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A SUBOPTIMAL ALGORITHM OF THE OPTIMAL BAYESIAN FILTER BASED UPON THE RECEDING HORIZON STRATEGY

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Abstract: The optimal Bayesian filter for a single target is known to provide the best tracking performance in a cluttered environment. However, its main drawback is the increase of memory size and computation quantity with time. In this paper, the inevitable problem of the optimal Bayesian filter is resolved in a suboptimal fashion by using a receding horizon strategy. As a result, the problem of memory and computational requirements is diminished. As *a priori* information, the horizon initial state is estimated from the validated measurements on the receding horizon. Consequently, the suboptimal algorithm proposed allows the real time implementation. *Copyright © 2002 IFAC*

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1. INTRODUCTION

In a cluttered environment, the target tracking problem naturally involves the uncertainty associated with measurements as well as the modeling inaccuracy. This uncertainty is related to the origin of measurements, because the measurements might not have originated from the target of interest (Bar-Shalom and Fortman, 1988). This problem was not recognized until the first paper of Sittler (1964) was published in 1964. The pioneering work of Sittler was motivated by the need to find a reasonable way of incorporating the measurements with uncertain origin into existing tracks. However, since his method was based on a non-Bayesian approach, the resulting state estimate and covariance do not account for the possibility that the determined decisions are incorrect.

The Bayesian procedures use the “nearest neighbor” of the predicted measurement, in which the Kalman filter is modified to account for the *a priori* probability that the measurement might be spurious. This filter utilizes only the sensor reports that are statistically close to the predicted track measurement for track updating and calculates its data association performance parameters based on averaging over *a priori* statistics. Singer and Sea (1973) extended the Bayesian approach to develop an optimal tracking filter within the class of nearest-neighbor filters that utilize *a priori* statistics for estimating correlation performance.

The need to incorporate all the observations lying in the neighborhood of the predicted measurement was pointed out by Bar-Shalom and Jaffer (1972), where a suboptimal algorithm using *a posteriori* probabilities was presented. In (Bar-Shalom and Jaffer, 1972), it was suggested that *a posteriori* correlation statistics, calculated on-line based on all reports in the vicinity of a track (i.e., all-neighbors approach) should be used to obtain the best possible tracking performance based on all available data provided by the surveillance sensor.

In (Singer et al., 1974), the theoretical formulation of an optimal filter using the *a posteriori* probability and all-neighbors class was completely carried out. This filter requires a growing memory and utilizes the data located around the vicinity of the track, accounting properly for the possibility that any particular report among these data may either be extraneous or have originated from the track. However, this filter is quite unsuitable for real-time application in dense multi-target environments.

Several approaches (Gelb, 1974; Kenefic, 1981; Bar-Shalom and Fortmann, 1988) for limiting memory growth and computation requirements, while still providing a reasonable approximation to the performance of the optimal filter, were proposed. In (Kenefic, 1981), the optimal *a posteriori* filter of Singer et al. (1974) was combined with an adaptive filter. The resulting filter requires an expanding memory. A (M, N) scan approximation, whereas an N scan approximation was used in Singer et al. (1974), was proposed in order to obtain an algorithm with stable memory requirements. In this approximation those measurement histories which were identical for the most recent N scans and those input histories which were identical for the most recent M scans were combined together into new histories.

The memory growth and computation problem for real-time implementation has been a critical issue in the Kalman filtering, too. Hence, finite memory filters (Jazwinski, 1972; Ling and Lim, 1999) were suggested to overcome the poor performance or divergence due to the modeling errors of the standard Kalman filter. Finite memory filters are also useful in situations in which a system model is valid over a finite interval. In (Kwon et al., 1999), a receding horizon Kalman FIR filter that combines the Kalman filter and the receding horizon strategy was presented. In their work it is shown that the suggested filter processes the unbiasedness property and the deadbeat property irrespective of any horizon initial condition. In (Han et al., 1999), a receding horizon Kalman FIR filter including the estimation of the horizon initial state was investigated. Also, an estimation and detection technique for the unknown inputs by using optimal FIR filter was presented in (Park et al., 2000).

The optimal Bayesian filter in a cluttered environment, though it has shown the better performance than other filters, has drawn little attention due to exponentially increasing memory and computation requirements. The main contributions of this paper are: A suboptimal approach using the measurements only in a receding horizon is derived. As a result, the increasing memory and computation requirements are diminished. Second, the horizon initial state is estimated from only validated measurements on the receding horizon. Third, the suboptimal algorithm solves the real-time implementation problem of the optimal Bayesian filter.

This paper is organized as follows: In Section 2, a suboptimal algorithm for the optimal Bayesian filter is derived, and the horizon initial state estimate and its covariance are obtained. *A posterior* probability of the validated measurements on the receding horizon is provided in Section 3. In Section 4, conclusions are stated.

2. NEW SUBOPTIMAL ALGORITHM

Consider the following state-space representation of the target motion and observation

$$x_{k+1} = F_k x_k + \omega_k , \tag{1}$$

$$y_k = H_k x_k + \nu_k \tag{2}$$

with ω_k and ν_k being zero-mean mutually independent white Gaussian noises with covariances Q_k and R_k, respectively. The suboptimal algorithm in the sequel does not use all measurements observed from the initial time up to the present time k, but uses only a set of measurements observed in some interval with fixed window-size N, i.e., on the receding horizon interval $[k-N, k]$.

Assumption 1: The possibility of a false track initiation is not considered in this paper. Hence, the horizon initial estimate and its covariance, as a prior information which will be estimated in the suboptimal algorithm, is assumed to be in a correct track. Let the set of validated measurements obtained at time k

$$Y_k = \{Y_{k,i}\}_{i=1}^{m_k},$$

where m_k is the number of measurements in the validation region. Let the set of measurements on the receding horizon $[k-N, k]$ at time k be denoted as

$$Y^k = \{Y_j\}_{j=k-N}^{k},$$

where a superscript k is used, while a subscript k was used in the set of validated measurements. A combination of measurements on the receding horizon $[k-N, k]$ at the k-th scan can be denoted as $Y^{k,l}$. Then, $Y^{k,l}$ is defined as follows:

$$Y^{k,l} \overset{\Delta}{=} \{y_{k-N,i_l}, \cdots, y_{k,i_l}\} = \{Y^{k-1,s}, y_{k,i_l}\},$$

where $Y^{k-1,s}$ stands for the combination of measurements up to time $k-1$ at the $(k-1)$-th scan. Denoting the event that the l-th history at time k is the correct sequence of measurements by $\theta^{k,l}$, it's *a posteriori* probability, conditioned on Y^k, is given by

$$\beta^{k,l} = P\{\theta^{k,l} \mid Y^k\}. \tag{3}$$

Now, the following theorem is stated.

Theorem 1: When the used measurements are restricted within a receding horizon, the state estimate and error covariance equations of the optimal Bayesian filter take the following forms:

$$\hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} F(I + P^s_{k_N+j|k} H'R^{-1}H)^{-1} \beta^{k,l} (\hat{x}^s_{k_N+j|k} + P^s_{k_N+j|k} H'R^{-1} y_{k_N+j,i_l}),$$

$$P_{k_N+j+1|k} = \sum_{l=1}^{L_k} \beta^{k,l} F(I + P^s_{k_N+j|k} H'R^{-1}H)^{-1} P^s_{k_N+j|k} F'$$

$$+ \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}^l_{k_N+j+1|k} \hat{x}'^l_{k_N+j+1|k} - \hat{x}_{k_N+j+1|k} \hat{x}'_{k_N+j+1|k} + Q.$$

Proof: The conditional mean of the state at time k can then be expressed as

$$\hat{x}_{k|k} \overset{\Delta}{=} E[x_k \mid Y^k] = \sum_{l=1}^{L_k} E[x_k \mid \theta^{k,l}, Y^k] P\{\theta^{k,l} \mid Y^k\} = \sum_{l=1}^{L_k} \hat{x}^l_{k|k} \beta^{k,l} \tag{4}$$

where $\hat{x}^l_{k|k} = E[x_k \mid \theta^{k,l}, Y^k]$ is the history-conditioned estimate. L_k is the total number of measurement histories at time k as

$$L_k = \prod_{j=1}^{k}(1+m_j), \tag{5}$$

where m_j is the number of measurements at time j. For each history, the state estimate conditioned upon the measurement history, $Y^{k,l}$, being correct is

$$\hat{x}_{k|k}^{l} = \hat{x}_{k|k-1}^{s} + K_k^l(y_{k,i_l} - \hat{y}_{k|k-1}^{s}), \tag{6}$$

where y_{k,i_l} is the measurement at time k in sequence l and $\hat{y}_{k|k-1}^{s}$ is the predicted measurement corresponding to history $Y^{k-1,s}$, with covariance S_k^s. The gain is

$$K_k^l = P_{k|k-1}^{s} H'(S_k^s)^{-1} \tag{7}$$

and the covariance of the history-conditioned updated state is

$$\begin{aligned} P_{k|k}^{l} &= E[(x_k - \hat{x}_{k|k}^{l})(x_k - \hat{x}_{k|k}^{l})' \mid \theta^{k,l}, Y^k] \\ &= [I - K_k^l H] P_{k|k-1}^{s}. \end{aligned} \tag{8}$$

The conditional mean of the state at time k of (4) can be expressed as

$$\hat{x}_{k|k} = \sum_{l=1}^{L_k} \hat{x}_{k|k-1}^{s}\beta^{k,l} + \sum_{l=1}^{L_k} P_{k|k-1}^{s} H'(S_k^s)^{-1}(y_{k,i_l} - \hat{y}_{k|k-1}^{s})\beta^{k,l} \tag{9}$$

And again,

$$\begin{aligned} \hat{x}_{k|k} &= \sum_{l=1}^{L_k} F(I + P_{k-1|k-1}^{s} H'R^{-1}H)^{-1}\beta^{k,l}\hat{x}_{k-1|k-1}^{s} \\ &\quad + \sum_{l=1}^{L_k} F(I + P_{k-1|k-1}^{s} H'R^{-1}H)^{-1} P_{k-1|k-1}^{s} H'R^{-1}\beta^{k,l} y_{k,i_l}. \end{aligned}$$

Therefore, (9) is rewritten as follows:

$$= \sum_{l=1}^{L_k} \Big[F(I + P_{k-1|k-1}^{s} H'R^{-1}H)^{-1}\beta^{k,l} (\hat{x}_{k-1|k-1}^{s} + P_{k-1|k-1}^{s} H'R^{-1} y_{k,i_l}) \Big].$$

The covariance associated with the combined estimate is

$$P_{k|k} = \sum_{l=1}^{L_k} \beta^{k,l} F(P_{k-1|k-1}^{-s} + H'R^{-1}H)^{-1}F' + \sum_{l=1}^{L_k} \beta^{k,l}\hat{x}_{k|k}^{l}\hat{x}_{k|k}'^{l} - \hat{x}_{k|k}\hat{x}_{k|k}' + Q \cdot \tag{10}$$

(10) can be then represented by

$$P_{k|k} = \sum_{l=1}^{L_k} \beta^{k,l} F(I + P_{k-1|k-1}^{s} H'R^{-1}H)^{-1} P_{k-1|k-1}^{s} F' + \sum_{l=1}^{L_k} \beta^{k,l}\hat{x}_{k|k}^{l}\hat{x}_{k|k}'^{l} - \hat{x}_{k|k}\hat{x}_{k|k}' + Q \cdot \tag{11}$$

The filter at time $k_N + j$ on the horizon $[k - N = k_N, k]$ is denoted as $\hat{x}_{k_N+j|k}$ for $0 \le j \le N-1$. Then the suboptimal algorithm on the receding horizon $[k_N, k]$ becomes the following form:

$$\hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} \Big[F(I + P_{k_N+j|k}^{s} H'R^{-1}H)^{-1}\beta^{k,l} (\hat{x}_{k_N+j|k}^{s} + P_{k_N+j|k}^{s} H'R^{-1} y_{k_N+j,i_l}) \Big] \tag{12}$$

where the error covariance is obtained from (11) as follows:

$$\begin{aligned} P_{k_N+j+1|k} &= \sum_{l=1}^{L_k} \beta^{k,l} F(I + P_{k_N+j|k}^{s} H'R^{-1}H)^{-1} P_{k_N+j|k}^{s} F' \\ &\quad + \sum_{l=1}^{L_k} \beta^{k,l}\hat{x}_{k_N+j+1|k}^{l}\hat{x}_{k_N+j+1|k}'^{l} - \hat{x}_{k_N+j+1|k}\hat{x}_{k_N+j+1|k}' + Q. \end{aligned} \tag{13}$$

Remark 1: It is noted that $\hat{x}_{k_N+j|k}$, in (12), for $k_N + j < k$ is an intermediate variable to compute $\hat{x}_{k|k}$ and cannot be used as a real estimate. Only the state estimate $\hat{x}_{k|k}$ is used as a real estimate of the real target x_k.

Remark 2: The suboptimal algorithm based upon the receding horizon in this paper will be called a suboptimal receding horizon Bayesian filter. As known in (12) and (13), the state estimate is obtained from the horizon initial state estimate and covariance and the measurement on the receding horizon $[k_N, k]$. However, since past measurements outside the horizon is discarded in this algorithm, it is needed to estimate the horizon initial condition without past information. So, the horizon initial state estimate and covariance are derived from the measurements on the receding horizon $[k_N, k]$. In addition, due to Assumption 1, the horizon initial track is assumed to be correct.

On the receding horizon $[k_N, k]$, to express the finite number of measurements in terms of the horizon initial state x_{k_N} the following equations are needed:

$$x_{k_N+1} = Fx_{k_N} + \omega_{k_N} \tag{14}$$

and

$$\begin{aligned} y_{k_N+1} &= HFx_{k_N} + H\omega_{k_N} + v_{k_N+1}, \\ y_{k_N} &= Hx_{k_N} + v_{k_N}. \end{aligned} \tag{15}$$

Consequently, the substitution of (14) into (15) yields:

$$Y^{k-1} = \overline{H}x_{k_N} + \overline{G}W^{k-1} + V^{k-1} \tag{16}$$

where

$$\begin{aligned} Y^{k_N+j} &\overset{\Delta}{=} [y_{k_N}' \; y_{k_N+1}' \cdots y_{k_N+j}']', \\ W^{k_N+j} &\overset{\Delta}{=} [\omega_{k_N}' \; \omega_{k_N+1}' \cdots \omega_{k_N+j}']', \\ V^{k_N+j} &\overset{\Delta}{=} [v_{k_N}' \; v_{k_N+1}' \cdots v_{k_N+j}']', \quad 0 \le j \le N-1 \end{aligned}$$

and $\overline{H}$ and $\overline{G}$ are as follows:

$$\overline{H} \overset{\Delta}{=} \begin{bmatrix} H \\ HF \\ HF^2 \\ \vdots \\ HF^{N-1} \end{bmatrix},$$

$$\overline{G} \overset{\Delta}{=} \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ H & 0 & \cdots & 0 & 0 \\ HF & H & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ HF^{N-2} & HF^{N-3} & \cdots & H & 0 \end{bmatrix}.$$

Next, let us obtain the horizon initial state estimate and covariance from (16). Denoting the horizon initial condition $\hat{x}_{k_N|k}$ of (12) as $\hat{x}_{k_N|z}$, it is defined as follows:

$$\hat{x}_{k_N|z} \overset{\Delta}{=} ZY^{k-1}, \tag{17}$$

where Z is a gain matrix of the horizon initial state estimator. But, in order to use the estimate $\hat{x}_{k_N|z}$ as the horizon initial condition, it must be an unbiased estimate of $\hat{x}_{k_N}$ as

$$E[\hat{x}_{k_N|z}] = E[ZY^{k-1}] = Z\overline{H}E[x_{k_N}] = E[x_{k_N}].$$

Therefore, to hold the above equality, the following constraint is needed.

$$Z\overline{H} = I. \tag{18}$$

That is, Z must be a pseudo-inverse of $\overline{H}$. Provided that the constraint in (18) holds, the error covariance of $\hat{x}_{k_N|z}$, $P_{k_N|z}$, can be used as a horizon initial covariance $P_{k_N|k}$ in (13). First, let us define $\tilde{x}_{k_N|z} \overset{\Delta}{=} x_{k_N} - \hat{x}_{k_N|z} = x_{k_N} - ZY^{k-1}$. Then,

$$\begin{aligned} P_{k_N|z} &= E[\tilde{x}\tilde{x}'] \\ &= ZE[\overline{G}W^{k-1}W'^{k-1}\overline{G}' + V^{k-1}V'^{k-1}]Z'. \end{aligned} \tag{19}$$

The covariance of (16) is obtained as follows:

$$\begin{aligned} E[YY'] &= E[\overline{G}W_{k-1}W'_{k-1}\overline{G}' + V_{k-1}V'_{k-1}] \\ &= \overline{G}[\mathrm{diag}(Q\,Q\cdots Q)]\overline{G}' + [\mathrm{diag}(R\,R\cdots R)] \end{aligned}$$

where Q and R are the system and measurement noise covariances. For simplicity, the last term of the above equation is defined as follows:

$$\overline{\Theta}_N \overset{\Delta}{=} \overline{G}[\mathrm{diag}(Q\,Q\cdots Q)]\overline{G}' + [\mathrm{diag}(R\,R\cdots R)].$$

The substitution of the above equation into (19) yields the following horizon initial covariance:

$$P_{k_N|z} = Z\overline{\Theta}_N Z'. \tag{20}$$

Now, subject to the constraint of (18), the gain matrix Z must be searched for the horizon initial information (17) and (20). In order to obtain optimal gain matrix Z, the following optimal criterion is required.

$$Z = \arg\min_z E[x_k - \hat{x}_{k|z}]'[x_k - \hat{x}_{k|z}]. \tag{21}$$

That is, the gain matrix Z that minimizes the estimate error covariance is needed. Due to the monotonicity of the state error covariance, the gain matrix Z that minimizes the state error covariance at current time k is the same as the gain matrix Z that minimizes the initial state error covariance at the horizon initial time k_N:

$$Z = \arg\min_z E[x_{k_N} - \hat{x}_{k_N|z}]'[x_{k_N} - \hat{x}_{k_N|z}].$$

First, the following cost function is established:

$$\begin{aligned} J(Z,\lambda) &= E[x_k - \hat{x}_{k|k}]^2 + \lambda'(\overline{H}'Z - I) \\ &= Z'\overline{\Theta}_N Z + \lambda'(\overline{H}'Z - I). \end{aligned}$$

That is, the minimization of $J(Z,\lambda)$ with respect to Z and λ is required, which is the vector of Lagrange multipliers. Using $\partial J/\partial Z = 0$ and $\partial J/\partial \lambda = 0$, the gain matrix Z that minimizes $J(Z,\lambda)$ is given as follows (Kay, 1993, p. 153):

$$Z = \overline{\Theta}_N^{-1}\overline{H}(\overline{H}'\overline{\Theta}_N^{-1}\overline{H})^{-1}. \tag{22}$$

Then, the receding horizon initial state estimate and its error covariance are derived as follows:

$$\hat{x}_{k_N|z} = ZY^{k-1}, \tag{23}$$

$$P_{k_N|z} = Z\overline{\Theta}_N Z', \tag{24}$$

where the gain matrix Z is $Z = (\overline{H}'\overline{\Theta}_N^{-1}\overline{H})^{-1}\overline{H}'\overline{\Theta}_N^{-1}$.

Remark 3: Using (23) and (24), the state estimate of the suboptimal receding horizon Bayesian filter is given by

$$\hat{x}_{k|k} = \hat{x}_{k_N+j+1|k}\,|_{j=N-1},$$

where the intermediate variable $\hat{x}_{k_N+j|k}$ is drived in the following iterative form:

$$\begin{aligned} \hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} \Big[& F(I + P^s_{k_N+j}H'R^{-1}H)^{-1}\beta^{k,l} \cdot \\ & \times (\hat{x}^s_{k_N+j|k} + P^s_{k_N+j}H'R^{-1}y_{k_N+j,i_l}) \Big]. \end{aligned}$$

3. POSTERIORI PROBABILITY OF THE FILTER

Now, we are in a position to deal with the *a posteriori* probability of (3). The following assumptions are needed.

Assumption 2: The number of false validated measurements is described by diffuse prior model.

Assumption 3: The false measurements are uniformly distributed in the gate.

First, the vector at each time on the receding horizon $[k_N, k]$ is denoted as

$$\mathbf{m}^k = [m_{k-N} \cdots m_k]. \tag{25}$$

Theorem 2: Let Assumptions 1-3 hold. Then, the *a posterior* probability of the validated measurements on the receding horizon $[k-N, k]$ is given as follows:

$$\beta^{k,l} = \begin{bmatrix} \dfrac{e^s}{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}}, & i_l = 1,\cdots,m_k \\ \dfrac{b^s}{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}}, & i_l = 0 \end{bmatrix} \times \beta^{k-1,s}$$

where

$$e^s_{j_l} \overset{\Delta}{=} \exp\{-\frac{1}{2}(y_{k,j_l} - \hat{y}^s_{k|k-1})'S_k^{-s}(y_{k,j_l} - \hat{y}^s_{k|k-1})\}$$

and

$$b^s \overset{\Delta}{=} (1 - P_D P_G)\,|2\pi S^s_k|^{1/2}\,\frac{m_k}{P_D V_k}.$$

P_G is the probability that the true measurement will fall in the gate and P_D is the target detection probability. V_k is the volume of the validation region at time k.

Proof: First, the computation of the probability $\beta^{k,l}$ conditioned on $\mathbf{m}^k$ can be expressed as

$$\begin{aligned}\beta^{k,l} &= P[\theta^{k,l} \mid Y^k, \mathbf{m}^k] \\ &= \frac{1}{c} p[Y_k \mid \theta_{k,i_l}, m_k, \theta^{k-1,s}, \mathbf{m}^{k-1}, Y^{k-1}] \\ &\quad \times P[\theta_{k,i_l} \mid \theta^{k-1,s}, Y^{k-1}, m_k, \mathbf{m}^{k-1}] \\ &\quad \times P[\theta^{k-1,s} \mid Y^{k-1}, m_k, \mathbf{m}^{k-1}]\end{aligned} \tag{26}$$

where $c = p[Y_k \mid m_k, Y^{k-1}, \mathbf{m}^{k-1}]$ is the normalization constant. Under Assumption 3, the first joint PDF of the validated measurements on the right hand side of (26) is

$$p[Y_k \mid \theta_{k,i_l}, m_k, \theta^{k-1,s}, Y^{k-1}, \mathbf{m}^{k-1}] = \begin{cases} V_k^{-m_k+1} P_G^{-1} f[y_{k,i_l}; \hat{y}^s_{k|k-1}, S^s_k], & i_l = 1, \cdots, m_k \\ V_k^{-m_k}, & i_l = 0. \end{cases} \tag{27}$$

The second density on the right hand side of (26) is

$$P[\theta_{k,i_l} \mid m_k, \theta^{k-1,s}, Y^{k-1}, \mathbf{m}^{k-1}] = P[\theta_{k,i_l} \mid m_k] = \gamma_{i_l}(m_k)$$

$$= \begin{cases} \frac{1}{m_k} P_D P_G \{P_D P_G + (1-P_D P_G) \frac{\mu_F(m_k)}{\mu_F(m_k-1)}\}^{-1}, & i_l = 1 \cdots m_k \\ (1-P_D P_G) \frac{\mu_F(m_k)}{\mu_F(m_k-1)} \{P_D P_G + (1-P_D P_G) \frac{\mu_F(m_k)}{\mu_F(m_k-1)}\}^{-1}, & i_l = 0 \end{cases} \tag{28}$$

where $\mu_F(m_k)$ is the probability mass function of the number of false measurements. Using the diffuse prior model, (28) is rewritten as follows:

$$\gamma_{i_l}(m_k) = \begin{cases} \frac{P_D P_G}{m_k}, & i_l = 1, \cdots, m_k \\ 1 - P_D P_G, & i_l = 0. \end{cases} \tag{29}$$

The third density in (26) is available from the previous step as follows:

$$\beta^{k-1,s} = P[\theta^{k-1,s} \mid Y^{k-1}, m_k, \mathbf{m}^{k-1}]. \tag{30}$$

The substitution of (27)-(30) into (26) gives the following form:

$$\beta^{k,l} = \begin{Bmatrix} V_k^{-m_k+1} P_G^{-1} f[y_{k,i_l} - \hat{y}^s_{k|k-1}; 0, S^s_k] \left(\frac{P_D P_G}{m_k}\right), \\ \qquad i_l = 1, \cdots, m_k \\ V_k^{-m_k}(1 - P_D P_G), \qquad i_l = 0 \end{Bmatrix} \times \beta^{k-1,s}$$

Using $V_k = C_{n_y} |\gamma S^s_k|^{1/2} = C_{n_y} \gamma^{n_y/2} |S^s_k|^{1/2}$, the above equation becomes

$$\beta^{k,l} = \begin{Bmatrix} (C_{n_y} \gamma^{n_y/2} |S^s_k|^{1/2})^{-m_k+1} \\ \times \frac{P_G^{-1} \exp\{-\frac{1}{2}(y_{k,i_l} - \hat{y}^s_{k|k-1})' S_k^{-s} (y_{k,i_l} - \hat{y}^s_{k|k-1})\} P_D P_G}{|2\pi S^s_k|^{1/2} m_k}, \\ (C_{n_y} \gamma^{n_y/2} |S^s_k|^{1/2})^{-m_k}(1 - P_D P_G), \\ i_l = 1, \cdots, m_k \\ i_l = 0 \end{Bmatrix} \times \beta^{k-1,s},$$

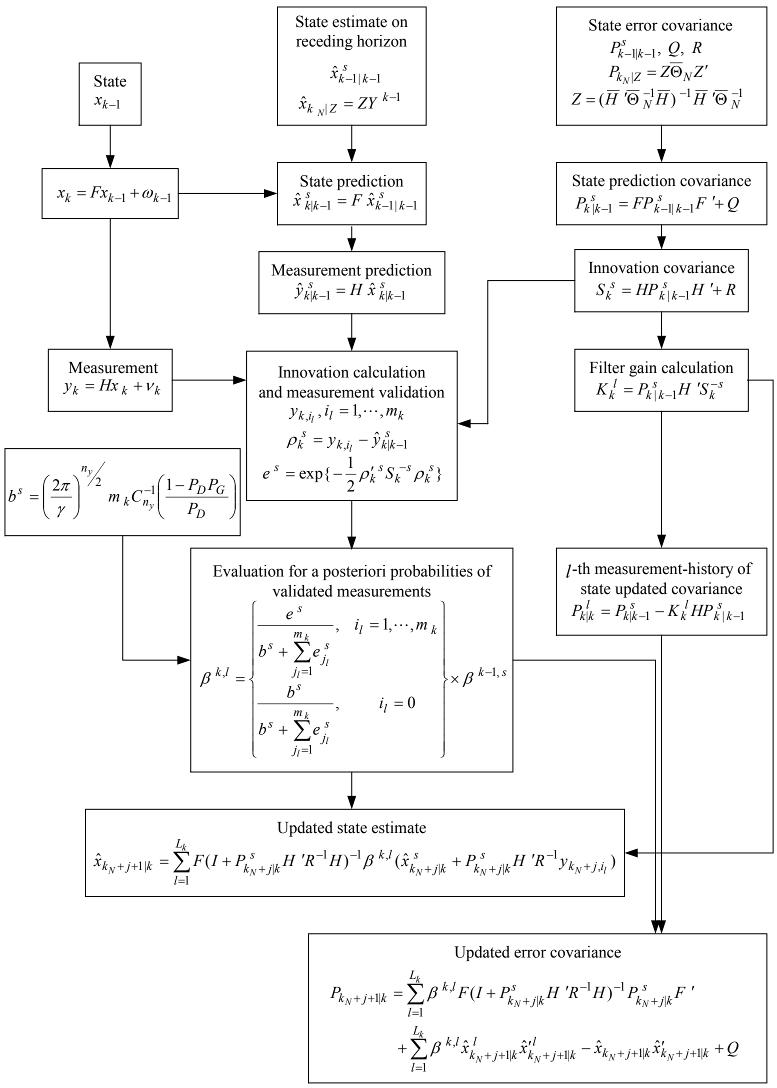

Fig. 1. One cycle of the suboptimal receding horizon Bayesian filter.

where C_{n_y} is the volume of the n_y-dimensional unit hypersphere and γ is the threshold of the gate. By normalizing this result, according to the value of i_l, $\beta^{k,l}$ is rewritten in the following two cases.

a) In case of $i_l = 1, \cdots, m_k$,

$$\beta^{k,l} = \frac{\exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}^s_{k|k-1})' S_k^{-s} (y_{k,i_l} - \hat{y}^s_{k|k-1})\right] \beta^{k-1,s}}{\sum_{i_l=1}^{m_k} \left\langle \begin{array}{l} \exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}^s_{k|k-1})' S_k^{-s} (y_{k,i_l} - \hat{y}^s_{k|k-1})\right] \\ \qquad + (1 - P_D P_G) \, | 2\pi S_k^s |^{1/2} \frac{m_k}{P_D V_k} \end{array} \right\rangle}.$$

b) In case of $i_l = 0$,

$$\beta^{k,l} = \frac{| 2\pi S_k^s |^{1/2} \frac{(1 - P_D P_G) m_k}{P_D V_k} \beta^{k-1,s}}{\sum_{i_l=1}^{m_k} \left\langle \begin{array}{l} \exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}^s_{k|k-1})' S_k^{-s} (y_{k,i_l} - \hat{y}^s_{k|k-1})\right] \\ \qquad + (1 - P_D P_G) \, | 2\pi S_k^s |^{1/2} \frac{m_k}{P_D V_k} \end{array} \right\rangle}.$$

Therefore, the *a posterior* probability conditioned on the validated measurements in the receding horizon $[k_N, k]$ is given by

$$\beta^{k,l} = \begin{bmatrix} \dfrac{e^s}{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}} & i_l = 1, \cdots, m_k \\ \dfrac{b^s}{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}} & i_l = 0 \end{bmatrix} \times \beta^{k-1,s}.$$

The scheme of the suboptimal receding horizon Bayesian filter derived in Theorem 1 and Theorem 2 is outlined in Fig. 1. Using the algorithm in Fig. 1, the computational complexity and storage requirements of the tracking filter can be substantially reduced compared to the standard optimal Bayesian filter.

4. CONCLUSIONS

In a cluttered environment, the use of the optimal Bayesian filter, as a possible solution to the target tracking problem, is often recommended. However, the computational burden and growing memory are known the main drawback of its use. The suboptimal algorithm proposed in this paper uses the measurements on the receding horizon and diminishes the computational complexity and storage requirement. Since the prior information outside the horizon was not available, the horizon initial state estimate and its covariance were obtained using the measurements in the receding horizon.

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