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NONLINEAR ADAPTIVE H_∞ CONTROL FOR ROBOTIC MANIPULATORS

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Abstract : A new class of adaptive nonlinear H_∞ control for robotic manipulators is proposed in this manuscript. Those control strategies are derived as solutions of particular nonlinear H_∞ control problems, where both disturbances and estimation errors of unknown system parameters are regarded as exogenous disturbances to the processes, and the $\mathcal{L}_2$ gains from those uncertainties to generalized outputs are prescribed explicitly. The resulting adaptive control systems are shown to be robust to the estimation errors in the adaptation schemes. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Recently, there has been much progress in the control of robotic manipulators. Some of those utilize useful physical properties of manipulators, such as positive realness or passivity, and the passivity-based control strategy has become one of the strong tools to deal with nonlinearity and uncertainty in manipulators and electrical-mechanical systems (Spong, 1992; Ortega and Spong, 1994). Additionally, nonlinear H_∞ control schemes of manipulators have been also discussed. Some of those utilize the solutions of corresponding Hamilton-Jacobi-Isaacs (HJI) equations (Chen, *et al*., 1994; Chen, *et al*., 1997; Nougata and Furuta, 1998), but there still remains the difficulty of solving nonlinear HJI equation. Another approach of those utilizes passivity and attains certain H_∞ control performance without HJI equation (Shen and Tamura, 1999). However, the control efforts are not penalized in the cost functionals, and sometimes, that approach gives rise to excessive control energy. Similar approach is also found (Battilotti and Lanaru, 1997), where HJI equation is introduced but no control efforts are penalized in the cost functionals.

In this manuscript, a new class of nonlinear H_∞ control schemes for robotic manipulators is proposed based on inverse optimality (Krstić and Deng, 1998; Miyasato, 1999). Those control schemes are derived as solutions of particular nonlinear H_∞ control problems, where both disturbances and estimation errors of unknown system parameters are regarded as exogenous disturbances to the processes (Miyasato, 2000). The resulting control systems are bounded with guaranteed certain H_∞ control performance; that is, the $\mathcal{L}_2$ gains from those uncertainties to generalized outputs (including control terms) are prescribed explicitly. Especially, the proposed control strategies are shown to be robust to the estimations errors of tuning parameters, and thus the good transient properties are attained even for the large estimation errors in adaptation schemes.

2. PROBLEM STATEMENT

Consider a robotic manipulator with n degrees of freedom described by the following equation:

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+G(\theta)=\tau+d, \quad (1)$$

where $\theta \in \mathbf{R}^n$ is a vector of joint angles, $M(\theta) \in \mathbf{R}^{n\times n}$ is a matrix of inertia, $C(\theta,\dot{\theta}) \in \mathbf{R}^{n\times n}$ is a matrix of Coriolis and centrifugal forces, $G(\theta) \in \mathbf{R}^n$ is a vector of gravitational torques, τ is a vector of input torques (control input), and d is a vector of external disturbance. It is assumed that the system parameters in $M(\theta)$, $C(\theta,\dot{\theta})$, $G(\theta)$ and external disturbance d are unknown. Only, θ, $\dot{\theta}$ and τ are assumed to be available for measurement.

It is known that robotic manipulator models with rotational joints have the following properties (Spong and Vidyasagar, 1989).

Properties of Robotic Manipulators.

1. $M(\theta)$ is a bounded, positive definite, and symmetric matrix.
$$0<\lambda_1 I \le M(\theta) \le \lambda_2 I < \infty, \quad \forall\theta. \tag{2}$$
2. $\dot{M}(\theta)-2C(\theta,\dot{\theta})$ is a skew symmetric matrix.
$$\xi^T\{\dot{M}(\theta)-2C(\theta,\dot{\theta})\}\xi = 0, \quad \forall\xi. \tag{3}$$
3. The left-hand side of (1) can be written into the following form,
$$M(\theta)a+C(\theta,\dot{\theta})b+G(\theta)=\Omega(\theta,\dot{\theta},a,b)^T\Phi, \tag{4}$$
where $\Omega(\theta,\dot{\theta},a,b)$ is a known function of θ, $\dot{\theta}$, a, b, and Φ is an unknown system parameter.

The control objective is to determine a suitable control input τ such that the joint angle vector θ follows the desired reference angle vector θ_d, while certain H_∞ control performance is attained.

3. TRACKING CONTROL SYSTEMS

The basic structure of the tracking control for robotic manipulators is shown (Shen and Tamura, 1999). First, set the input torque τ in the following form, where the reference angle vector θ_d is utilized.

$$\begin{aligned}\tau &= u+\hat{M}(\theta)\ddot{\theta}_d+\hat{C}(\theta,\dot{\theta})\dot{\theta}_d+\hat{G}(\theta)\\ &= u+\Omega(\theta,\dot{\theta},\ddot{\theta}_d,\dot{\theta}_d)^T\hat{\Phi}. \end{aligned}\tag{5}$$

$\hat{\Phi}$ is an estimate of the unknown parameter Φ, and u is a stabilizing control signal to be determined later. Denote the tracking error by

$$e \equiv \theta-\theta_d, \tag{6}$$

then, the next equation is derived.

$$\begin{aligned}&M(\theta)\ddot{e}+C(\theta,\dot{\theta})\dot{e}=u+d+\{\hat{M}(\theta)-M(\theta)\}\ddot{\theta}_d\\ &\quad +\{\hat{C}(\theta.\dot{\theta})-C(\theta,\dot{\theta})\}\dot{\theta}_d+\{\hat{G}(\theta)-G(\theta)\}.\end{aligned}\tag{7}$$

The augmented error signal s is introduced such as

$$s\equiv\dot{e}+\lambda e, \quad (\lambda>0). \tag{8}$$

The overall robotic system is rewritten by utilizing e and s as follows:

$$\dot{e} = s-\lambda e, \tag{9}$$
$$\begin{aligned}M\dot{s} &= (\lambda M-C)s-\lambda(\lambda M-C)e+u+d\\ &\quad +\{\hat{M}-M\}\ddot{\theta}_d+\{\hat{C}-C\}\dot{\theta}_d+\{\hat{G}-G\}.\end{aligned}\tag{10}$$

Define a positive function V by

$$V=\frac{1}{2}s^TM(\theta)s+\frac{1}{2}||e||^2, \tag{11}$$

and take the time derivative of it along the trajectory of s, e, and θ.

$$\begin{aligned}\dot{V} &= -\lambda||e||^2+s^T(\Phi_1 e+\Phi_2 s+e+u+d)\\ &\quad +s^T\{(\hat{M}-M)\ddot{\theta}_d+(\hat{C}-C)\dot{\theta}_d\\ &\quad +(\hat{G}-G)\}, \end{aligned}\tag{12}$$
$$\begin{aligned}\Phi_1 &= -\lambda^2 M+\lambda C,\\ \Phi_2 &= \lambda M.\end{aligned}\tag{13}$$

The control signals are obtained in the following forms, where v is also a stabilizing control signal to be determined later.

$$u = -\hat{\Phi}_1 e-\hat{\Phi}_2 s-e+v, \tag{14}$$
$$\begin{aligned}\hat{\Phi}_1 &= -\lambda^2\hat{M}+\lambda\hat{C},\\ \hat{\Phi}_2 &= \lambda\hat{M},\end{aligned}\tag{15}$$
$$\begin{aligned}\tau &= \hat{M}(\ddot{\theta}_d+\lambda^2 e-\lambda s)+\hat{C}(\dot{\theta}_d-\lambda e)+\hat{G}-e+v\\ &= \Omega(\theta,\dot{\theta},a,b)^T\hat{\Phi}-e+v,\end{aligned}\tag{16}$$
$$\begin{aligned}a &\equiv \ddot{\theta}_d+\lambda^2 e-\lambda s,\\ b &\equiv \dot{\theta}_d-\lambda e.\end{aligned}\tag{17}$$

The substitution of the control signal (14) into $\dot{V}$ (12) yields

$$\begin{aligned}\dot{V} &= -\lambda||e||^2+s^Tv+s^Td\\ &\quad +s^T\{(\hat{M}-M)\ddot{\theta}_d+(\hat{C}-C)\dot{\theta}_d+(\hat{G}-G)\\ &\quad -(\hat{\Phi}_1-\Phi_1)e-(\hat{\Phi}_2-\Phi_2)s\}\\ &= -\lambda||e||^2+s^Tv+s^Td+s^T\Omega(\theta,\dot{\theta},a,b)^T\tilde{\Phi},\end{aligned}\tag{18}$$
$$\tilde{\Phi} \equiv \hat{\Phi}-\Phi, \tag{19}$$

where a and b are defined by (17). The control signals (14), (16) and $\dot{V}$ (18) are fundamental formulas, from which all control schemes in this manuscript are deduced.

4. NONLINEAR ADAPTIVE CONTROL

First, it is assumed that $d=0$, and the conventional adaptive scheme (Narandra and Annaswamy, 1989; Shen and Tamura, 1999) is applied to the control of robotic manipulators. Determine the stabilizing control signal v and adaptive laws such as

$$v = -Ks, \quad (K=K^T>0), \tag{20}$$
$$\dot{\hat{\Phi}} = -\Gamma\Omega(\theta,\dot{\theta},a,b)s, \quad (\Gamma=\Gamma^T>0). \tag{21}$$

Define a positive function W by

$$W=V+\frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}, \tag{22}$$

and take the time derivative of it. Then it follows that

$$\dot{W}=-\lambda||e||^2-s^TKs\le 0, \tag{23}$$

and the next theorem is obtained.

Theorem 1 : *The adaptive control system of robotic manipulators (1) defined by (14), (16), (20), (21) is uniformly bounded, and the tracking errors e, s converge to zero asymptotically.*

$$\lim_{t\to\infty} e = 0, \quad \lim_{t\to\infty} s = 0. \tag{24}$$

5. NONLINEAR ADAPTIVE H_∞ CONTROL

The proposed nonlinear adaptive H_∞ control schemes are shown. First, the nonlinear H_∞ control schemes are discussed, where the tuning laws of $\hat{\Phi}$ are not specified. Next, the projection-type adaptive laws are introduced for the tuning of $\hat{\Phi}$, and the nonlinear adaptive H_∞ control schemes are presented. It is shown that both control systems are uniformly bounded, and attain certain H_∞ control performance.

5.1 Nonlinear H_∞ Control.

In this section, $\hat{\Phi}$ is an arbitrary bounded design parameter. The proposed nonlinear H_∞ control schemes are derived as solutions of certain H_∞ control problems, where $d \neq 0$ and parameter error $\tilde{\Phi}$ are regarded as external disturbances to the process. For that purpose, consider the following virtual process.

$$\dot{x} = f(x) + g_{11}(x)d + g_{12}(x)\tilde{\Phi} + g_2 v, \tag{25}$$

$$x = \begin{bmatrix} e \\ s \end{bmatrix}, \quad f(x) = \begin{bmatrix} -\lambda e \\ -M^{-1}Cs \end{bmatrix},$$
$$g_{11}(x) = g_2(x) = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$
$$g_{12}(x) = \begin{bmatrix} 0 \\ M^{-1}\Omega^T \end{bmatrix}. \tag{26}$$

It should be noted that the time derivative of V (11) along the trajectory of the virtual system (25), (26) is the same as (18). For the virtual system, consider the next Hamilton-Jacobi-Isaacs equation (HJI equation), where the solution V is given by (11).

$$\frac{\partial V}{\partial t} + \mathcal{L}_f V + \frac{1}{4}\left\{ \frac{||\mathcal{L}_{g_{11}}V||^2}{\gamma_1^2} + \frac{||\mathcal{L}_{g_{12}}V||^2}{\gamma_2^2} - \mathcal{L}_{g_2}V\, R^{-1}(\mathcal{L}_{g_2}V)^T \right\} + q(x) = 0. \tag{27}$$

The positive function $q(x)$ and positive definite symmetric matrix R are to be obtained from (27) based on inverse optimality, for the given solution V (11) and the positive constants γ_1, γ_2. The substitution of the solution V (11) into HJI equation (27) yields the next relation.

$$-\lambda||e||^2 + \frac{||s||^2}{4\gamma_1^2} + \frac{s^T\Omega^T\Omega s}{4\gamma_2^2} - \frac{1}{4}s^T R^{-1} s + q(x) = 0. \tag{28}$$

Then $q(x)$ and R are given by

$$q = \lambda||e||^2 + s^T\left(\frac{1}{4}R^{-1} - \frac{1}{4\gamma_1^2}I - \frac{1}{4\gamma_2^2}\Omega^T\Omega\right)s, \tag{29}$$

$$R = \left(\frac{1}{\gamma_1^2}I + \frac{1}{\gamma_2^2}\Omega^T\Omega + K\right)^{-1}, \quad (K = K^T > 0), \tag{30}$$

and the input signal (stabilizing control signal) v is obtained as a solution of the corresponding H_∞ control problem in the following way:

$$v = -\frac{1}{2}R^{-1}(\mathcal{L}_{g_2}V)^T = -\frac{1}{2}R^{-1}s. \tag{31}$$

Then, the next theorem is obtained for the original robotic manipulators (1).

Theorem 2 : *The nonlinear control system of robotic manipulators (1) defined by (14), (16), (31), (30) is uniformly bounded for arbitrary bounded design parameters $\hat{\Phi}$ $(\hat{\Phi}_1, \hat{\Phi}_2)$, and arbitrary bounded disturbance d. Additionally, v is an optimal control signal which minimizes the following cost functional J.*

$$J = \sup_{d,\tilde{\Phi}\in\mathcal{L}^2}\left\{\int_0^t (q + v^T R v)d\tau + V(t) - \gamma_1^2\int_0^t ||d||^2 d\tau - \gamma_2^2\int_0^t ||\tilde{\Phi}||^2 d\tau\right\}. \tag{32}$$

Furthermore, the next inequality holds.

$$\int_0^t (q + v^T R v)d\tau + V(t) \le \gamma_1^2\int_0^t ||d||^2 d\tau + \gamma_2^2\int_0^t ||\tilde{\Phi}||^2 d\tau + V(0). \tag{33}$$

Proof. By considering HJI equations (27), (28), $\dot{V}$ is evaluated as follows:

$$\begin{aligned}\dot{V} &= -\lambda||e||^2 + s^T v + s^T d + s^T\Omega^T\tilde{\Phi} \\ &= -\frac{||s||^2}{4\gamma_1^2} - \frac{s^T\Omega^T\Omega s}{4\gamma_2^2} + \frac{1}{4}s^T R^{-1} s - q \\ &\quad + s^T\Omega^T\tilde{\Phi} + s^T v + s^T d \\ &= \left(v + \frac{1}{2}R^{-1}s\right)^T R\left(v + \frac{1}{2}R^{-1}s\right) - v^T R v \\ &\quad -\gamma_1^2\left\|d - \frac{s}{2\gamma_1^2}\right\|^2 + \gamma_1^2||d||^2 \\ &\quad -\gamma_2^2\left\|\tilde{\Phi} - \frac{\Omega s}{2\gamma_2^2}\right\|^2 + \gamma_2^2||\tilde{\Phi}||^2 - q. \end{aligned} \tag{34}$$

Then, it is shown that v (31) is an optimal solution to J, and that the inequality (33) holds. The substitution of v (31) into (34) yields the next relation,

$$\dot{V} \le -q - v^T R v + \gamma_1^2||d||^2 + \gamma_2^2||\tilde{\Phi}||^2, \tag{35}$$

from which, uniform boundedness of the control system is derived.

Remark 1. ¿From the inequality (33), it is seen that the time average $\lim_{T\to\infty}\frac{1}{T}\int_0^T(q + v^T R v)dt$ can be made arbitrarily small by decreasing design parameters γ_1, γ_2.

Remark 2. In the proposed control strategy (14), (16), (31), (30), let $\gamma_2 \to \infty$. Then, the resulting control scheme is a usual nonlinear H_∞ control structure, where $\mathcal{L}_2$ gain from disturbance d to the generalized output $\sqrt{q + v^T R v}$ is prescribed.

5.2 Nonlinear Adaptive H_∞ Control.

Next, the adaptive H_∞ control scheme is proposed, where $\hat{\Phi}$ are tuned adaptively. For $d \neq 0$, the projection-type adaptive laws are introduced such as

$$\begin{aligned}
&\text{if } \|\hat{\Phi}(t)\| = N_\Phi \ \& \ \hat{\Phi}(t)^T \Gamma \Omega(t) s(t) < 0 \\
&\quad \dot{\hat{\Phi}}(t) = -\Gamma\Omega(t)s(t) + \Gamma \frac{\hat{\Phi}(t)\hat{\Phi}(t)^T}{\hat{\Phi}(t)^T \Gamma \hat{\Phi}(t)} \Gamma \Omega(t) s(t), \\
&\text{otherwise} \\
&\quad \dot{\hat{\Phi}}(t) = -\Gamma \Omega(t) s(t), \qquad (36)
\end{aligned}$$

where $\|\Phi\| \leq N_\Phi$ and $\|\hat{\Phi}(0)\| \leq N_\Phi$, and N_Φ is assumed to be known. Then, for the same W (22)

$$W = V + \frac{1}{2}(\hat{\Phi} - \Phi)^T \Gamma^{-1} (\hat{\Phi} - \Phi),$$

$\dot{W}$ is obtained such as

$$\dot{W} \quad \leq \quad -\lambda \|e\|^2 + s^T v + s^T d. \qquad (37)$$

That relation (37) corresponds to the virtual system (25), (26) with $\tilde{\Phi} = 0$. However, the same discussion as *5.1* can be also applied to this case, and the next theorem is derived for the same v (31), (30) and the adaptive laws (36).

Theorem 3 : *The nonlinear adaptive control system of robotic manipulators (1) defined by (14), (16), (31), (30), (36) is uniformly bounded for arbitrary bounded disturbance d. Additionally, v is an optimal control signal which minimizes the following cost functional J.*

$$\begin{aligned}
J \ = \ \sup_{d, \tilde{\Phi} \in \mathcal{L}^2} \Bigg\{ \int_0^t (q + v^T R v) d\tau + W(t) \\
-\gamma_1^2 \int_0^t \|d\|^2 d\tau \Bigg\}. \qquad (38)
\end{aligned}$$

Furthermore, the next inequality holds.

$$\begin{aligned}
&\int_0^t (q + v^T R v) d\tau + W(t) \\
&\qquad \leq \gamma_1^2 \int_0^t \|d\|^2 d\tau + W(0). \qquad (39)
\end{aligned}$$

Especially, when $d \in \mathcal{L}^2$, the tracking errors e, s converge to zero asymptotically.

$$\lim_{t\to\infty} e = 0,$$
$$\lim_{t\to\infty} s = 0.$$

Proof. The projection-type adaptive laws attain $\hat{\Phi} \in \mathcal{L}^\infty$. Then, the boundedness of adaptive systems is derived from Theorem 2. The optimality of v and inequality (39) are easily deduced from replacing $\tilde{\Phi}$ and V by 0 and W, respectively, in (34).

Remark 3. Similar to the previous case, the time average $\lim_{T\to\infty} \frac{1}{T}\int_0^T (q + v^T R v) dt$ can be made arbitrarily small by decreasing γ_1.

Remark 4. Theorem 2 also holds for the adaptive control scheme (14), (16), (31), (30), (36). Hence, the proposed adaptive system is robust to the estimation errors of tuning parameters $\tilde{\Phi}$, and this leads to good transient property with less control efforts.

Remark 5. The proposed adaptive control schemes are closely related to the work (Tomei, 1999), where similar nonlinear damping terms are introduced. However, the present method is derived as a solution for certain H_∞ control problems and the control efforts are also penalized.

6. SIMULATION STUDIES

A numerical simulation study is performed to show the effectiveness of the proposed adaptive control scheme. A SICE-DD arm (the standard manipulator model in the **S**ociety of **I**nstrument and **C**ontrol **E**ngineers (SICE)) with two-degree of freedom is considered. Physical parameters of the manipulator are written as follows:

$$\begin{aligned}
&m_1 = 12.27\,\text{kg}, \quad m_2 = 2.083\,\text{kg}, \\
&I_1 = 0.1149\,\text{kg}\cdot\text{m}^2, \quad I_2 = 0.0114\,\text{kg}\cdot\text{m}^2, \\
&l_1 = 0.2\,\text{m}, \quad l_2 = 0.2\,\text{m}, \\
&r_1 = 0.063\,\text{m}, \quad r_2 = 0.080\,\text{m}.
\end{aligned}$$

The desired trajectories are given by

$$\begin{aligned}
&\theta_{d1}(t) = 6\pi \cdot \left(\frac{1}{2}t^2 - \frac{1}{3}t^3\right), \\
&\theta_{d2}(t) = \pi - 2\theta_{d1}(t), \\
&\theta_{d1}(0) = 0, \\
&\theta_{d2}(0) = \pi, \\
&\theta_{d1}(1) = \pi, \\
&\theta_{d2}(1) = -\pi, \\
&\dot{\theta}_{d1}(0) = \dot{\theta}_{d1}(1) = \dot{\theta}_{d2}(0) = \dot{\theta}_{d2}(1) = 0.
\end{aligned}$$

For comparison with the conventional adaptive control scheme, it is assumed that $d = 0$.

The design parameters are chosen such that

$$\Gamma = I, \qquad \lambda = 1, \qquad K = I.$$

The simulation results are shown in the followings, where $e_1 = \theta_1 - \theta_{d1}$, $e_2 = \theta_2 - \theta_{d2}$, and

- Case 1: conventional adaptive control scheme (Theorem1)

$$v = -Ks,$$
$$\int_0^1 e_1(t)^2 dt = 0.21286\text{E}-02,$$
$$\int_0^1 e_2(t)^2 dt = 0.24662\text{E}-02,$$
$$\int_0^1 ||e(t)||^2 dt = 0.45948\text{E}-02,$$
$$\int_0^1 \tau_1(t)^2 dt = 0.58322\text{E}+01,$$
$$\int_0^1 \tau_2(t)^2 dt = 0.37925\text{E}+00,$$
$$\int_0^1 ||\tau(t)||^2 dt = 0.62115\text{E}+01,$$

- Case 2: usual H_∞ control scheme with $\gamma_1 = 0.1$, $\gamma_2 \to \infty$ (Theorem 3)

$$v = -\frac{1}{2}\left(\frac{1}{\gamma_1^2}I + K\right)s,$$
$$\int_0^1 e_1(t)^2 dt = 0.14007\text{E}-02,$$
$$\int_0^1 e_2(t)^2 dt = 0.11984\text{E}-02,$$
$$\int_0^1 ||e(t)||^2 dt = 0.25992\text{E}-02,$$
$$\int_0^1 \tau_1(t)^2 dt = 0.52174\text{E}+01,$$
$$\int_0^1 \tau_2(t)^2 dt = 0.36290\text{E}+00,$$
$$\int_0^1 ||\tau(t)||^2 dt = 0.55803\text{E}+01,$$

- Case 3 : proposed control scheme with $\gamma_1 = 0.1$, $\gamma_2 = 0.1$ (Theorem 3)

$$v = -\frac{1}{2}\left(\frac{1}{\gamma_1^2}I + \frac{1}{\gamma_2^2}\Omega^T\Omega + K\right)s,$$
$$\int_0^1 e_1(t)^2 dt = 0.30649\text{E}-03,$$
$$\int_0^1 e_2(t)^2 dt = 0.24101\text{E}-03,$$
$$\int_0^1 ||e(t)||^2 dt = 0.54750\text{E}-03,$$
$$\int_0^1 \tau_1(t)^2 dt = 0.43353\text{E}+01,$$
$$\int_0^1 \tau_2(t)^2 dt = 0.32726\text{E}+00,$$
$$\int_0^1 ||\tau(t)||^2 dt = 0.46626\text{E}+01.$$

7. CONCLUSION

A design method of adaptive nonlinear H_∞ control for robotic manipulators is presented in this manuscript. The proposed control schemes are derived as solutions of particular nonlinear H_∞ control problems, where both disturbances and estimation errors of unknown system parameters are regarded as exogenous disturbances to the processes. ¿From the several numerical simulation studies (Case 1 ∼ Case 3), it is seen that the proposed control strategy (Case 3) has better transient properties with less control energy compared with the conventional adaptive control (Case 1) or even usual nonlinear H_∞ control schemes (Case 2). Those are owing to that the control efforts are also penalized in the corresponding H_∞ control problems. Nevertheless, there is no necessity of solving nonlinear HJI equation in the proposed control schemes.

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