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AN EXTENDED ROBUST H_∞ FILTER DESIGN FOR SDINS IN-FLIGHT ALIGNMENT

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Abstract: In this paper, a robust filter for an in-flight alignment (IFA) is presented to effectively eliminate system errors in the case where a strapdown inertial navigation system (SDINS) has large initial attitude errors. First, an extended robust H_∞ filter is proposed for a general nonlinear uncertain system. We also analyze the characteristics of the proposed filter, such as an H_∞ performance criterion, using the Lyapunov function method. Analysis results show that the proposed filter has robustness against disturbances, such as process and measurement noises, and against parameter uncertainties. Then the IFA for the SDINS is designed using the presented filter. Simulation results demonstrate that the proposed filter effectively improve the performance. *Copyright © 2002 IFAC*

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1. INTRODUCTION

A strapdown inertial navigation system(SDINS) requires accurate alignment in order to achieve good performance. However, for systems that require rapid reaction time or have low-grade inertial sensors, an initial alignment that is performed prior to launch must have large attitude errors. In these cases, IFA is important to remove the effects of initial alignment errors and other types of navigation errors. IFA utilizes acceleration data and angular rate data from inertial measurement unit, together with data provided by external sensors such as GPS, radar, or odometer (Farrel 99, Siouris 93, Weinreb 78, Bar-Itzhack 92, Yu 99, Yu 01). When an aided SDINS is developed for IFA, the main concerns are the filter and the SDINS error model since they play an important role in achieving accurate alignment. A considerable amount of effort has been made to develop effective error models. However, in case of system with large attitude errors, it is difficult to accurately linearize the SDINS error model (Yu 99). Therefore, error models include significant parameter uncertainty. This uncertainty degrades performance of the filter. During the last four decades, the Kalman filter and the extended Kalman filter(EKF) have been widely used in the SDINS IFA. They require not only a precise system model, but also the statistical property of noise to achieve accurate performance. However, model uncertainty and incomplete statistical information often occur in real applications and make it difficult to precisely estimate the system states, potentially leading to very large estimation errors. These difficulties can be overcome by studying a robust filter (Yu 01, Yaesh 93, Shaked 95).

Recently, a robust filter has received considerable attention. It has robustness against 1) statistical incompleteness of system noise, such as process noise and measurement noise, 2) system modeling uncertainty, and 3) sensitivity caused by parameter variation of the system model. They can be categorized as H_2 filters, H_∞ filters, and mixed H_2/H_∞ filters (Nagpal 91, de Souza 95, Bolzern 97). In case of given statistical information and modeling uncertainty, the H_2 filter has been usually constructed to estimate system errors. A guaranteed cost minimization has been widely used as the performance index of H_2 filter and upper bound minimization or minimum variance of estimation errors are used as the cost. Especially, the upper bound minimization has less assumptions and simple structure in developing robust filter compared to other performance indices. The H_∞ filter minimizes the H_∞ norm of the transfer function between the noise and the estimation error. Thus, the H_∞ filter is usually employed when the energy of the system noise is bounded and the statistical properties of the noise are unknown. This filter minimizes the highest energy gain of the estimation error for all initial conditions and noises. In particular, a robust H_∞ filter, a robust filter with a modified H_∞ performance, can be established for a system with model uncertainty as well as unknown statistical noise properties (Shaked 95).

For a nonlinear system, a second-order nonlinear filter and an extended Kalman filter have been utilized. Because the second-order nonlinear filter considers higher-order terms in the computation of its covariance, it is suitable for a highly nonlinear system. However, it has large computational complexity. Thus, the extended Kalman filter has been widely used for real system applications. Since the extended Kalman filter uses a linearized model of a nonlinear system with an abbreviation of higher-order terms, excessive estimation errors occur when it is applied to a highly nonlinear system. In addition, the extended Kalman filter requires a statistical information about noise, such as white Gaussian noise, which can hardly be obtained in real applications. Therefore, several studies have been conducted on the nonlinear robust filter. The H_∞ nonlinear filter with Hamilton-Jacobi inequality is the result of one such study, but its computation procedures for obtaining a filter are complicated and it is very difficult to use in real applications. To simplify complicated computation procedures, an approximation solution to the robust filtering problem has recently been developed based on a linearization method. The robust filter derived based on this approach is called the extended robust filter or extended H_∞ filter (Shaked 95, Einicke 99). In (James and Petersen 98), the nonlinear state estimation with similar characteristics is especially proposed for a nonlinear uncertain system with uncertainties described by an integral quadratic constraint.

In this paper, a new robust filter for nonlinear uncertain systems is presented. The derivation is similar to that of (James and Petersen 98). The robust filter is constructed with local linearization of the system at the reference point. This approach extends the extended Kalman filter to a robust filter. By introducing a state estimation set that is the solution of Hamiton-Jacobi-Bellman partial differential equation and by solving locally the filtering problem, the robust filter is derived. Then the characteristics of filter are analyzed. The proposed filter is applied to the SDINS IFA.

2. NONLINEAR ROBUST H_∞ FILTER

Consider a nonlinear uncertain system described by

$$\begin{aligned}\dot{x}(t) &= f(x(t)) \\ &\quad + B_1(t)\Delta_1(t)N(x(t)) + B_2(t)w_0(t)\end{aligned} \tag{1}$$

$$y(t) = h(x(t)) + \Delta_2(t)N(x(t)) + v_0(t) \tag{2}$$

where $B_1(t)\Delta_1(t)N(x(t))$ and $\Delta_2(t)N(x(t))$ represent the system uncertainties. $B_1(t)$ and $N(x(t))$ are known matrices. $\Delta_1(t)$ and $\Delta_2(t)$ are unknown matrices satisfying the condition $\left\| \begin{matrix} Q_1^{-1/2}\Delta_1(t) \\ R_1^{-1/2}\Delta_2(t) \end{matrix} \right\| \le 1$ where Q_1 and R_1 are bounded positive definite matrices. $w_0(t)$ is the process noise and $v_0(t)$ is the measurement noise. They belong to the set of L_2 norm and the statistical properties are unknown. In addition, $w_0(t)$ and $v_0(t)$ are noises satisfying the bound

$$\begin{aligned}\Phi(x(0)) + \int_0^T [w_0^T(t)Q_2^{-1}w_0(t) \\ + v_0^T(t)R_2^{-1}v_0(t)]\, dt \le d\end{aligned}$$

where $0 \le t \le T$ and d is an assigned positive real number. Converting the uncertainties to the fictitious L_2 noises and introducing a freedom parameter, the uncertain system (1) and (2) can be transformed into as an auxiliary system,

$$\dot{x}(t) = f(x(t)) + B(t)w(t) \tag{3}$$

$$y(t) = h(x(t)) + v(t) \tag{4}$$

where $B(t) = \left[\varepsilon B_1(t)\ B_2(t)\right]$,

$n(t) = \varepsilon^{-1}N(x(t))$, $\quad w(t) = \begin{bmatrix} \Delta_1(t)n(x(t)) \\ w_0(t) \end{bmatrix}$, and

$v(t) = \left[I\ I\right]v_1 = \left[I\ I\right]\begin{bmatrix} \varepsilon\Delta_2(t)n(x(t)) \\ v_0(t) \end{bmatrix}$.

ε is a freedom parameter. By the combination of the state variables and $n(t)$, the filter output $z(x(t))$ is of the form

$$z(x(t)) = \left[(L(t)x(t))^T \quad (\gamma n(t))^T\right]^T \tag{5}$$

where γ is a given positive real value that indicates the level of noise attenuation in H_∞ filter design. To construct a robust filter, it is assumed that the system (3) and (4) satisfies Assumptions 1-6.

Assumption 1: Every function shown in (3)-(5) belongs to C^1 and the first derivative is bounded.

Assumption 2: The matrix $N(x(t))$ is bounded.

Assumption 3: The functions Φ, L_1, and L_2 belong to C^1 and are bounded nonnegative functions. They also satisfy

$$\left|\phi(x_2) - \phi(x_1)\right| \le \theta(1 + |x_2| + |x_1|)|x_2 - x_1| \tag{6}$$

where $\theta > 0$ and $\phi = \Phi, L_1$, or L_2.

Assumption 4: The function L_1 satisfies a coercivity condition,

$L_1(w,v) \geq c|w|$ where $c > 0$.

Assumption 5: The matrix B is of full rank.

Assumption 6: The matrix $L(t)$ is bounded by

$$l_1 I \leq L(t)^T L(t) \leq l_2 I, \qquad \forall t$$

where l_1 and l_2 are positive real numbers.

2.1 Extended Robust H_∞ Filter

In this section, a robust filter with a modified H_2 filter structure and a modified H_∞ performance index is derived based on a local solution of the filter problem. Similar to the development of the well-known the extended Kalman filter, we derive the filter by linearizing the system in the neighborhood of the estimated trajectory, $\hat{x}$.

We consider a system that satisfies integral quadratic constraint given by

$$\begin{aligned}(x(0)-x_0)^T M(x(0)-x_0) + \int_0^T L_1(w,v)\, dt \\ \leq d + \int_0^T L_2(n,z)\, dt\end{aligned} \tag{7}$$

where $L_1(w,v) = w^T Q^{-1} w + v^T R^{-1} v$

$$= w^T \begin{bmatrix} Q_1^{-1} & 0 \\ 0 & Q_2^{-1} \end{bmatrix} w + v_1^T \begin{bmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{bmatrix} v_1$$

$L_2(n,z) = n^T n + \gamma^{-2}(z_1 - z)^T (z_1 - z)$, and $z_1 = z(\hat{x})$. For the system (3) and (4) with (7), a partial differential equation is generally given by

$$\frac{\partial}{\partial t} V + \max_w \begin{bmatrix} \nabla_x V(f(x) + Bw) - L_1(w,v) \\ + L_2(n(x), z(x)) \end{bmatrix} = 0 \tag{8}$$

where $V(x,t)$ denotes a value function and $V(x,0) = \Phi(0)$. Assumptions 1-5 ensure that $V(x,t)$ is finite (McEneancy 95). To derive a robust filter, we consider a system that satisfies an integral quadratic constraint given by

$$\begin{aligned}&(x(0)-x_0)^T M(x(0)-x_0) \\ &+ \frac{1}{2}\int_0^T \left[w(t)^T Q^{-1} w(t) + v(t)^T R^{-1} v(t)\right] dt \\ &\leq d + \frac{1}{2}\int_0^T \left[n^T n + \gamma^{-2}(z_1 - z)^T (z_1 - z)\right] dt.\end{aligned} \tag{9}$$

Using (8) and (9), the partial differential equation is obtained as

$$\begin{aligned}&\frac{\partial}{\partial t} V + \nabla_x V f(x) + \frac{1}{2}\nabla_x V B Q B^T \nabla_x V^T \\ &- \frac{1}{2}(y - h(x))^T R^{-1}(y - h(x)) + \frac{1}{2} n(x)^T n(x) \\ &+ \frac{1}{2}\gamma^{-2}(z_1 - z(x))^T (z_1 - z(x)) = 0.\end{aligned} \tag{10}$$

where $V(x,0) = (x - x_0)^T M (x - x_0)$.

The $\hat{x}(t)$ as an estimate value of the state variable $x(t)$ is defined to be

$$\hat{x}(t) = \arg\min_x V(x,t). \tag{11}$$

Equation (11) satisfies two conditions:

$$\nabla_x V(\hat{x}(t),t) = 0 \tag{12}$$

$$\nabla_x^2 V(\hat{x}(t),t)\dot{\hat{x}}(t) + \frac{\partial}{\partial t}\nabla_x V(\hat{x}(t),t)^T = 0 \tag{13}$$

The gradient of (10) with respect to x is given by

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla_x V^T + \nabla_x f(x)^T \nabla_x V^T + \nabla_x^2 V B Q B^T \nabla_x V^T \\ &+ \nabla_x^2 V f(x) + \nabla_x h(x)^T R^{-1}(y - h(x)) \\ &+ \nabla_x n(x)^T n(x) - \gamma^{-2}\nabla_x z(x)^T (z_1 - z(x)) = 0.\end{aligned} \tag{14}$$

Using (12) and (13) and evaluating at $x = \hat{x}$, (14) is simplified as

$$\begin{aligned}&\nabla_x^2 V(\hat{x},t)\dot{\hat{x}}(t) = \nabla_x^2 V(\hat{x},t) f(\hat{x}(t)) \\ &+ \nabla_x h(\hat{x}(t))^T R^{-1}(y - h(\hat{x}(t))) + \nabla_x n(\hat{x}(t))^T n(\hat{x}(t)) \\ &- \gamma^{-2}\nabla_x z(\hat{x}(t))^T (z_1 - z(\hat{x}(t))) = 0.\end{aligned} \tag{15}$$

Furthermore, suppose that the matrix $\nabla_x^2 V(\hat{x},t)$ is nonsingular for all t, the dynamic equation of state estimate satisfying (11) can be written as

$$\begin{aligned}\dot{\hat{x}}(t) = f(\hat{x}(t)) + (\nabla_x^2 V(\hat{x}(t),t))^{-1}[\nabla_x h(\hat{x}(t))^T R^{-1} \\ (y - h(\hat{x}(t))) + \nabla_x n(\hat{x}(t))^T n(\hat{x}(t))].\end{aligned} \tag{16}$$

In addition, the gradient of (14) with respect to x is expressed as

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla_x^2 V + \nabla_x f(x)^T \nabla_x^2 V^T + \nabla_x^2 f(x)^T \nabla_x V^T \\ &+ \nabla_x^2 V \nabla_x f(x) + \nabla_x^3 V f(x) + \nabla_x^3 V B Q B^T \nabla_x V^T \\ &+ \nabla_x^2 V B Q B^T \nabla_x^2 V(x) - \nabla_x h(x)^T R^{-1} \nabla_x h(x)) \\ &+ \nabla_x^2 h(x)^T R^{-1}(y - h(x)) + \nabla_x^2 n(x)^T n(x) \\ &+ \nabla_x n(x)^T \nabla_x n(x) - \gamma^{-2}\nabla_x^2 z(x)^T (z_1 - z(x)) \\ &- \gamma^{-2}\nabla_x z(x)^T \nabla_x (z_1 - z(x)) = 0.\end{aligned} \tag{17}$$

Using (12) and (13) and evaluating at $x = \hat{x}$, (17) is simplified as

$$\begin{aligned}&\dot{\Pi} + \nabla_x f(\hat{x})^T \Pi + \Pi \nabla_x f(\hat{x}) \\ &+ \Pi B Q B^T \Pi - \nabla_x h(\hat{x})^T R^{-1} \nabla_x h(\hat{x}) \\ &+ \nabla_x n(\hat{x})^T \nabla_x n(\hat{x}) + \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x}) = 0\end{aligned} \tag{18}$$

where $\Pi = \nabla_x^2 V$ and $\Pi(0) = M$. The corresponding differential equation for $P(t) = \Pi(t)^{-1}$ from (18) is

$$\begin{aligned}&\dot{P}(t) = P(t)\nabla_x f(\hat{x})^T + \nabla_x f(\hat{x}) P(t) + B Q B^T \\ &- P(t)[\nabla_x h(\hat{x})^T R^{-1} \nabla_x h(\hat{x}) \\ &- \nabla_x n(\hat{x})^T \nabla_x n(\hat{x}) - \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})] P(t).\end{aligned} \tag{19}$$

From these results, a robust filter can be summarized as

$$\begin{aligned}\dot{\hat{x}}(t) = f(\hat{x}(t)) + P(t)\nabla_x h(\hat{x}(t))^T R^{-1}(y - h(\hat{x}(t)) \\ + \varepsilon^{-2} P(t)\nabla_x N(\hat{x}(t))^T N(\hat{x}(t))\end{aligned} \tag{20}$$

$$z(\hat{x}(t)) = \begin{bmatrix} (L(t)\hat{x}(t))^T & (\frac{\gamma}{\varepsilon} N(\hat{x}(t))^T \end{bmatrix}^T \tag{21}$$

$$
\begin{aligned}
&\dot{P}(t) = P(t)\nabla_x f(\hat{x})^T + \nabla_x f(\hat{x})P(t) + \varepsilon^2 B_1 Q_1 B_1^T \\
&+ B_2 Q_2 B_2^T - P(t)[\nabla_x h(\hat{x})^T R^{-1}\nabla_x h(\hat{x}) \\
&- \varepsilon^{-2}\nabla_x N(\hat{x})^T \nabla_x N(\hat{x}) - \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})]P(t)
\end{aligned} \tag{22}
$$

where $P(0) = M(0)^{-1}$, $\hat{x}(0) = x_0$, and M is a matrix which reflects the initial errors of the system.

The proposed filter has the structure of an H_2 filter but with (21) and $\gamma^{-2}P(t)\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})P(t)$ in (22). However, by virtue of (21) and $\gamma^{-2}P(t)\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})P(t)$, this filter is robust against the disturbances and the uncertainties, as shown in the next section.

2.2 Analysis of Extended Robust H_∞ Filter

In this section, the analytical performances of the filter proposed in section 2.1 are investigated. We will consider H_∞ performance index, such as the energy ratio between the noise and the estimation error, as an important property of the filter.

Now, a modified H_∞ performance index is derived. The estimate errors can be defined to be

$$
\zeta(t) = x(t) - \hat{x}(t) \tag{23}
$$

and the dynamic equation of the estimated errors $\zeta(t)$ is expressed as

$$
\begin{aligned}
\dot{\zeta}(t) &= (A(t) - K(t)C(t))\zeta(t) + B(t)w(t) \\
&- P(t)\nabla_x n(\hat{x}(t))^T n(\hat{x}(t)) + \varphi(x(t), \hat{x}(t)) \\
&- K(t)\chi(x(t), \hat{x}(t)) - K(t)v(t)
\end{aligned} \tag{24}
$$

where $A(t) = \frac{\partial f}{\partial x}(\hat{x}(t))$, $C(t) = \frac{\partial h}{\partial x}(\hat{x}(t))$, and $K(t) = P(t)C(t)^T R^{-1}$. Nonlinear functions $\varphi(x(t), \hat{x}(t))$ and $\chi(x(t), \hat{x}(t))$ are defined as

$$
\begin{aligned}
f(x(t)) - f(\hat{x}(t)) &= A(t)(x(t) - \hat{x}(t)) + \varphi(x(t), \hat{x}(t)) \\
h(x(t)) - h(\hat{x}(t)) &= C(t)(x(t) - \hat{x}(t)) + \chi(x(t), \hat{x}(t))
\end{aligned}
$$

where $\varphi(x(t), \hat{x}(t))$ and $\chi(x(t), \hat{x}(t))$ are higher-order terms in the estimation errors. We make Assumption 7 and Assumption 8.

Assumption 7: $n(t) = \varepsilon^{-1}N(x(t)) = \varepsilon^{-1}N(t)x(t)$.

Assumption 8: There exist positive real numbers, $\varepsilon_\varphi, \varepsilon_\chi, k_\varphi$, and k_χ, to bound the nonlinear terms $\varphi(x(t), \hat{x}(t))$ and $\chi(x(t), \hat{x}(t))$ as follows:

$$
\begin{aligned}
&\|\varphi(x(t), \hat{x}(t))\| \le k_\varphi \|x(t) - \hat{x}(t)\|^2, \qquad \|\zeta\| \le \varepsilon_\varphi \\
&\|\chi(x(t), \hat{x}(t))\| \le k_\chi \|x(t) - \hat{x}(t)\|^2, \qquad \|\zeta\| \le \varepsilon_\chi
\end{aligned}
$$

Lemma 1 [Yu 01]**:** Suppose that A1 and A8 are satisfied. For estimated errors $\|\zeta\| \le \varepsilon_1$, there exist real numbers ε_1 and k such that

$$
\begin{aligned}
&(x(t) - \hat{x}(t))^T P(t)^{-1}[\varphi(x(t), \hat{x}(t)) \\
&- K\chi(x(t), \hat{x}(t))] \le k\|x(t) - \hat{x}(t)\|^3
\end{aligned} \tag{25}
$$

where $\varepsilon_1 = \min(\varepsilon_\varphi, \varepsilon_\chi)$, $k = \frac{k_\varphi}{p_1} + \frac{c_2 k_\chi}{r}$, $\|C(t)\| \le c_2$, p_1 is the lower bound of $P(t)$, and $rI \le R$.

Suppose that a Lyapunov function is chosen as

$$
V(\zeta(t)) = \zeta^T(t)P(t)^{-1}\zeta(t) \tag{26`}
$$

where $P(t)$ is the solution of (22). Differentiating $V(\zeta(t))$ over time yields

$$
\begin{aligned}
\dot{V}(\zeta(t)) &= \dot{\zeta}^T(t)P(t)^{-1}\zeta(t) + \zeta^T(t)\dot{P}(t)^{-1}\zeta(t) \\
&+ \zeta^T(t)P(t)^{-1}\dot{\zeta}(t).
\end{aligned} \tag{27}
$$

Substituting (22) and (24) in (27), it is easy to show that (27) becomes

$$
\begin{aligned}
&\dot{V}(\zeta(t)) = \zeta^T(t)[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + w^T Q^{-1} w - s^T s \\
&+ v^T R^{-1} v - \eta^T R^{-1}\eta + 2\varphi^T P(t)^{-1}\zeta - 2(K(t)\chi)^T P(t)^{-1}\zeta \\
&+ \varepsilon^{-2}\{\zeta^T[-2\nabla_x N(\hat{x})^T \nabla_x N(\hat{x})]\zeta \\
&- \zeta^T \nabla_x N(\hat{x})^T N(\hat{x}) - (\nabla_x N(\hat{x})^T N(\hat{x}))^T \zeta\}
\end{aligned} \tag{28}
$$

where $s = Q^{-\frac{1}{2}}w - (B(t)Q^{\frac{1}{2}})^T P(t)^{-1}\zeta$ and $Q = Q^{\frac{1}{2}}Q^{\frac{1}{2}T}$, $\eta = v + C(t)\zeta$, and $R = R^{\frac{1}{2}}R^{\frac{1}{2}T}$.

Utilizing the Assumption 7 and the triangle inequality property, (28) can be expressed as

$$
\begin{aligned}
&\dot{V}(\zeta(t)) \le \zeta^T(t)[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + w^T Q^{-1} w \\
&+ v^T R^{-1} v + 2\varphi^T P(t)^{-1}\zeta - 2(K(t)\chi)^T P(t)^{-1}\zeta \\
&+ \varepsilon^{-2}N(x)^T N(x)
\end{aligned} \tag{29}
$$

Applying Lemma 1 to (29), we obtain the following inequality,

$$
\begin{aligned}
\dot{V}(\zeta(t)) &\le \zeta^T[-\gamma^{-2}L(t)^T L(t)]\zeta + 2k\|\zeta\|^3 \\
&+ w^T Q^{-1} w + v^T R^{-1} v + \varepsilon^{-2}N(x)^T N(x).
\end{aligned} \tag{30}
$$

Provided that the estimate errors satisfy $\|\zeta\| \le \varepsilon_2$, (30) can be modified to

$$
\begin{aligned}
\dot{V}(\zeta(t)) &\le \zeta^T[-\gamma^{-2}L(t)^T L(t)]\zeta + 2k\|\zeta\|^3 \\
&\quad + w^T Q^{-1} w + v^T R^{-1} v + \varepsilon^{-2}N(x)^T N(x) \\
&\le -\frac{\gamma^{-2}l_1}{2}\|\zeta\|^2 + w^T Q^{-1} w + v^T R^{-1} v \\
&+ \varepsilon^{-2}N(x)^T N(x) \\
&\le -\frac{\gamma^{-2}l_1}{2l_2}\zeta^T[L(t)^T L(t)]\zeta + w^T Q^{-1} w \\
&+ v^T R^{-1} v + \varepsilon^{-2}N(x)^T N(x)
\end{aligned} \tag{31}
$$

where $\varepsilon_2 = \min(\varepsilon_1, \frac{\gamma^{-2}l_1}{4k})$.

Finally, the cost function of the derived filter is obtained as follows. By integrating both sides of (31), the modified H_∞ performance index J is expressed as

$$J = \frac{\|L\zeta\|_2^2}{\|w\|_{2Q^{-1}}^2 + \|v\|_{2R^{-1}}^2 + \|N(x)\|_{2\varepsilon^{-2}}^2 + \zeta(0)^T P(0)^{-1}\zeta(0)} < \gamma^2 \mu^{-1} = \gamma_t^2 \tag{32}$$

where $\mu = \frac{l_1}{2l_2}$. The cost function J of the robust filter is less than γ_t^2. As μ is less than 1, the new value γ_t is always greater than γ. γ_t is not only an index of disturbance attenuation level, but also an important parameter describing filter's estimation ability in the worst case. Decreasing γ_t means that robustness of the filter increases. Equation (32) shows that the proposed filter guarantees robustness against the noises, including process noise and measurement noise and the system model uncertainty. On the contrary, when the extended Kalman filter or the H_2 filter is applied to the nonlinear system, the cost function, such as (32), cannot be defined since the value of γ is ∞. Therefore they cannot guarantee robustness against noise and uncertainty and have the effect of disturbance attenuation.

3. APPLICATION TO SDINS IFA

To verify the performance of the proposed filter, an SDINS IFA with velocity-aiding is designed. GPS or Doppler radar can be used for the aiding source. The inertial navigation system is constructed in a local-level frame (NED frame). The error models, such as latitude error δL, longitude error δl, height error δh, velocity error δv^n, and attitude error q^n, are adopted from references (Yu 99, Yu 01). When error models are derived, it is desirable to reduce uncertainty. So we use a multiplicative quaternion error, q^n that is much simpler than an equivalent tilt angle as an attitude error. These error models are given by

$$\delta\dot{L} = \frac{R_{mm}\rho_E}{R_m + h}\delta L + \frac{\rho_E}{R_m + h}\delta h + \frac{1}{R_m + h}\delta v_N \tag{33}$$

$$\delta\dot{l} = \frac{\rho_N}{\cos L}(\tan L - \frac{R_{tt}}{R_t + h})\delta L - \frac{\rho_N \sec L}{R_t + h}\delta h + \frac{\sec L}{R_t + h}\delta v_E \tag{34}$$

$$\delta\dot{h} = -\delta v_D \tag{35}$$

$$\delta\dot{v}^n = \Delta C_b^n f^b - [2w_{ie}^n + w_{en}^n]\times\delta v^n + C_b^n \delta f^b + v^n \times (2\delta w_{ie}^n + \delta w_{en}^n) + \delta g^n \tag{36}$$

$$\dot{q}^n = -w_{in}^n \times q^n - \frac{1}{2}[C_b^n \delta w_{ib}^b - \delta w_{in}^n] \tag{37}$$

where $R_{mm} = \frac{\partial R_m}{\partial L}$, $R_{tt} = \frac{\partial R_t}{\partial L}$, δf^b is an accelerometer error, δw_{ib}^b is a gyroscope error, δg^n is a gravity error, $\delta v^n = [\delta v_N\ \delta v_E\ \delta v_D]^T$, R_m is the semimajor axis of the earth, R_t is the semiminor axis of the earth, $w_{en}^n = [\rho_N\ \rho_E\ \rho_D]^T$, $w_{in}^n = w_{ie}^n + w_{en}^n$, and $q^n = [q_{1n}\ q_{2n}\ q_{3n}]^T$ (Yu 99, Yu 01). In the velocity error model, $\Delta C_b^n f^b$ is strongly related to attitude errors. In the case where the system has only large attitude errors, it is given by

$$\begin{aligned}\Delta C_b^n f^b &= -2[q_{0n}I + R_Q][R_Q]C_b^n f^b \\ &= 2[q_{0n}I + R_Q][C_b^n f^b \times]q^n \\ &= 2[C_b^n f^b \times]q^n + 2[c_n I + R_Q][C_b^n f^b \times]q^n\end{aligned} \tag{38}$$

where $q_{0n} = 1 + c_n$ and $[R_Q]$ denotes a skew-symmetric matrix of q^n. Equation (38) shows that $\Delta C_b^n f^b$ is composed of the linear term and the uncertainty. Thus the system equations involve uncertainty. Convert the uncertainty into a factorized form, as $B_1(t)\Delta_1(t)N(x(t))$ in (1) is not difficult. Measurement of the velocity-aided SDINS IFA is given as

$$y(t) = C(t)x(t) + v(t) \tag{39}$$

where $C(t) = [0_{3\times3},\ I_{3\times3},\ 0_{3\times9}]$. 15 state variables are composed of position (3), velocity (3), attitude (3), accelerometer bias (3), and gyroscope bias (3). The major errors of the inertial sensors considered in the simulation are gyroscope bias error(3 deg/hr), gyroscope scale factor error(500 ppm), accelerometer bias error(1000 ug), and accelerometer scale factor error(200 ppm). The gyroscope bias and accelerometer bias are assumed to be random constants. The initial attitude errors are assumed to be 10 degrees horizontal plane attitude error and 20 degrees heading error. Monte Carlo simulations are performed for 1000 seconds and the filter update period is 0.1 second. The results are compared with the results of the corresponding EKF. The results are shown in Fig 1 and Fig 2. The proposed filter reduces the attitude error and the position error compared with the EKF. As shown in the figures, the proposed filter reduces the heading error by about 50 % and the position error by about 30 % more than the EKF. In addition, the convergence of the attitude error is much faster than that of the EKF. The simulation results have shown that it is possible to further improve IFA accuracy by employing the proposed filter, which can consider uncertainties of the SDINS.

4. CONCLUSION

The extended robust H_∞ filter for SDINS in-flight alignment with large initial attitude errors has been presented. The extended robust H_∞ filter has been derived by considering a nonlinear uncertain system and by introducing the notion of a local solution to the filtering problem. The proposed filter possesses the H_∞ performance criterion. Thus, it is robust against noise and uncertainty. The derivation method and the characteristic analysis method of the proposed filter are developed in general. Thus, they can be extended to any other nonlinear uncertain system problem. The simulation results for a velocity-aided SDINS IFA have demonstrated that the proposed filter is more effective in estimating the attitude error and position error than the EKF.

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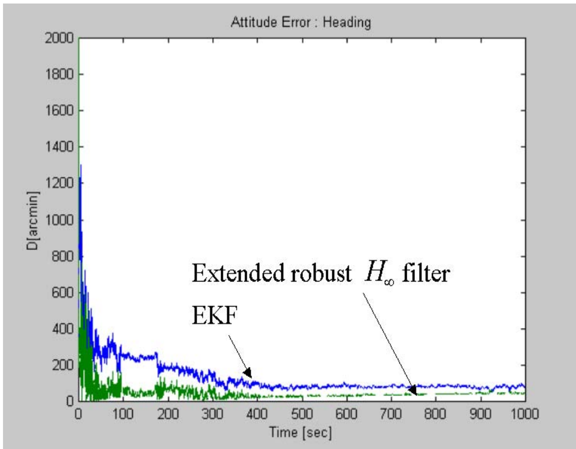

Fig.1. Heading error of filters

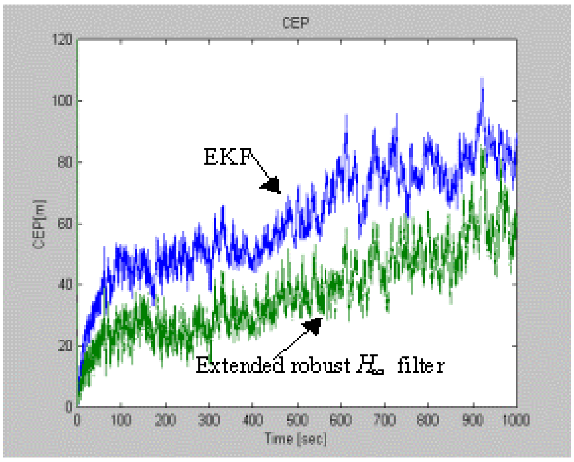

Fig.2. Position error of filters