ECONOMIC LOAD ALLOCATION

Ján Šomvársky and Vladimír Havlena

Honeywell Prague Laboratory, Honeywell Intl.
Pod vodárenskou věží 4, CZ-182 08 Prague, Czech Republic
somvarsky@htc.honeywell.cz, havlena@htc.honeywell.cz

Abstract: The presented algorithm provides solution of the economic load allocation (ELA) problem, minimizing total steam production costs, as well as dynamic allocation. The total fuel feed trajectory demand is provided by a predictive master pressure controller which controls pressure in a common steam header. Application of a model-based predictive controller as the master pressure controller provides additional features to the allocation algorithm: fuel flow at the end of the prediction horizon is used as an estimate of the steady state value for evaluation of the economic allocation, while the transient part can be allocated independently. *Copyright © 2002 IFAC.*

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1. INTRODUCTION

If steam is generated by several units (boilers) supplying steam to a common header, pressure in the header should be controlled by just one controller, a Master Pressure Controller (MPC), providing total energy input (fuel feed), followed by an allocation module which divides it into fuel feed demands (set points) for individual boilers. Note that individual pressure controllers with integral action for each boiler running in parallel can lead to instabilities. The allocation should be cost optimized in varying economic conditions (prices of fuels and electricity, environmental limits) while taking into account total production demands, and a number of other constraints, e.g., technological constraints, runtime hours and life time consumption.

The underlying optimization problem is well known and has been solved in a variety of ways and complexity levels since 60's. With most packagers, boiler steam load allocation is realized through biasing the appropriate boiler masters. As the steam demand of the header changes, the allocator adjusts the biasing to assure the minimum cost solution is obtained. This approach has weak tie to real-time control system as it provides steam flow biases instead directly fuel feed allocation with negative impact on performance namely in transient states.

The presented algorithm provides real-time online allocation of time varying total fuel feed demand among boilers, taking into account constraints. The resulting *dynamic allocation* is composed of the *target (steady state) allocation* (set by operator or provided by the optimization routine), and of the dynamic part proportional to the boilers' *dynamic weights* (set by operator).

The allocation module is intended for the use with a model-based *predictive* MPC, like (Havlena and Findejs, 2000), in which case new possibilities are provided: steady state load balance based on the steady state fuel flow at the end of prediction horizon can be used as a target value for computation of target economic allocation, and dynamic deviations in the transient part of prediction period are allocated proportionally to the dynamic weights (set by operator).

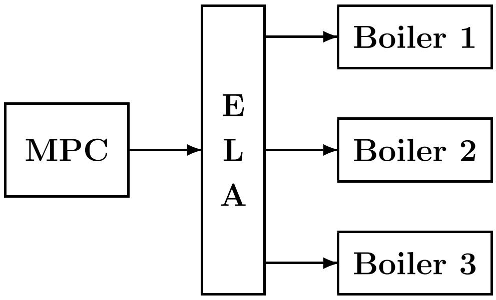

Fig. 1. Master pressure controller (MPC) provides total fuel feed demand *trajectory*, and economic load allocation (ELA) module allocates it among boilers.

If a predictive MPC is used, in addition to the absolute (technological) constraints, also rate of change constraints are introduced with positive impact on the stability of combustion process and thermal stress and life time of the technology.

The algorithm is modular, e.g., the target allocation can be set manually or provided by the optimization routine using boilers' cost curves which can be evaluated from efficiency curves, etc. Modular structure allows us to add new features (e.g., to take into account burner configuration, fuel mixing, etc.) without the necessity to change the basic algorithm. The off-line what-if analysis as a decision support tool is also possible using existing routines.

2. DYNAMIC ALLOCATION

A master pressure controller (MPC), controlling the pressure in a common steam header to which several units (boilers) supply steam, provides total energy load (fuel feed) demand. It should be divided into fuel feed demands (set points) for individual boilers (Fig. 1). If MPC is a predictive controller, the total energy load demand, $\mathbf{F}^{\text{tot}} = (F^{\text{tot}}(1), F^{\text{tot}}(2), \ldots, F^{\text{tot}}(K))$, is a trajectory, i.e., a sequence of values corresponding to a sequence of prediction times up to prediction horizon (numbered by $k = 0, 1, \ldots, K$).

The dynamic allocation trajectories are calculated in two steps: first, the unconstrained allocation trajectories, $F_i^{\text{unconstr}}(k)$, are determined, and then, they are modified to satisfy constraints and approach the unconstrained allocation as much as possible (in sense of minimum least squares of deviations) obtaining dynamic allocation trajectories, $F_i^{\text{dyn}}(k)$.

The unconstrained allocation trajectories, $F_i^{\text{unconstr}}(k)$, are defined by means of two sets of parameters: (1) the target allocation, $F_i^{\text{targ}}(k)$, and (2) the dynamic weights, $w_i^{\text{dyn}}(k)$, (both possibly trajectories) as follows:

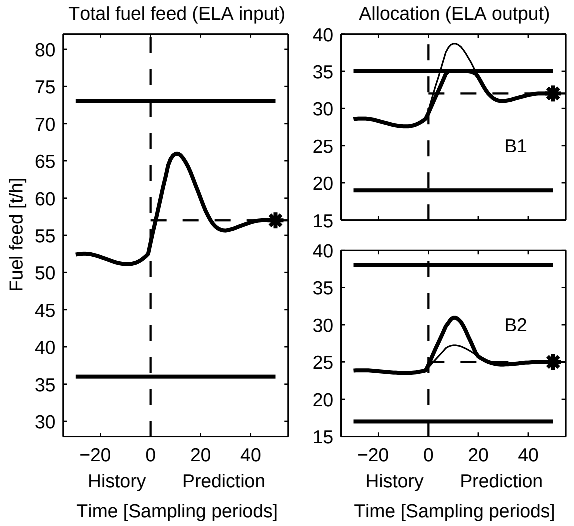

Fig. 2. Example of dynamic fuel feed allocation: historical (realized) values and predicted trajectories. Total fuel feed, F^{tot}, is allocated among two boilers, B1 and B2: thin curves – F_i^{unconstr}, thick curves – F^{tot}, F_i^{dyn}, horizontal thick lines – absolute limits (low and high), horizontal dashed lines – F_i^{targ}, '$*$' – steady state fuel feeds, $i = 1, 2$, $w_1^{\text{dyn}} : w_2^{\text{dyn}} = 3 : 1$.

$$F_i^{\text{unconstr}}(k) = F_i^{\text{targ}}(k) + w_i^{\text{dyn}}(k)\,\Delta F^{\text{tot}}(k), \quad (1)$$

where

$$\Delta F^{\text{tot}}(k) = F^{\text{tot}}(k) - \sum_{i=1}^{N} F_i^{\text{targ}}(k) \qquad (2)$$

for each boiler, $i = 1, 2, \ldots, N$, and each trajectory point, $k = 0, 1, \ldots, K$.

The target allocation corresponds to values around (or near) which the unconstrained allocation varies, and the dynamic weights are a measure of boilers' sensitivities to the changes of total load demand (Fig. 2). So, the changes in total energy load demand are divided into the boiler loads proportionally to the dynamic weights, i.e., the allocated value is composed of target value plus portion from transient deviation. The expected balance condition that the sum over boilers of allocated values is equal to total fuel feed demand, $\sum_{i=1}^{N} F_i^{\text{unconstr}}(k) = F^{\text{tot}}(k)$, is fulfilled for each trajectory point k, assuming the dynamic weights are normalized to 1 ($\sum_{i=1}^{N} w_i^{\text{dyn}}(k) = 1$).

The target allocation is set directly (by operator) or evaluated from the *offsets* (set by operator) or computed by the optimization routine (described in the next section). The target allocation is assumed to be constant during prediction period (independent of k) unless it is significantly changed (by operator). In this case, instead of using the new value for the whole prediction period, linear interpolation from the old value at $k = 0$ to the new one at $k = K$ (ramp) is used in order to assure bumpless operation.

If the target allocation is set directly, i.e. it is time-invariant up to the next operator intervention, its sum over boilers cannot follow the time-varying target total fuel feed demand (value at the end of prediction horizon). This appreciable balance condition can be fulfilled, if F_i^{targ} are evaluated using *offsets*, F_i^{off}:

$$F_i^{\text{targ}} = F_i^{\text{off}} + F^{\text{av}} \tag{3}$$

where F^{av} is determined from the balance condition:

$$F^{\text{tot}}(K) = \sum_{i=1}^{N} F^{\text{targ}} = \sum_{i=1}^{N} F_i^{\text{off}} + NF^{\text{av}} \tag{4}$$

Thus, even if offsets (defining differences between boiler loads) are constant, the $\sum_{i=1}^{N} F_i^{\text{targ}}$ is time varying and equal to the $F^{\text{tot}}(K)$. Alternatively, the offsets of steam flows can be set, and converted to fuel feeds assuming steady state.

The dynamic weights are set by operator. They are assumed to be constant during prediction period (independent of k) unless it is significantly changed (by operator). In this case, they are ramped using the same algorithm as for $F_i^{\text{targ}}(k)$, and normalized to 1.

However, the allocation $F_i^{\text{unconstr}}(k)$ does not need to satisfy constraints. ELA *active* constraints are obtained as intersection of technology, operator and algorithm-generated constraints propagated from slave controllers on all levels of sub cascade. Constraints generated by algorithm can be time varying, hence, also ELA active constraints can be trajectories. There are absolute constraints (low and high limits, $F_i^{\min}(k)$ and $F_i^{\max}(k)$, respectively), and rate of change ones (decremental and incremental limits, $F_i^-(k)$ and $F_i^+(k)$, respectively). While absolute constraints must not be violated, rate of change ones can be, but it should be highly penalized (to avoid undesired thermal stress which has impact on the life time of the equipment).

The final dynamic allocation trajectories, $F_i^{\text{dyn}}(k)$, must satisfy total fuel feed demand (as $F_i^{\text{unconstr}}(k)$ do)

$$\sum_{i=1}^{N} \mathbf{F}_i = \mathbf{F}^{\text{tot}} \tag{5}$$

and, in addition, the absolute (hard) constraints, and the rate of change constraints, considered as *soft*, i.e., they can be violated by arbitrary (but highly penalized) values $z_i(k)$:

$$\mathbf{F}_i^{\min} \le \mathbf{F}_i^{\text{dyn}} \le \mathbf{F}_i^{\max}, \tag{6}$$

$$-\mathbf{F}_i^- \le \boldsymbol{\Delta}\mathbf{F}_i^{\text{dyn}} - \mathbf{z}_i \le \mathbf{F}_i^+, \tag{7}$$

where

$$\mathbf{F}_i^{\text{dyn}} = \begin{pmatrix} F_i^{\text{dyn}}(0) \\ \vdots \\ F_i^{\text{dyn}}(K) \end{pmatrix}, \qquad i = 1, \ldots, N \tag{8}$$

$$\mathbf{F}^{\text{dyn}} = \begin{pmatrix} \mathbf{F}_1^{\text{dyn}} \\ \vdots \\ \mathbf{F}_N^{\text{dyn}} \end{pmatrix} \tag{9}$$

and same for $\mathbf{F}_i^{\min}$, $\mathbf{F}_i^{\max}$, $\mathbf{F}_i^-$, $\mathbf{F}_i^+$, $\mathbf{z}_i$, $i = 1, \ldots, N$, and $\mathbf{F}^{\text{tot}}$, all with dimension $(K+1)$, and $\mathbf{F}^{\min}$, $\mathbf{F}^{\max}$, $\mathbf{F}^-$, $\mathbf{F}^+$ and $\mathbf{z}$, all with dimension $N(K+1)$. The difference vector, $\boldsymbol{\Delta}\mathbf{F}_i^{\text{dyn}}$, is defined as follows:

$$\boldsymbol{\Delta}\mathbf{F}_i^{\text{dyn}} = \mathbf{D}\mathbf{F}_i^{\text{dyn}} - \mathbf{F}_i^{\text{act}} \tag{10}$$

with $(K+1) \times (K+1)$ difference matrix $\mathbf{D}$, and $(K+1)$-dimensional vectors $\mathbf{F}_i^{\text{act}}$ of boilers' actual fuel feeds in the first components and zeros elsewhere:

$$\mathbf{D} = \begin{pmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{pmatrix} \qquad \mathbf{F}_i^{\text{act}} = \begin{pmatrix} F_i^{\text{act}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{11}$$

Variables $z_i(k)$ are introduced in order to penalize violation of the rate of change constraints. If $z_i(k)$ is equal to zero (no penalty), the $\Delta F_i^{\text{dyn}}(k)$ must lie within rate of change limits according to the inequalities (7). If $\Delta F_i^{\text{dyn}}(k)$ is not within corresponding limits, the variable $z_i(k)$ is equal to the deviation of $\Delta F_i^{\text{dyn}}(k)$ from range $\langle -F_i^-(k), F_i^+(k) \rangle$, and limit violation penalty defined as norm of $\mathbf{z}$ becomes nonzero.

The dynamic allocation, $\mathbf{F}^{\text{dyn}}$, is obtained by minimizing penalty for deviation $\mathbf{F}^{\text{dyn}}$ from unconstrained allocation $\mathbf{F}^{\text{unconstr}}$, and for violation of rate of change limits, i.e., by minimizing the function

$$f(\mathbf{F}, \mathbf{z}) = \left|\left|\mathbf{F} - \mathbf{F}^{\text{unconstr}}\right|\right|^2_{\mathbf{Q}^{(1)}} + ||\mathbf{z}||^2_{\mathbf{Q}^{(2)}} \tag{12}$$

with respect to variables $\mathbf{F}^{\text{dyn}}$ and $\mathbf{z}$, subjected to constraints (5)–(7). It is a quadratic programming problem with dimension $2N(K+1)$.

The square $N(K+1) \times N(K+1)$ norm matrices $\mathbf{Q}^{(1)}$ and $\mathbf{Q}^{(2)}$ can be chosen as diagonal ones with elements depending only on boiler i, not on trajectory point k:

$$\mathbf{Q}^{(j)} = \begin{pmatrix} \mathbf{q}_1^{(j)} & & \\ & \ddots & \\ & & \mathbf{q}_N^{(j)} \end{pmatrix} \qquad j = 1, 2 \tag{13}$$

$$\mathbf{q}_i^{(j)} = w_i^{(j)} \mathbf{I} \tag{14}$$

with $(K+1) \times (K+1)$ unit matrix $\mathbf{I}$ and penalty weights $w_i^{(1)}$ and $w_i^{(2)}$, $i = 1, \ldots, N$.

Note that only the allocation for the first trajectory point $k = 0$ is realized. In the next control loop, the whole dynamic allocation trajectories are computed anew (receding horizon concept).

3. ECONOMIC STEADY STATE ALLOCATION

The objective of economic load allocation algorithm is to optimize the resource allocation for steam, power, heat, compressed air, chilled water or other media produced by parallel resources (steam is assumed in this paper). The criterion is to minimize production costs over the effective range of production while taking into account total production demands for individual products, and a number of other constraints.

3.1 *Steam/fuel conversion*

The ELA algorithm divides total *steam flow* demand into *steam flows* for individual boilers It is the significant difference from the dynamic allocation algorithm which works with *manipulated variables* (fuel feeds). Thus, total fuel feed demand should be converted to steam flow, and optimized steam flow allocation to fuel feeds.

The relation between instantaneous values of fuel feed and of steam flow is well-defined only in steady state operation or in sense of averaged values over long enough time period which compensates dynamical response. We use the simple steam/fuel conversion

$$F_i = \frac{S_i}{k_i\,\eta_i(S_i)} \tag{15}$$

with conversion factor

$$k_i = \frac{\bar{S}_i}{\bar{F}_i\,\eta_i(\bar{S}_i)} \tag{16}$$

where $\bar{F}_i$ and $\bar{S}_i$ are moving average values of realized fuel feed and steam flow over defined time period and are regularly updated. η_i is the efficiency curve of i-th boiler. The conversion factor defined by equation (16) is relatively stable with varying boiler load.

3.2 *Efficiency curves*

Efficiency curves of boilers are measured and calculated according to standard direct or indirect methods (ASME, 1970). At a fixed level of steam flow, efficiency is affected by excess of air. In such case, a set of efficiency curves is obtained, each for a different value of air excess determined by the content of O_2 in flue gases. If air-to-fuel-ratio optimization is used, different excess air levels

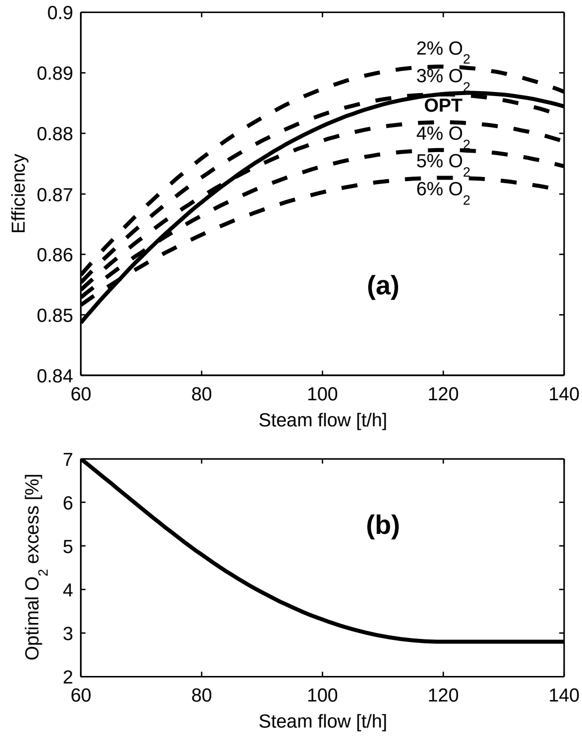

Fig. 3. Efficiency curves for different values of air excess (expressed in terms of O_2 content in flue gases) and for optimized air excess (a), and optimum O_2 content as function of steam flow.

are achieved at different load levels. From the set of efficiency curves, resulting efficiency curve can be constructed: for each load level, the efficiency corresponding to optimum air excess is taken (Fig. 3).

Efficiency curves can be constructed for different boiler configurations (e.g. the number of operating burners). If configuration changes, efficiency (and cost) curve is switched.

3.3 *Cost curves*

The ELA algorithm is based on cost curves of individual boilers. Cost curve is a function of *steam flow* and depends on additional time-varying parameters (tariffs, prices of fuel, fuel quality) as well as on time-invariant ones (e.g. efficiency curve). Total cost curve, the sum of boiler cost curves

$$C^{\text{tot}}(S_1, \ldots, S_N) = \sum_{i=1}^{N} C_i(S_i) \tag{17}$$

serves for determining the optimum allocation. A boiler cost curve is evaluated in terms of efficiency curve as

$$C_i(S_i) = \kappa_i\,\frac{S_i}{\eta_i(S_i)}, \tag{18}$$

where the price factor κ_i can be determined from the difference of specific enthalpy of output steam

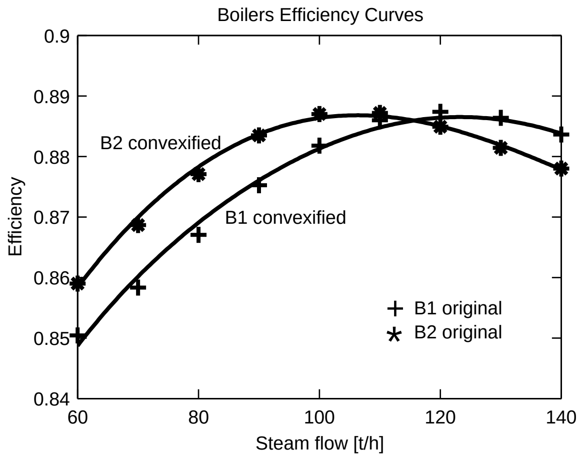

Fig. 4. Efficiency curves original (measured) and those obtained from convexified cost curves.

and input water Δh_i, calorific value of fuel q_i, and price of fuel p_i, as

$$\kappa_i = p_i \frac{\Delta h_i}{q_i}. \tag{19}$$

3.4 *Convexification of cost curves*

Boilers' cost curves are be convexified in order to ensure unambiguous solution of the optimal allocation problem, minimizing the cost function (17).

Boiler cost curve in practice is determined by a discrete set of points (steam flows and corresponding values). Convexification means to calculate new values (costs) such that the first and the second derivatives (calculated by difference method) are greater than the given threshold values while minimizing the norm of deviations of new values from old ones, and possibly norm of third derivatives (in order to get smoother cost curve).

Convexified cost curve points are locally (say, at S^0) approximated by a quadratic polynomial, $p_0(S^0)S^2 + p_1(S^0)S + p_2(S^0)$. Relevance of points far from S^0 is suppressed by a weighting function.

Convexification is of crucial importance as it ensures unambiguity of solution whereas shifts from original values are relatively small in comparison with experimental errors (see Fig. 4).

3.5 *Optimization algorithm*

Cost optimal steam flow allocation, S_i^{opt}, is obtained by minimizing total cost curve (17) under constraints

$$\sum_{i=1}^{N} S_i^{\text{opt}} = S^{\text{tot}}, \tag{20}$$

$$S_i^{\min} \le S_i^{\text{opt}} \le S_i^{\max} \; . \tag{21}$$

To solve this problem, total cost curve is approximated by a quadratic form, and total steam flow and steam flow limits are calculated.

In the first step, target fuel feed allocation calculated in the previous run of optimization routine, $F_i^{\text{targ,old}}$, is converted to steam flows, S_i^{old}, using implicit equation (15). New value of total steam flow demand, S^{tot}, should be proportional to the target value of total fuel feed, $F^{\text{tot}}(K)$:

$$S^{\text{tot}} = F^{\text{tot}}(K) \sum_{i=1}^{N} S_i \Bigg/ \sum_{i=1}^{N} F_i^{\text{targ,old}} \; , \tag{22}$$

where the proportionality constant is an effective conversion factor for given allocation being in accordance with the MPC algorithm.

Starting estimation of new steam flow allocation, S_i^0, is chosen to be proportional to old one but satisfying total steam flow demand:

$$S_i^0 = S_i^{\text{old}} \, S^{\text{tot}} \Bigg/ \sum_{i=1}^{N} S_i^{\text{old}} \; . \tag{23}$$

If allocation S_i^0 does not lie within steam flow limits set by operator or converted from operator fuel feed limits, the new constrained allocation, S_i^1, is determined using similar procedure as for calculation of F_i^{dyn} from F_i^{unconstr}.

Coefficients $p_{0,i}$, $p_{1,i}$ and $p_{2,i}$ of quadratic approximations of boilers cost curves at points S_i^1 are then calculated, allowing to express approximated total cost curve as quadratic form

$$C^{\text{tot}}(\mathbf{S}) = \sum_{i=1}^{N} C_i(S_i) =$$

$$\mathbf{S}^T \begin{pmatrix} p_{0,1} & & \\ & \ddots & \\ & & p_{0,N} \end{pmatrix} \mathbf{S} + \mathbf{S}^T \begin{pmatrix} p_{1,1} \\ \vdots \\ p_{1,N} \end{pmatrix} + \sum_{i=1}^{N} p_{2,i} \; . \tag{24}$$

Steam flow limits $S_i^{\min}$ and $S_i^{\max}$ are set to intersection of operator ranges, and in addition restricted to defined ranges around S_i^1 in order to prevent too abrupt reallocations.

Optimal allocation is basically related to incremental cost curves (Fig. 5).

Finally, the solution S_i^{opt} is converted into the new target fuel feed allocation, $F_i^{\text{targ,new}}$. The optimized total fuel flow should be less than (or equal to) the original one: $\sum_{i=1}^{N} F_i^{\text{targ,new}} \le F^{\text{tot}}(K)$.

The existence of unambiguous solutions is obvious requirement assuring stability of operation. It is fulfilled if cost curves are increasing convex functions. But the convexity of cost curves (measured or evaluated from efficiency curves) is not assured. In such case, they are modified to be convex.

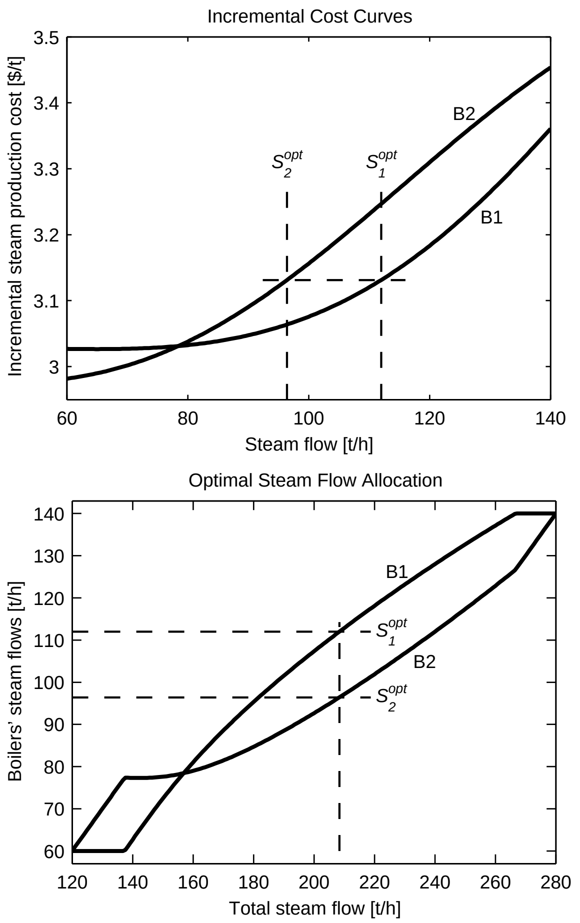

Fig. 5. Incremental cost curves determine optimal allocation: boilers' steam flows corresponding to equal incremental costs minimize total cost. This common value depends on total steam flow. Allocation curves can be constructed in such a way for whole range.

Even if unambiguous solution exists, numerical problems (speed of convergence) can arise. For this reason, the non linear programming problem is transformed into sequence of quadratic programming (QP) problems — sequential quadratic programming (SQP) method — with local approximation of cost curves by quadratic functions in each iteration step. SQP is implemented as iterations spread in time (IST) where one iteration (QP problem) is solved in one control loop.

3.6 *Virtual units*

Boilers can be grouped to form virtual units. The necessary condition is the availability of total cost curve of unit. Operation of several virtual units can be optimized by another (ELA) optimizer, and in this way, hierarchical control system can be built up.

Total cost curve of virtual unit is constructed from the cost curves of its members (boilers) in similar way as optimized efficiency curve in previous paragraph: total cost for each total load level is set to that at optimized allocation.

4. CONCLUSIONS

Presented ELA algorithm fully exploits advanced features of the predictive control technology. Using the whole predicted trajectory of fuel flow, the economic allocation based on steady state steam loads and transient optimization based e.g. on boiler dynamics considerations are fully separated.

Operation on the trajectories also enables the algorithm to avoid future unfeasible solutions of the allocation problem by constraint back propagation.

The solution was implemented and tested at a combined heat and power plant with three boilers 125 t/h.

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