Copyright © 2002 IFAC
15th Triennial World Congress, Barcelona, Spain

OBSERVER-BASED CONTROLLER FOR INDUCTION MOTORS

J. De Leon-Morales *, **R. Alvarez-Salas** **,[1], **J. M. Dion** **
and **L. Dugard** **

* *University of Nuevo Leon, Department of Electrical Engineering,*
P.O. Box 148-F, 66450 San Nicolas de Los Garza, N. L.
MEXICO
Tel. (52) 83 76 45 14 - Fax: (52) 83 76 45 14
E-mail: jleon@ccr.dsi.uanl.mx
** *Lab. d'Automatique de Grenoble, CNRS/INPG UMR 5528,*
ENSIEG-BP 46, 38402 Saint Martin d'Hères Cedex
FRANCE
Tel. (33) 4 76 82 62 36 - FAX: (33) 4 76 82 63 88
E-mail: Luc.Dugard@inpg.fr

Abstract: This paper deals with the observation and control of a class of nonlinear systems. A cascade observer for a class of state affine nonlinear systems is proposed. Considering an output feedback tracking controller, a stability analysis of the resulting closed-loop system is given. The proposed observed-based controller is then shown to be closed loop stable and is applied to an induction motor industrial setup to show the proposed methodology.

Keywords: Induction motor, nonlinear control, nonlinear observer.

1. INTRODUCTION

The use of induction motors is widespread in industry, due to their reliability, ruggedness, and low cost. However, they are difficult to control for several reasons. They are nonlinear, coupled, multivariable processes. The rotor electrical state variables are usually unavailable for measurement, and the motor parameters can vary considerably from their nominal values, which degrades the control performances.

In this paper, one considers the problem of designing a control input in order to track a desired output reference when the state is not fully measurable. Several solutions have been proposed by using nonlinear techniques to design controls laws, *e.g.* differential geometric approach (Marino *et al.*, 1993), sliding-modes methods (Utkin *et al.*, 1999), backstepping (Dawson *et al.*, 1998), passivity (Ortega *et al.*, 1998) and adaptive techniques (Marino *et al.*, 1999). For nonlinear systems with stable zero dynamics, and assuming that all components of the state are measurable, a state feedback controller can be designed such that the state is bounded and the tracking error converges to zero.

However, the vector state, in general, is not completely measurable and it should be estimated. Nonlinear observer design for a particular class of state affine nonlinear systems is considered. Moreover, the stability analysis of the closed-loop system, when the controller is a function of the estimated state, is performed for particular classes of nonlinear systems.

[1] Supported by CONACYT México

The paper is organized as follows: an observer for a class of nonlinear systems in cascade is introduced in section 2. A feedback control for this class is given in section 3. In section 4, a stability analysis of the closed loop system is performed. The induction motor case is considered in section 5. In section 6, the proposed observer-based controller is applied to an industrial experimental induction motor setup. Finally, some conclusions are given.

2. NONLINEAR OBSERVER

In this section, one presents a cascade observer for a class of nonlinear systems. Consider the following multi-variable nonlinear system:

$$\Sigma_{NL}: \begin{cases} \dot{X} = f(X) + g(X)u \\ y = h(X) \end{cases} \quad (1)$$

where $X \in R^N$ is the state vector of the system, $u \in R^M$ is the control vector and $y \in R^P$ is the output vector. Assume there exists a change of coordinates which transforms (1) into another system represented by:

$$\begin{aligned} \dot{X}_1 &= A_1(u,y)X_1 + g_1(u,y,X_1) \quad (2) \\ \dot{X}_i &= A_i(u,y)X_i + g_i(u,y,X_1,\cdots,X_i),\ i=2,\ldots,n \\ y_i &= C_iX_i,\ i=1,\ldots,n. \end{aligned}$$

where $X_i = col(x_{1,i}, x_{2,i}, ..., x_{r_i,i}) \in R^{n_i}$, $\sum_{i=1}^{n} n_i = N$, $y_i \in R^{p_i}$, $\sum_{i=1}^{n} p_i = P$, the matrices $A_i \in R^{n_i \times n_i}$, $i = 1, 2, ..., n$, are

$$A_i(u,y) = \begin{pmatrix} 0_{p_i} & \alpha_{1i}(u,y) & 0_{p_i} & \cdots & 0_{p_i} \\ 0_{p_i} & 0_{p_i} & \alpha_{2i}(u,y) & \cdots & 0_{p_i} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{p_i} & 0_{p_i} & 0_{p_i} & \ddots & \alpha_{(r_i-1)i}(u,y) \\ 0_{p_i} & 0_{p_i} & 0_{p_i} & \cdots & 0_{p_i} \end{pmatrix}$$

where $\alpha_{ji} \in R^{p_i \times p_i}$, $j = 1, 2, \ldots, r_i - 1$ and 0_{p_i} is the $p_i \times p_i$ null matrix. The vector fields $g_i \in R^{p_i}$, $i = 1, 2, \ldots, r_i$, are

$$g_i(u,y,X_1,\cdots,X_i) = \begin{pmatrix} g_{1i}(v_i, x_{1,i}) \\ g_{2i}(v_i, x_{1,i}, x_{2,i}) \\ \vdots \\ g_{n_i i}(v_i, x_{1,i}, x_{2,i} \ldots x_{n_i,i}) \end{pmatrix}$$

where $v_i = (u, y, X_1, \cdots, X_{i-1})$.

The output matrix is $C_i = \begin{pmatrix} I_{p_i} & 0_{p_i} & \cdots & 0_{p_i} \end{pmatrix}$ where I_{p_i} denotes the $p_i \times p_i$ null matrix.

Moreover, it is assumed that each subsystem is observable and verifies the following assumptions:

H1 *There exist positive constants* c_{1i}, c_{2i}, $0 < c_{1i} < c_{2i} < \infty, i = 1, ..., n$; *such that for all* $X_i \in R^{n_i}$; $0 < c_{1i} I_{p_i} \leq \alpha_{ji}^T(u,y)\alpha_{ji}(u,y) \leq c_{2i} I_{p_i} \leq \infty, j = 1, ..., r_i - 1$.

H2 *The functions* $g_i(u, y, X_1, ..., X_i), i = 1, ..., n$ *are globally Lipschitz w. r. t.* $(X_1, ..., X_i)$ *and uniformly w. r. t. u and y.*

H3 $\sup_{\theta_i \geq 1} \left\| \dot{\Gamma}_i(u,y)\Gamma_i^{-1}(u,y) \right\| \leq L_{ii}$, *where* L_{ii} *is a positive constant and*

$$\Gamma_i(u,y) = diag\{I_{p_i}, \alpha_{1i}(u,y), ..., \prod_{j=1}^{r_i-1} \alpha_{ji}(u,y)\}$$

for $i = 1, .., n$.

An observer for systems in cascade is given by

$$\begin{aligned} \dot{Z}_1 &= A_1(u,y)Z_1 + g_1(u,y,Z_1) \quad (3) \\ &\quad + M_1(u,y)C_1(Z_1 - X_1) \\ \dot{Z}_i &= A_i(u,y)Z_i + g_i(u,y,Z_1,\cdots,Z_i) \\ &\quad + M_i(u,y)C_i(Z_i - X_i),\ i=2,\ldots,n \end{aligned}$$

where $M_i(u,y) = \Gamma_i^{-1}(u,y)\Delta_{\theta i}^{-1}K_i, i = 1, ..., n$ are the gains of the observer, $\Delta_{\theta i} = diag\{\frac{1}{\theta_i}I_{p_i}, \frac{1}{\theta_i^2}I_{p_i}, ..., \frac{1}{\theta_i^{n_i}}I_{p_i}\}$ with $\theta_i > 0, K_i$ is such that the matrix $(\bar{A}_i - K_iC_i)$ is stable and $\bar{A}_i = \begin{pmatrix} 0_{p_i} & I_{p_i} & \cdots & 0_{p_i} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{p_i} & 0_{p_i} & \ddots & I_{p_i} \\ 0_{p_i} & 0_{p_i} & \cdots & 0_{p_i} \end{pmatrix}$.

From **H1**, the matrix Γ_i is nonsingular.

Theorem 1: *Assume that the system (2) satisfies assumptions H1, H2 and H3. Then, there exist* $\theta_{0i} > 0$, $i = 1, ..., n$ *such that for all* $\theta_i > \theta_{0i}$, *the system (3) is an exponential observer for system (2).*

Proof: See (DeLeon *et al.*, 2000).

3. FEEDBACK CONTROL DESIGN

Consider nonlinear system (1), suppose that the system has relative degree r and is observable. From Frobenius' Theorem, there exists a diffeomorphism $\xi = T(x) = col(T_1(X) - \mathcal{Y}_R, T_2(X))$ that transforms the original nonlinear system into the following one:

$$\Sigma_\xi: \begin{cases} \dot{\xi}_r = A\xi_r + B\{\alpha(\xi_r, \xi_{n-r}) + \beta(\xi_r, \xi_{n-r})u\} \\ \dot{\xi}_{n-r} = Q(\xi_r, \xi_{n-r}) \\ y = \xi_1 \end{cases} \quad (4)$$

which is state feedback linearizable with index r, where $\xi = col(\xi_r, \xi_{n-r}) \in R^n$, $\mathcal{Y}_R = col(y_R, y_R^{(1)}, ..., y_R^{(r-1)})$, y_R is the desired reference signal, and $T_1(X) = (h(X), L_f h(X), ..., L_f^{r-1}h(X))$, $T_2(X) = (\varphi_{r+1}(X), ..., \varphi_n(X))$. The functions φ_i are such that $L_g\varphi_i(X) = \langle d\varphi_i(X), g\rangle = 0$, for $i = r+1, ..., n$.

We assume the zero dynamics of the system given by $\dot{\xi}_{n-r} = Q(0, \xi_{n-r})$ stable. Then, there exists a smooth feedback control $u = u(\xi_r, \xi_{n-r}) = \beta^{-1}(\xi_r, \xi_{n-r})(-\alpha(\xi_r, \xi_{n-r}) + v)$, with $v = -K\xi_r$, such that $(A - BK)$ is Hurwitz, making the origin $(\xi_r, \xi_{n-r}) = 0$ asymptotically stable.

4. ANALYSIS OF STABILITY OF THE CLOSED-LOOP SYSTEM

In this section, the stability of the system (1) with the control law function of the estimated state given by the observer (3) is studied.

Let us consider the estimation error equations

$$\dot{e} = \left(A(u,y) - \Gamma^{-1}(u,y)\Delta_\theta^{-1}KC\right)e + \Upsilon(u,y,e,Z)$$

where the measurable outputs are $y = CX = col(y_1, y_2 ..., y_n) = col(C_1X_1, C_2X_2, ..., C_nX_n)$, the estimation error $e = col(e_1, ..., e_n)$, the state estimate $Z = col(Z_1, Z_2, ..., Z_n)$, and

$$\begin{aligned} A(u,y) &= diag\left(A_1(u,y) - \Gamma_1^{-1}(u,y)\Delta_{\theta 1}^{-1}K_1C_1, ...,\right. \\ &\left. A_n(u,y) - \Gamma_n^{-1}(u,y)\Delta_{\theta n}^{-1}K_nC_n\right) \\ \Upsilon(u,y,e,Z) &= col(g_1(u,y,Z_1) - g_1(u,y,Z_1 - e_1), ..., \\ &g_n(u,y,Z_1, .., Z_n) - g_n(u,y,Z_1 - e_1, ...Z_n - e_n)). \end{aligned}$$

Now, define the augmented system

$$\begin{cases} \dot{X} = f(X) + g(X)u(Z), \\ \dot{e} = \left(A(u,y) - \Gamma^{-1}(u,y)\Delta_\theta^{-1}KC\right)e + \Upsilon(u,y,e,Z) \end{cases}$$

and using the following change of coordinates

$$\begin{aligned} \xi &= col(\xi_r, \xi_{n-r}) = col\left(T_1(X) - \mathcal{Y}_R, T_2(X)\right) \\ \epsilon &= col(\epsilon_1, ..., \epsilon_n) \\ &= col(\Gamma_1(u,y)\Delta_{\theta 1}e_1, ..., \Gamma_n(u,y)\Delta_{\theta n}e_n) \end{aligned}$$

the above system can be rewritten in the form

$$\Sigma_A : \begin{cases} \dot{\xi}_r = A\xi_r + \dfrac{\partial T_1(X)}{\partial X}g(X)\left[u(Z) - u(X)\right], \\ \dot{\xi}_{n-r} = Q(\xi_r, \xi_{n-r}) \\ \dot{\epsilon} = \tilde{A}\epsilon + \tilde{\Upsilon}(u,y,\epsilon,\xi_r,\xi_{n-r}). \end{cases}$$

where

$$\begin{aligned} &\tilde{A} = diag\left(\theta_1\tilde{A}_1, ..., \theta_n\tilde{A}_n\right), \\ &\tilde{\Upsilon} = col\left(\dot{\Gamma}_1(u,y)\Gamma_1^{-1}(u,y)\epsilon_1,\right. \\ &+G_1(u,y,Z_1) - G_1(u,y,X_1), ..., +G_n(u,y,Z_1, ..., Z_n) \\ &\left.-G_n(u,y,X_1, ..., X_n) + \dot{\Gamma}_n(u,y)\Gamma_n^{-1}(u,y)\epsilon_n\right) \end{aligned}$$

with $\tilde{A}_i = \{\bar{A}_i - K_iC_i\}$ and $G_i(u,y,Z_1, ..., Z_i) - G_i(u,y,X_1, ..., X_i) = \Gamma_i(u,y)\Delta_{\theta i}\{g_i(u,y,Z_1, ..., Z_i) - g_i(u,y,X_1, ..., X_i)\}$, $i = 1, ..., n$.

For the stability analysis of Σ_A, the following assumptions are introduced:

H4 *The zero dynamics $\dot{\xi}_{n-r} = Q(0, \xi_{n-r})$ is stable.*

H5 *Set $Z, X \in B(0, \rho)$ the ball centered in 0 and radius $\rho > 0$. There exist a scalar nonnegative locally Lipschitz constant function $\bar{L}(\rho) = L_1(\rho)L_2(\rho)L_3(\rho)$ such that*

$$\left\|\frac{\partial T_1(X)}{\partial X}g(X)\left[u(Z) - u(X)\right]\right\| \leq \bar{L}(\rho)\left\|e\right\|$$

where $\|g(X)\| \leq L_1(\rho)$, $\left\|\dfrac{\partial T_1(X)}{\partial X}\right\| \leq L_2(\rho)$, $\|u(Z) - u(X)\| \leq L_3(\rho)\|e\|$ and $L_i(\rho), i = 1, 2, 3,$ are Lipschitz constants.

H6 *The Lyapunov function candidate*

$$L(\xi_r, \epsilon) = W(\xi_r) + V(\epsilon_1, \epsilon_2) \quad (5)$$

where $W(\xi_r) = \xi_r^TP\xi_r = [T_1(X) - \mathcal{Y}_R]^TP[T_1(X) - \mathcal{Y}_R]$ and $V(\epsilon) = V(\epsilon_1, \epsilon_2) = V_1(\epsilon_1) + V_2(\epsilon_2)$, satisfies the following inequalities

$$\beta_1\left\|\xi_r(t)\right\|^2 \leq W(\xi_r(t)) = \xi_r^TP\xi_r \leq \beta_2\left\|\xi_r(t)\right\|^2,$$

$$\frac{\partial W}{\partial \xi_r}A\xi_r \leq -\beta_3\left\|\xi_r(t)\right\|^2$$

where $A^TP + PA = -Q$.

The Lyapunov function $V(\epsilon)$ is such that

$$\alpha_1\left\|\epsilon\right\|^2 \leq V(\epsilon) \leq \alpha_2\left\|\epsilon\right\|^2 \quad (6)$$

$$\frac{\partial V}{\partial \epsilon}\left(\tilde{A}\epsilon + \tilde{\Upsilon}(u,y,\epsilon,\xi_r,,\xi_{n-r})\right) \leq -\alpha_3\left\|\epsilon(t)\right\|^2$$

where $\alpha_i, \beta_i, i = 1, 2, 3,$ are positive constants. Let be $\mu_1 = \dfrac{\beta_1}{\beta_2}$, $\mu_2 = \dfrac{\alpha_1}{\alpha_2}$.

Theorem 2: *Consider the system Σ_{NL} (1) obtained via change of coordinates and an input-output state feedback linearizing control u depending on the state estimated given by the observer (3). Under the assumptions H1, H2, H3, H4, H5 and H6, the dynamics of the whole system Σ_A is locally asymptotically stable. More precisely, the dynamics of the tracking error ξ and the estimation error ϵ converge exponentially to zero and an estimation of the attraction region is given by $\left\{\xi_r \in R^{n-r} : \|\xi_r(0)\| \leq \sqrt{\mu_1}\frac{\rho}{2}\right\} \times \left\{e \in R^n : \|e(0)\| \leq \sqrt{\mu_2}\frac{\rho}{2}\right\}$.*

Proof: For the sake of simplicity, one assumes that $i = 2$. Taking the Lyapunov function candidate

$$L(\xi_r, \epsilon) = W(\xi_r) + V(\epsilon_1, \epsilon_2)$$

Set $-[T_1(X) - \mathcal{Y}_R]^TQ[T_1(X) - \mathcal{Y}_R] \leq -\eta W(\xi_r)$, where η is a positive constant.

The derivative of $W(\xi_r)$ w. r. t. time, it follows that

$$\begin{aligned} \frac{dW(\xi_r)}{dt} &\leq -\eta W(\xi_r) \\ &+ 2\sqrt{W(\xi_r)}\left\|\frac{\partial T_1(X)}{\partial X}\right\|\left\|g(X)\right\|\left\|u(Z) - u(X)\right\|. \end{aligned} \quad (7)$$

From assumption **H5**:

$$\frac{d\sqrt{W(\xi_r)}}{dt} = -\frac{\eta}{2}\sqrt{W(\xi_r)} + \bar{L}(\rho)\left\|e\right\|. \quad (8)$$

Since the derivative of V is given by

$$\begin{aligned}\dot{V} &= \dot{V}_1(\epsilon_1) + \dot{V}_2(\epsilon_2)\\ &\leq -\left(\theta_1 k_1 - L_{11} - N_{11}\right)\left\|\epsilon_1\right\|_{P_1}^2\\ &\quad -\left(\theta_2 k_2 - L_{22} - N_{22}\right)\left\|\epsilon_2\right\|_{P_2}^2 + H_2 M_2 \left\|\epsilon_1\right\|_{P_1}\left\|\epsilon_2\right\|_{P_2}\\ &\leq -\delta_1\left\|\epsilon_1\right\|_{P_1}^2 - \delta_2\left\|\epsilon_2\right\|_{P_2}^2 + H_2 M_2\left\|\epsilon_1\right\|_{P_1}\left\|\epsilon_2\right\|_{P_2}\end{aligned}$$

for $\delta_1 = \theta_1 k_1 - L_{11} - N_{11} > 0$ and $\delta_2 = \theta_2 k_2 - L_{22} - N_{22} > 0$ where $\left\|G_1(u,y,Z_1) - G_1(u,y,X_1)\right\| \leq N_{11}\left\|\epsilon_1\right\|$, $\left\|G_2(u,y,Z_1,Z_2) - G_2(u,y,X_1,X_2)\right\| \leq N_{21}\left\|\epsilon_1\right\| + N_{22}\left\|\epsilon_2\right\|$, N_{11}, N_{21} and N_{22} are Lipschitz constants, H_2 is a constant from assumption **H1** and k_2 is a positive constant. Given that $\left\|\epsilon_1\right\|\left\|\epsilon_2\right\| \leq \frac{a^2}{2}\left\|\epsilon_1\right\|^2 + \frac{1}{2a^2}\left\|\epsilon_2\right\|^2$ for any $a > 0$. It follows that

$$\begin{aligned}\dot{V}(\epsilon) = \dot{V}(\epsilon_1) + \dot{V}(\epsilon_2) &\leq -\left(\delta_1 - H_2 N_{21}\frac{a^2}{2}\right)\left\|\epsilon_1\right\|_{P_1}^2\\ &\quad -\left(\delta_2 - H_2 N_{21}\frac{1}{2a^2}\right)\left\|\epsilon_2\right\|_{P_2}^2.\end{aligned}$$

Choosing a, θ_1 and θ_2 such that the above inequality verifies

$$\dot{V}(\epsilon(t)) = -b(\theta)\left\|\epsilon\right\|_P^2 \leq -c(\theta)V(\epsilon(t))$$

with $\theta = (\theta_1, \theta_2)$, where $b(\theta)$ and $c(\theta)$ are positive functions of θ. Then,

$$V(\epsilon(t)) \leq V(\epsilon(0))\exp(-c(\theta)t) \quad (9)$$

and with (6), one obtains

$$\left\|\epsilon(t)\right\| \leq \sqrt{\frac{V(\epsilon(0))}{\alpha_1}}\exp(-c(\theta)\frac{t}{2}).$$

Replacing the above term in (8) and integrating, it follows that

$$\begin{aligned}\sqrt{W(\xi_r)} &\leq \sqrt{W(0)}\exp(-\eta\frac{t}{2})\\ &\quad + \bar{L}(\rho)\sqrt{\frac{V(\epsilon(0))}{\alpha_1\gamma}}\exp(-\eta\frac{t}{2})\{1 - \exp(-\gamma\frac{t}{2})\}\end{aligned}$$

with $\gamma = \frac{c(\theta) - \eta}{2}$.

Choosing $c(\theta) > \eta$, which means that the observer dynamics is faster than the system dynamics, then

$$\begin{aligned}\sqrt{W(\xi_r)} &\leq \sqrt{W(0)}\exp(-\eta\frac{t}{2}) \quad (10)\\ &\quad + \bar{L}(\rho)\sqrt{\frac{V(\epsilon(0))}{\alpha_1\gamma}}\exp(-\eta\frac{t}{2}).\end{aligned}$$

From (9) and (10) it is concluded that $L(\xi_r, \epsilon)$ is Lyapunov stable.

Now, the stability domain of D of the system Σ_A is characterized. From assumption **H6**, $\alpha_1, \alpha_2, \beta_1, \beta_2 > 0$ exist and satisfy the following inequalities

$$\alpha_1\left\|\epsilon(t)\right\|^2 \leq V(\epsilon) \leq \alpha_2\left\|\epsilon(t)\right\|^2 \quad (11)$$

$$\beta_1\left\|\xi_r(t)\right\|^2 \leq W(\xi_r) \leq \beta_2\left\|\xi_r(t)\right\|^2 \quad (12)$$

Choosing

$$\left\|\xi_r(0)\right\| \leq \sqrt{\frac{\beta_1}{\beta_2}}\frac{\rho}{2} = \sqrt{\mu_1}\frac{\rho}{2}; \quad \left\|\epsilon(0)\right\| \leq \sqrt{\frac{\alpha_1}{\alpha_2}}\frac{\rho}{2} = \sqrt{\mu_2}\frac{\rho}{2}. \quad (13)$$

Replacing (11) and (13) in (9):

$$\left\|\epsilon(t)\right\| \leq \sqrt{\frac{V(e(0))}{\alpha_1}}\exp(-c(\theta)\frac{t}{2}) \leq \sqrt{\frac{\alpha_2}{\alpha_1}}\left\|\epsilon(0)\right\| \leq \frac{\rho}{2}.$$

Similarly for $\left\|\xi_r(t)\right\|$, one obtains

$$\sqrt{\beta_1}\left\|\xi_r(t)\right\| \leq \sqrt{W(\xi_r)} \leq \sqrt{\beta_2}\left\|\xi_r(0)\right\| + \frac{\bar{L}(\rho)}{\gamma}\sqrt{\frac{\alpha_2}{\alpha_1}}\left\|\epsilon(0)\right\|.$$

Hence, for $\rho > 0$, $\exists\ c(\theta_0) > 0$, such that $\forall\ c(\theta) > c(\theta_0)$ and

$$\frac{\bar{L}(\rho)}{(c(\theta) - \eta)\sqrt{\beta_1}} \leq \frac{1}{2}.$$

Using (12) and (13) in (10), it follows that

$$\left\|\xi_r(t)\right\| \leq \frac{1}{\sqrt{\beta_1}}\left\{\sqrt{\beta_2}\left\|\xi_r(0)\right\| + \sqrt{\frac{\beta_1}{\mu_2}}\left\|\epsilon(0)\right\|\right\} \leq \rho.$$

This proves the attractivity of the origin of the system.

5. APPLICATION TO AN INDUCTION MOTOR

The controller and observer are designed using the standard (α, β) non linear model (Marino *et al.*, 1993):

$$\Sigma_{NL} : \dot{X} = f(X) + gu \quad (14)$$

$$f(X) = \begin{bmatrix} -\gamma i_{s\alpha} + \frac{K}{T_r}\phi_{r\alpha} + p\omega K\phi_{r\beta} \\ -\gamma i_{s\beta} - p\omega K\phi_{r\alpha} + \frac{K}{T_r}\phi_{r\beta} \\ \frac{M}{T_r}i_{s\alpha} - \frac{1}{T_r}\phi_{r\alpha} - p\omega\phi_{r\beta} \\ \frac{M}{T_r}i_{s\beta} + p\omega\phi_{r\alpha} - \frac{1}{T_r}\phi_{r\beta} \\ \frac{pM}{JL_r}(\phi_{r\alpha}i_{s\beta} - \phi_{r\beta}i_{s\alpha}) - \frac{f}{J}\omega - \frac{1}{J}\tau_L \end{bmatrix}$$

$$g = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0\ 0\ 0 \\ 0 & \frac{1}{\sigma L_s} & 0\ 0\ 0 \end{bmatrix}^T$$

where $X = col\,[i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta}, \omega]$ and $u = col\,[u_{s\alpha}, u_{s\beta}]$. The states of the system are the two phases components of the stator current and of the rotor flux and mechanical speed. The inputs are the stator voltages. It is assumed that the unknown load torque is constant (not measured) and the nominal values of the rotor resistance and the other parameters of the model are known. The variables and parameters of the motor model are given in Table 1.

Tab. 1: Model variables and parameters

M, L_r, L_s mutual, rotor, stator inductances		
$i_{s\alpha}, i_{s\beta}$	stator currents	$T_r := \frac{L_r}{R_r}$
$\phi_{r\alpha}, \phi_{r\beta}$	rotor fluxes	
ω	mechanical speed	$K := \frac{M}{\sigma L_s L_r}$
$u_{s\alpha}, u_{s\beta}$	stator voltages	
R_r, R_s	rotor, stator resistances	$\sigma := 1 - \frac{M^2}{L_s L_r}$
τ_L	load torque	$\gamma := \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$
J	inertia	
f	viscous damping coefficient	
p	pole pair number	

5.1 *Observer design*

The design of the cascade observer (3) for the induction motor is introduced ($i = 2$). The observer of the electrical subsystem is given by

$$\dot{Z}_1 = \begin{pmatrix} 0_{2\times2} & KN(\omega) \\ 0_{2\times2} & 0_{2\times2} \end{pmatrix} Z_1 + \begin{pmatrix} -\gamma I_{2\times2} & 0_{2\times2} \\ \frac{M}{T_r} I_{2\times2} & N(\omega) \end{pmatrix} Z_1 + \begin{pmatrix} \frac{1}{\sigma L_s} I_{2\times2} \\ 0_{2\times2} \end{pmatrix} u - \begin{pmatrix} \theta_1 k_1 I_{2\times2} & 0 \\ \frac{\theta_1^2 k_2}{K} N^{-1}(\omega) & 0 \end{pmatrix} (Z_1 - X_1)$$

where $X_1 = col\,(i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta})$, the output is $y_1 = col\,(i_{s\alpha}, i_{s\beta})$, $Z_1 = col\left(\hat{i}_{s\alpha}, \hat{i}_{s\beta}, \hat{\phi}_{r\alpha}, \hat{\phi}_{r\beta}\right)$, $N(\omega) = \begin{pmatrix} \frac{1}{T_r} & p\omega \\ -p\omega & \frac{1}{T_r} \end{pmatrix}$ and (k_1, k_2, θ_1) are constant design parameters.

Now, the observer of the mechanical subsystem is given by

$$\dot{Z}_2 = \begin{pmatrix} 0 & \frac{1}{J} \\ 0 & 0 \end{pmatrix} Z_2 + \begin{pmatrix} \hat{\psi} \\ 0 \end{pmatrix} - \begin{pmatrix} \theta_2 l_1 & 0 \\ -\theta_2^2 l_2 J & 0 \end{pmatrix} (Z_2 - X_2)$$

with $X_2 = col\,(\omega, \tau_L)$, the output is $y_2 = \omega$, $Z_2 = col\,(\hat{\omega}, \hat{\tau}_L)$, $\hat{\psi} = \frac{pM}{JL_r}(\hat{\phi}_{r\alpha} i_{s\beta} - \hat{\phi}_{r\beta} i_{s\alpha}) - \frac{f}{J}\omega$ and (l_1, l_2, θ_2) are constant design parameters.

5.2 *Control design*

A multivariable input-output linearizing state feedback is designed for the system (14). The tracking control is given by

$$u = \begin{pmatrix} u_{s\alpha} \\ u_{s\beta} \end{pmatrix} = D^{-1}(Z) \begin{pmatrix} -L_f^2 h_1(Z) + v_a \\ -L_f^2 h_2(Z) + v_b \end{pmatrix}$$

$$D(Z) = \begin{pmatrix} -\frac{pK}{J}\hat{\phi}_{r\beta} & \frac{pK}{J}\hat{\phi}_{r\alpha} \\ 2R_r K \hat{\phi}_{r\alpha} & 2R_r K \hat{\phi}_{r\beta} \end{pmatrix}$$

such that the output $y = col(h_1, h_2)$ asymptotically tracks a reference signal $\mathcal{Y}_R$, where $h_1 = \omega$, $h_2 = \phi_{r\alpha}^2 + \phi_{r\beta}^2$ are considered as the outputs to be controlled and $\mathcal{Y}_R = col(\omega_r(t),\ \|\phi_r(t)\|^2)$ are the reference signals to be tracked for the speed and for the square of the flux norm, respectively. It is assumed that the reference signal and its derivatives are bounded for all $t \geq 0$ and

$$\begin{aligned} v_a &= k_{a1}\left(\omega_r(t) - \omega\right) + k_{a2}\int \left(\omega_r(t) - \omega\right) dt \\ &\quad + k_{a3}\frac{d}{dt}\left(\omega_r(t) - \omega\right) + \ddot{\omega}_r(t) \\ v_b &= k_{b1}\left(\|\phi_r(t)\|^2 - \left(\hat{\phi}_{r\alpha}^2 + \hat{\phi}_{r\beta}^2\right)\right) \\ &\quad + k_{b2}\int \left(\|\phi_r(t)\|^2 - \left(\hat{\phi}_{r\alpha}^2 + \hat{\phi}_{r\beta}^2\right)\right) dt \\ &\quad + k_{b3}\frac{d}{dt}\left(\|\phi_r(t)\|^2 - \left(\hat{\phi}_{r\alpha}^2 + \hat{\phi}_{r\beta}^2\right)\right) + \left\|\ddot{\phi}_r(t)\right\|^2 \end{aligned}$$

where (k_{a1}, k_{a2}, k_{a3}) and $(k_{ba1}, k_{b2}, k_{b3})$ are constant design parameters.

Using Theorem 2 closed-loop stability is ensured.

6. EXPERIMENTAL RESULTS

In order to illustrate the performance of the proposed scheme, some experimental results on a 4-pole, 7.5 kW, three-phase induction motor are shown. The induction motor load is simulated by a 7.5 kW DC motor fed by an inverter with current circulation which provides four quadrants operation (see (Lubineau *et al.*, 2000)).

The controller is tested on a wide operating domain with the following benchmark: the speed increases with constant acceleration up to nominal speed. A torque load is applied at nominal speed. Then the rotation sense is inverted. A constant torque of same sign as before is then applied to show the behavior of controller when the motor is set in energy generation operating point. Torque disturbance is maintained until speed is set back to zero and the flux reference is constant. In experiments, the observer parameters are $k_1 = 0.7$, $k_2 = 0.12$, $\theta_1 = 4.5$, $l_1 = 11.0$, $l_2 = 30$ and $\theta_2 = 3$. The parameters of the controller are chosen as

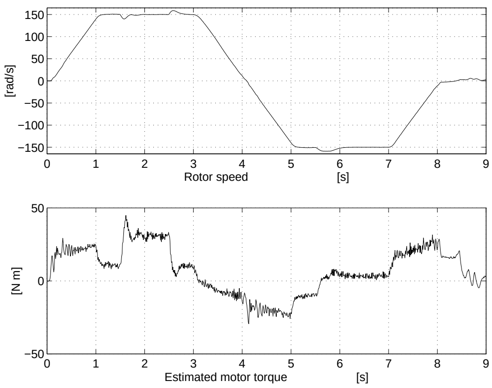

Fig. 1. Rotor speed and motor torque.

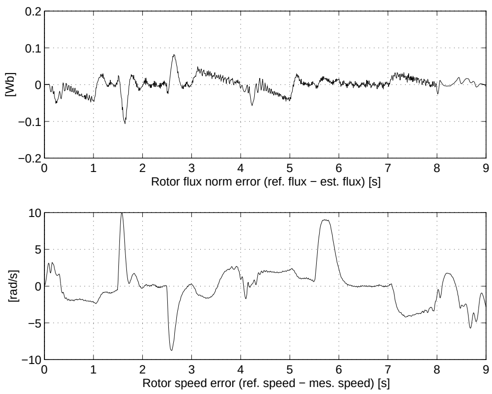

Fig. 2. Tracking errors.

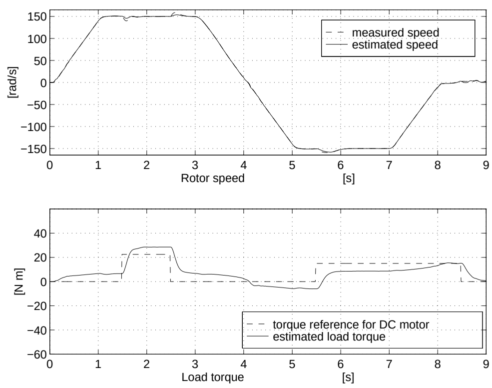

Fig. 3. Estimated speed and load torque.

follows: $k_{a1} = 3100$, $k_{a2} = 15000$, $k_{a3} = 170$, $k_{ba1} = 22500$, $k_{b2} = 225000$, and $k_{b3} = 310$. Figure 1 shows rotor speed and motor torque. It includes torque due to acceleration and torque due to load. Figure 2 shows tracking errors on flux and speed. The flux tracking error is given between the reference and the estimate. The speed error is less than 7% of nominal speed despite torque disturbances. Finally, the observer performance is illustrated. Figures 3 and 4 show the estimated speed and load torque, the latter one corresponds to the case $\dot{\omega} = 0$ and $\omega = \omega_{nom}$.

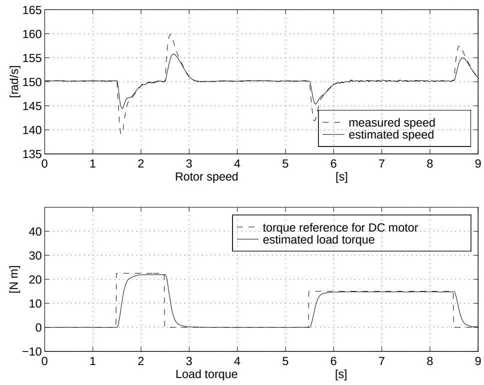

Fig. 4. Estimated speed and load torque ($\dot{\omega} = 0$, $\omega = \omega_{nom}$).

7. CONCLUSIONS

In this paper, an observer-based controller for a class of nonlinear systems is proposed and shown to be closed-loop stable. It is applied on an industrial 7.5 kW induction motor. The real-time experiments show the efficiency of the developed controller for a broad domain of operating conditions including low and high speed, as well as motor and generator behavior, with respect to standard methods.

8. REFERENCES

Dawson, D. M., J. Hu and T. C. Burg (1998). *Nonlinear Control of Electric Machinery.* Marcel Dekker.

DeLeon, J., A. Glumineau and G. Schreier (2000). Cascade nonlinear observer for induction motors. *IEEE Conf. on Control Applications. Anchorage, Alaska,* pp. 623–628.

Lubineau, D., J. M. Dion, L. Dugard and D. Roye (2000). Design of an advanced nonlinear controller for induction motor and experimental validation on an industrial benchmark. *EPJ Applied Physics* **9**, 165–175.

Marino, R., S. Peresada and P. Tomei (1999). Global adaptive output feedback control of current-fed induction motors with unknown rotor resistance. *IEEE Trans. Automat. Contr.* **44**(5), 967–983.

Marino, R., S. Peresada and P. Valigi (1993). Adaptive input-output feedback linearizing control of induction motors. *IEEE Trans. Automat. Contr.* **38**(2), 208–221.

Ortega, R., A. Loria, P. J. Nicklasson and H. Sira-Ramírez (1998). *Passivity-based Control of Euler-Lagrange Systems.* Springer Verlag.

Utkin, V., J. Guldner and J. Shi (1999). *Sliding Mode Control in Electromechanical Systems.* Taylor & Francis.