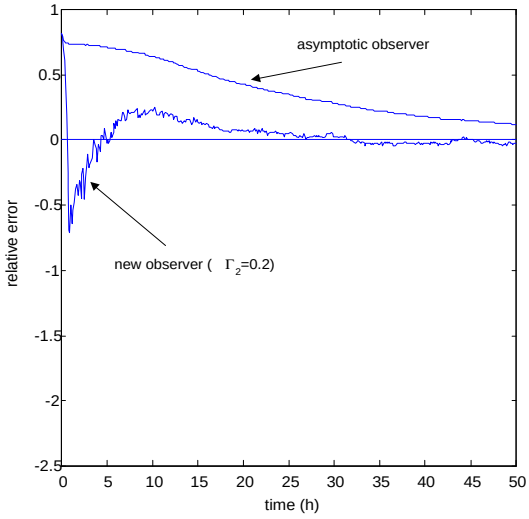

Figure 8. The estimation error of the acetoclastic methanogenic bacteria concentration by asymptotic observer and new observer dealing with modeling errors.

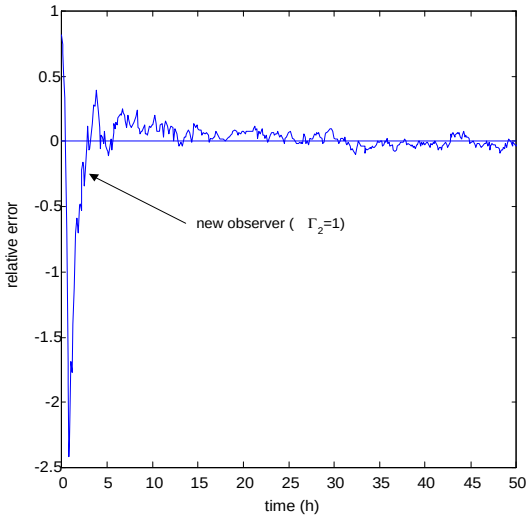

Figure 9. The estimation error of the acetoclastic methanogenic bacteria concentration by new observer dealing with modeling errors.