Copyright © 2002 IFAC
15th Triennial World Congress, Barcelona, Spain

ADAPTIVE EXTREMUM SEEKING CONTROL OF CONTINUOUS STIRRED TANK BIOREACTORS

T. Zhang * **M. Guay** *,[1] **D. Dochain** **

* *Department of Chemical Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6*
** *CESAME, UCL, 4-6 Avenue G. Lemaître B-1348 Louvain-la-Neuve, Belgium*

Abstract: In this paper, we present an adaptive extremum seeking control scheme for continuous stirred tank bioreactors. The proposed adaptive extremum seeking approach utilizes the structure information of the kinetics of the bioreactors to construct a seeking algorithm that drives the system states to the desired set-points that extremize the value of an objective function. Lyapunov's stability theorem is used in the design of the extremum seeking controller structure and the development of the parameter learning laws. Simulation experiment is given to show the effectiveness of the proposed approach.

Keywords: Extremum seeking, Lyapunov function, parameter estimation, persistence of excitation

1. INTRODUCTION

Most adaptive control schemes documented in the literature ((Landau 1979), (Goodwin and Sin 1984), (Astrom and Wittenmark 1995), (Narendra and Annaswamy 1989), (Ioannou and Sun 1996) and (Krstic *et al.* 1995)) are developed for regulation to known set-points or tracking known reference trajectories. In some applications, however, the control objective could be to optimize an objective function which can be a function of unknown parameters, or to select the desired states to keep a performance function at its extremum value. Self-optimizing control and extremum seeking control are two methods to handle these kinds of optimization problems. The task of extremum seeking is to find the operating set-points that maximize or minimize an objective function. Since the early research work on extremum control in the 1920's (Leblanc 1922), many successful applications of extremum control approaches have been reported (e.g., (Vasu 1957), (Astrom and Wittenmark 1995), (Sternby 1980) and (Drkunov *et al.* 1995)). Recently, Krstic et. al ((Krstic 2000), (Krstic and Wang 2000)) presented several extremum control schemes and stability analysis for extremum-seeking of linear unknown systems and a class of general nonlinear systems ((Krstic 2000), (Krstic and Wang 2000) and (Krstic and Deng 1998)).

The implications for the chemical and biochemical industries are clear. In these sectors, it is recognized that even small performance improvements in key process control variables may result in substantial economic benefits. As an example, the potential benefits of extremum seeking techniques in the maximization of biomass production rate in well-mixed biological processes has been demonstrated in (Wang *et al.* 1999).

In this study, we investigate an alternative extremum seeking scheme for continuous stirred tank bioreactors. The proposed scheme utilizes explicit structure information of the objective function that depends on system states and unknown plant parameters. The scheme presented in this paper is based on Lyapunov's stability theorem. As a result, the global stability is ensured during the seeking of the extremum of the nonlinear continuous stirred tank bioreactors. It is also

[1] Research supported by the Natural Sciences and Engineering Research Council of Canada and the Canadian Foundation for Innovation.

shown that once a certain level of persistence of excitation (PE) condition is satisfied, the convergence of the extremum seeking mechanism can be guaranteed. The paper is organized as follows. Section 2 presents some notations and the problem formulation. In Section 3, an parameter estimation algorithm is developed. Section 4 presents the adaptive extremum seeking controller and the stability and convergence of the closed-loop extremum seeking system. Numerical simulation is shown in Section 5 followed by brief conclusions in Section 6.

2. PROBLEM

Consider the following microbial growth models

$$\dot{x} = \mu(x,s)x - ux \tag{1}$$
$$\dot{s} = -k_1\mu(x,s)x + u(s_0 - s) \tag{2}$$
$$y = k_2\mu(x,s)x \tag{3}$$

where states $x \in [0,+\infty)$ and $s \in [0,+\infty)$ denote biomass and substrate concentrations, respectively, $u \geq 0$ is the dilution rate, y is the production rate of the reaction product, s_0 denotes the concentration of the substrate in the feed, and $k_1, k_2 > 0$ are yield coefficients. We consider the case where only s and y are measurable, the biomass concentration x is not available for feedback control.

In this work, we consider the extremum seeking problem for plant (1)-(2) with growth rate $\mu(x,s)$ expressed by Monod's model. This model is given by

$$\mu(x,s) = \mu(s) = \frac{\mu_m s}{K_s + s} \quad \text{(Monod)} \tag{4}$$

where $\mu_m > 0$ is the maximum value of the specific growth rate, and $K_s > 0$ is the saturation constant for the Monod growth rate model. Monod's model is one of the most commonly used model for growth kinetics. However, it is important to note that the scheme developed in this paper is not limited to this model and can be easily extended to the plants with other growth rate representations. The control objective is to design a controller, u, such that the production rate y achieves its maximum.

We first calculate the system's equilibria corresponding to a constant dilution rate u_e. There are two equilibria in this case. The first is $x_e = 0$ and $s_e = s_0$ which is called the wash-out equilibrium. The second is

$$s_e = \frac{K_s u_e}{\mu_m - u_e}, \;\; x_e = \frac{s_0 - s_e}{k_1}$$

At the steady-state, the production rate can be expressed by

$$y_e = \frac{k_2\mu_m s_e(s_0 - s_e)}{k_1(K_s + s_e)} \tag{5}$$

From (2) and (4), we have

$$\frac{\partial y_e}{\partial s_e} = \frac{-k_2\mu_m}{k_1(K_s+s_e)^2}\left(s_e^2 + 2K_s s_e - s_0 K_s\right) \tag{6}$$

and

$$\frac{\partial^2 y_e}{\partial s_e^2} = \frac{-2k_2\mu_m}{k_1(K_s+s_e)^3}\left(K_s^2 + s_0 K_s\right) \tag{7}$$

It is shown that $\frac{\partial^2 y_e}{\partial s_e^2} < 0, \forall s_e \geq 0$. Hence, at the system equilibrium, $y_e(s)$ has a maximum

$$y^* = y_e(s^*) = \frac{k_2\mu_m s^* x^*}{K_s + s^*} \tag{8}$$

with

$$s^* = \sqrt{K_s^2 + s_0 K_s} - K_s \tag{9}$$
$$x^* = \frac{s_0 - s^*}{k_1}. \tag{10}$$

From the above analysis, we know that if the substrate concentration s can be stabilized at the set-point s^* then the production rate y is maximized. However, since the exact values of the Monod's model parameters K_s and μ_m are usually unknown, the desired set-point s^* is not available. In this work, an adaptive extremum seeking algorithm is developed to search this unknown set-point such that the production rate, y, is optimized.

Assumption 1: The upper bound of K_s is known, i.e., $K_s \leq K_{s0}$ with known constant $K_{s0} > 0$.

3. ESTIMATION

In this section, we develop the parameter estimation algorithm for the unknown parameters k_1/k_2, K_s and μ_m. It follows from (3) that $\mu(s)x = y/k_2$. Equations (1)-(2) can be re-expressed as

$$\dot{x} = \frac{1}{k_2}y - ux \tag{11}$$
$$\dot{s} = -\frac{k_1}{k_2}y + u(s_0 - s) \tag{12}$$

By (3)-(4) and (12)-(13), the time derivative of y is

$$\dot{y} = \frac{k_2 K_s \mu_m x}{(K_s+s)^2}\dot{s} + k_2\mu(s)[\frac{1}{k_2}y - ux]$$

Since the biomass concentration x is not measurable, we re-express $\dot{y}$ by replacing x with $y/k_2\mu(s)$ as follows

$$\dot{y} = -uy + \frac{\mu_m s^2 y - \frac{k_1 K_s}{k_2}y^2 + K_s u y(s_0 - s)}{s(K_s + s)} \tag{13}$$

Let $\theta = [\theta_s \; \theta_\mu \; \theta_k]^T$ with $\theta_\mu = \frac{\mu_m}{K_s}$, $\theta_s = \frac{1}{K_s}$, $\theta_k = \frac{k_1}{k_2}$. Equations (13) and (14) can be re-written as

$$\dot{s} = -\theta_k y + u(s_0 - s) \tag{14}$$

$$\dot{y} = -uy + \frac{\theta_\mu s^2 y - \theta_k y^2 + (s_0 - s)uy}{s(1+\theta_s s)} \tag{15}$$

Let $\hat{\theta}$ denote the estimate of the true parameter θ, and $\hat{s}$ and $\hat{y}$ be the predictions of s and y by using the estimated parameter $\hat{\theta}$, respectively. The predicted states $\hat{s}$ and $\hat{y}$ are generated by

$$\dot{\hat{s}} = -\hat{\theta}_k y + u(s_0 - s) + k_s e_s \tag{16}$$

$$\dot{\hat{y}} = -uy + \frac{\hat{\theta}_\mu s^2 y - \hat{\theta}_k y^2 + (s_0 - s)uy}{s(1+\hat{\theta}_s s)} + k_y e_y \tag{17}$$

with $k_s, k_y > 0$, the prediction errors $e_s = s - \hat{s}$ and $e_y = y - \hat{y}$. It follows from (15)-(18) that

$$\dot{e}_s = -k_s e_s - \tilde{\theta}_k y \tag{18}$$

$$\dot{e}_y = -k_y e_y + \frac{\tilde{\theta}\Phi(s,y,\hat{\theta})y}{(1+\theta_s s)(1+\hat{\theta}_s s)} \tag{19}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\Phi(s,y,\hat{\theta}) = [\phi_s \ \phi_\mu \ \phi_k]^T$ with

$$\begin{aligned}
\phi_s &= -(s_0 - s)u - \hat{\theta}_\mu s^2 + \hat{\theta}_k y \\
\phi_\mu &= (1+\hat{\theta}_s s)s \\
\phi_k &= -(1+\hat{\theta}_s s)\frac{y}{s}
\end{aligned}$$

By $\theta_s = \frac{1}{K_s}$, the desired set-point (10) can be re-expressed as $s^* = \left(\sqrt{1+s_0\theta_s} - 1\right)/\theta_s$. Since the parameter θ_s is unknown, we first design a controller to make the substrate concentration s follow $\left(\sqrt{1+s_0\hat{\theta}_s} - 1\right)/\hat{\theta}_s$ that is an estimate of s^*. Later, an excitation signal is designed and injected into the adaptive system such that the estimated parameter $\hat{\theta}_s$ converges to its true value. The extremum seeking control objective can be achieved when the substrate concentration s is stabilized at the optimal operating point s^*.

Define

$$z_s = s - \frac{1}{\hat{\theta}_s}\left(\sqrt{1+s_0\hat{\theta}_s} - 1\right) + d(t) \tag{20}$$

where $d(t) \in C^1$ is an excitation signal that will be assigned later. We consider a Lyapunov function candidate

$$V = \frac{z_s^2}{2} + \frac{1}{2}\left(\frac{\tilde{\theta}_\mu^2}{\gamma_\mu} + \frac{\tilde{\theta}_s^2}{\gamma_s} + \frac{\tilde{\theta}_k^2}{\gamma_k}\right) + \frac{e_s^2}{2} + (1+\theta_s s)\frac{e_y^2}{2} \tag{21}$$

with constants $\gamma_\mu, \gamma_s, \gamma_k > 0$. Taking the time derivative of V and substituting (15) and (19)-(20) leads to

$$\begin{aligned}
\dot{V} = {} & z_s \left[\beta(\hat{\theta}_s)\dot{\hat{\theta}}_s + \dot{d}(t) - \theta_k y + u(s_0 - s)\right] \\
& - \frac{\tilde{\theta}_\mu \dot{\hat{\theta}}_\mu}{\gamma_\mu} - \frac{\tilde{\theta}_s \dot{\hat{\theta}}_s}{\gamma_s} - \frac{\tilde{\theta}_k \dot{\hat{\theta}}_k}{\gamma_k} - \tilde{\theta}_k y e_s - k_s e_s^2 \\
& + \frac{\tilde{\theta}^T \Phi(s,y,\hat{\theta}) y e_y}{1+\hat{\theta}_s s} - k_y(1+\theta_s s)e_y^2 \\
& + \frac{\theta_s}{2}[-\theta_k y + u(s_0 - s)]e_y^2
\end{aligned} \tag{22}$$

where

$$\beta(\hat{\theta}_s) = \frac{2+s_0\hat{\theta}_s}{2\hat{\theta}_s^2\sqrt{1+s_0\hat{\theta}_s}} - \frac{1}{\hat{\theta}_s^2} \tag{23}$$

We consider the following parameter updating laws

$$\dot{\hat{\theta}}_s = \begin{cases} \dfrac{\gamma_s \phi_s y e_y}{1+\hat{\theta}_s s}, & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \text{ and } \phi_s y e_y \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

$$\dot{\hat{\theta}}_\mu = \gamma_\mu s y e_y \tag{25}$$

$$\dot{\hat{\theta}}_k = -\gamma_k y\left(\frac{y}{s}e_y + z_s + e_s\right) \tag{26}$$

with the initial condition $\hat{\theta}_s(0) \geq 1/K_{s0} > 0$. Substituting the updating laws (25)-(27) into (23), we obtain

$$\begin{aligned}
\dot{V} \leq {} & z_s(\gamma_s \beta_a(y,s,\hat{\theta}_s)e_y + \dot{d}(t) - \hat{\theta}_k y \\
& + [1+\gamma_s \beta_b(y,s,\hat{\theta}_s)e_y]u(s_0 - s)) \\
& - k_s e_s^2 - (1+\theta_s s)\left[k_y - \frac{\theta_s(s_0 - s)u}{2(1+\theta_s s)}\right]e_y^2
\end{aligned} \tag{27}$$

where

$$\beta_a(y,\ s,\hat{\theta}_s) = \begin{cases} \dfrac{(-\hat{\theta}_\mu s^2 + \hat{\theta}_k y)y}{1+\hat{\theta}_s s}\beta(\hat{\theta}_s), & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \\ & \text{and } \phi_s y e_y \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

$$\beta_b(y,\ s,\hat{\theta}_s) = \begin{cases} -\dfrac{y}{1+\hat{\theta}_s s}\beta(\hat{\theta}_s), & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \\ & \text{and } \phi_s y e_y \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

4. CONTROLLER DESIGN

Considering the following extremum seeking controller

$$u = -\frac{\gamma_s \beta_a(y,s,\hat{\theta}_s)e_y + \dot{d}(t) - \hat{\theta}_k y + k_z z_s}{[1+\gamma_s \beta_b(y,s,\hat{\theta}_s)e_y](s_0 - s)} \tag{30}$$

with $k_z > 0$ and the gain function

$$k_y = k_{y0} + \frac{(s_0 - s)K_{s0}|u|}{2(1+K_{s0}s)} \tag{31}$$

with $k_{y0} > 0$, we have

$$\dot{V} \leq -k_z z_s^2 - k_s e_s^2 - k_{y0} e_y^2 \tag{32}$$

In order to avoid the singularity that may happen in equation (31) when $1 + \gamma_s \beta_b(y, s, \hat{\theta}_s) e_y$ approaches zero, a small learning gain γ_s should be used to ensure that

$$1 + \gamma_s \beta_b(y, s, \hat{\theta}_s) e_y > 0 \tag{33}$$

Following LaSalle-Yoshizawa's Theorem, it is concluded that $\hat{\theta}$, z_s, e_s and e_y are bounded, and

$$\lim_{t\to\infty} z_s = 0, \qquad \lim_{t\to\infty} e_s = 0, \qquad \lim_{t\to\infty} e_y = 0 \tag{34}$$

It should be noticed that the convergence of the state errors e_s and e_y does not mean that the estimated parameters converge to their true values as $t \to \infty$. In the following, we investigate the condition that guarantees the parameter convergence.

By LaSalle's Invariance Principle, the error vector $(z_s, e_s, e_y, \tilde{\theta})$ converges to the largest invariant set M of the dynamic system (19)-(20) and (25)-(27) contained in the set $E = \{(z_s, e_s, e_y, \tilde{\theta}) \in R^5 | z_s = e_s = e_y = 0\}$. The purpose of the following is to study the invariant set M to obtain the condition under which parameter convergence can be achieved. Since e_s and e_y converge to zero, we know that $\int_0^\infty \dot{e}_s dt = e_s(\infty) - e_s(0) = -e_s(0)$ and $\int_0^\infty \dot{e}_y dt = e_y(\infty) - e_y(0) = -e_y(0)$. This implies that $\dot{e}_s$ and $\dot{e}_y$ are integrable. It follows from the error equations (19)-(20) that $\ddot{e}_s$ and $\ddot{e}_y$ are functions of $y, s, \hat{y}, \hat{s}$, $\hat{\theta}$, d and its time derivatives. Since $\hat{\theta}, e_s, e_y \in L_\infty$, and the excitation signal d and $\dot{d}$ are bounded, we know that $\ddot{z}_x$ and $\ddot{e}_y$ are bounded. This implies the uniform continuity of $\dot{e}_s$ and $\dot{e}_y$. By Barbalat's Lemma (Ioannou and Sun 1996), we conclude that $\dot{e}_s, \dot{e}_y \to 0$ as $t \to \infty$.

On the invariant set M, we have $e_s = e_y \equiv 0$ and $\dot{e}_s = \dot{e}_y \equiv 0$. By setting $e_s = e_y = \dot{e}_s = \dot{e}_y = 0$, equations (19)-(20) lead to $\tilde{\theta}_k y = 0$ and

$$\frac{\tilde{\theta}^T \Phi(s, y, \hat{\theta}) y}{(1 + \theta_s s)(1 + \hat{\theta}_s s)} = 0, \quad (z_s, e_s, e_y, \tilde{\theta}) \in M \tag{35}$$

Since $s > 0$ and $\hat{\theta}$ are bounded, we know that

$$\tilde{\theta}_a^T \Phi_a(s, y, \hat{\theta}) y = 0, \quad (z_s, e_s, e_y, \tilde{\theta}) \in M \tag{36}$$

where $\tilde{\theta}_a = [\tilde{\theta}_s \ \tilde{\theta}_\mu]^T$ and $\Phi_a(s, y, \hat{\theta}) = [\phi_s \ \phi_\mu]^T$. Therefore, the largest invariant set M in E is

$$M = \left\{ (z_s, e_s, e_y, \ \tilde{\theta}) \in R^6 \middle| z_s = e_s = e_y = 0, \right.$$
$$\left. \tilde{\theta}_a^T \Phi_a(s, y, \hat{\theta}) y = 0, \ \tilde{\theta}_k y = 0 \right\}$$

It follows from (37) that $\forall (z_s, e_s, e_y, \tilde{\theta}) \in M$

$$\Psi(t) = \tilde{\theta}_a^T \Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta}) y^2 \tilde{\theta}_a = 0, \tag{37}$$

If $\Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta}) y^2$ is positive definite, then we may conclude that $\tilde{\theta} = 0$. However, it is impossible to satisfy this condition because the matrix $\Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta}) y^2$ is singular at any given time. We consider the condition

$$\lim_{t\to\infty} \frac{1}{T_0} \int_t^{t+T_0} \left[\tilde{\theta}_a^T \Psi(t) \tilde{\theta}_a \right] d\tau = 0 \tag{38}$$

with positive constant T_0. It can be shown from (25)-(27) and $\lim_{t\to\infty} e_s, e_y = 0$ that $\lim_{t\to\infty} \dot{\hat{\theta}} = 0$, which implies that $\tilde{\theta}$ converges to a constant as $t \to \infty$. Therefore, $\forall (z_s, e_s, e_y, \tilde{\theta}) \in M$

$$\tilde{\theta}_a^T \left\{ \lim_{t\to\infty} \frac{1}{T_0} \int_t^{t+T_0} \Psi(t) d\tau \right\} \tilde{\theta}_a = 0, \tag{39}$$

As a result, we show that if the dither signal $d(t)$ is designed such that the following condition holds

$$\lim_{t\to\infty} \frac{1}{T_0} \int_t^{t+T_0} \Psi(t) d\tau \geq c_0 I \tag{40}$$

for some $c_0 > 0$ and

$$s \in \Omega_s = \left\{ s \middle| s = \frac{1}{\theta_s} \left(\sqrt{1 + s_0 \bar{\theta}_s} - 1 \right) - d(t), \right.$$
$$\left. \bar{\theta}_s \geq 1/K_{s0} \right\} \tag{41}$$

then, the parameter error $\tilde{\theta}$ converges to zero asymptotically.

Theorem 4.1. For the system (1)-(3), if

i) the learning rate γ_s is chosen small enough such that (34) holds, and
ii) the dither signal $d(t)$ satisfies the PE condition (41),

then, the extremum seeking controller (31) with adaptive laws (25)-(27) guarantees that the production rate y converges to an adjustable neighborhood of its maximum y^*.

Proof: Since the PE condition (41) is satisfied, we have $\lim_{t\to\infty} \hat{\theta}_s = \theta_s$ and $\lim_{t\to\infty} \hat{\theta}_k = \theta_k$. By $\lim_{t\to\infty} z_s = 0$ and $\lim_{t\to\infty} e_y = 0$, we see from (21) and (31) that

$$\lim_{t\to\infty} s = s^* - \lim_{t\to\infty} d(t) \tag{42}$$

$$\lim_{t\to\infty} u = \lim_{t\to\infty} \frac{\theta_k y - \dot{d}(t)}{s_0 - s} \tag{43}$$

Hence, by (3) and (12) we know that when $t \to \infty$, the following equation holds

$$\dot{x} = \left[s_0 - s + \frac{\dot{d}(t)}{\mu(s)} - k_1 x\right] \frac{\mu(s)x}{s_0 - s}$$

From (11) and (43), the above equation can be further expressed as

$$\dot{x} = \left[x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1\mu(s)} - x\right] \frac{k_1\mu(s)x}{s_0 - s}$$

Since $x, \mu(s)$ and $s_0 - s$ are positive definite, we see that (i) $\dot{x} < 0$ when $x > x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1\mu(s)}$, (ii) $\dot{x} > 0$ when $x < x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1\mu(s)}$. This implies that the biomass concentration x converges to a neighborhood of x^*. The size of the neighborhood depends on the external dither signal $d(t)$ and its changing rate. For easy presentation, we denote

$$\lim_{t\to\infty} x = x^* + \varepsilon(d, \dot{d}) \tag{44}$$

where $\varepsilon(d, \dot{d})$ represents the effect of the dither signal. It is clear that $\varepsilon(d, \dot{d}) \to 0$ when $d(t), \dot{d}(t) \to 0$.

Using the Mean Value Theorem (Ortega and Rheinboldt 1970), we may re-expressing the production rate y in (3) as

$$y = k_2\mu(s^*)x + k_2(s - s^*)x \int_0^1 \frac{\partial\mu(s_\lambda)}{\partial s_\lambda} d\lambda$$

where $s_\lambda = \lambda s + (1 - \lambda)s^*$. Considering (9), (43) and (45), we have

$$\begin{aligned} \lim_{t\to\infty} y =& \; y^* + k_2\mu(s^*)\varepsilon(d, \dot{d}) \\ & - \lim_{t\to\infty} \left[k_2 d(t) \int_0^1 \frac{\partial\mu(s_\lambda)}{\partial s_\lambda} d\lambda\right] \end{aligned} \tag{45}$$

The above equation implies that the production rate y converges to a neighborhood of the desired production rate y^*, whose size is adjustable by tuning the amplitudes of the injected dither signal $d(t)$ and its time derivative. **Q.E.D.**

5. SIMULATION RESULTS

To show the effectiveness of the proposed design, a simulation study is performed using the experimental conditions provided in the work (Wang et al. 1999). The following parameters and initial states are used in the simulation experiment.

$$\begin{aligned} & K_s = 0.2, \;\; \mu_m = 1.0, \;\; Y = 0.5, \;\;\; k_1 = 2.0, \\ & k_2 = 1.0, \;\; s_0 = 10.0, \;\;\; x(0) = 3.0, \\ & s(0) = 0.9. \end{aligned}$$

We suppose that the upper bound of K_s is known as $K_{s0} = 0.5$. The design parameters in the adaptive controller (31) and the adaptive laws (25)-(27) are

$$\begin{gathered} \gamma_s = 2.0, \quad \gamma_\mu = 20.0, \quad \gamma_k = 2.0 \\ \hat{\theta}_s(0) = 8.0, \quad \hat{\theta}_\mu(0) = 2.0, \quad \hat{\theta}_k(0) = 4.0 \end{gathered}$$

The dither signal is chosen as $d(t) = 2.2 - \cos(0.5t) - \cos(0.3t)$.

Figures 1 and 2 present the simulation result of the adaptive extremum seeking controller. It is shown from Figure 1 that the production rate reaches a neighborhood of its maximum value 3.77 very quickly. Due to the injection of the excitation signal $d(t)$, the production rate keeps oscillating below the optimal point. This oscillation is necessary to ensure the convergence to the true parameter values. It is interesting to note that, in this case, the maximum is reached before the parameters converge to their true values. This shows the effectiveness of this technique. The convergence of the algorithm can be confirmed by inspection of a simulation performed over a longer time period (i.e. $t = 800$), shown in Figure 3.

6. CONCLUSION

We have solved a class of extremum seeking control problems for continuous stirred tank bioreactors represented by Monod's growth model with unknown parameters. The proposed extremum seeking controller drives biomass and substrate concentrations to unknown desired set-points that optimize the production rate. A persistence of excitation condition is derived to ensure the convergence of the production rate of the bioreactor to a neighborhood of its maximum.

7. REFERENCES

Astrom, K.J. and B. Wittenmark (1995). Adaptive Control, 2nd Edition. Addison-Wesley. Reading, MA.

Drkunov, S., U. Ozguner, P. Dix and B. Ashrafi (1995). Abs control using optimum search via sliding modes. IEEE Trans. Contr. Syst. Tech. **3**, 79–85.

Findeisen, W., F.N. Bailey, M. Brdys, K. Malinowski, P. Tatjewski and A. Wozniak (1980). Control and coordination in Hierarchical Systems. John Wiley & sons.

Goodwin, G.C. and K.S. Sin (1984). Adaptive Filtering Prediction and control. Prentice-Hall. Englewood Cliffs, NJ.

Ioannou, P. A. and J. Sun (1996). Robust Adaptive Control. Prentice-Hall. Englewood Cliffs, NJ.

Krstic, M. (2000). Performance improvement and limitations in extremum seeking control. Systems & Control Letters **5**, 313–326.

Krstic, M. and H. Deng (1998). Stabilization of Nonlinear Uncertain Systems. Springer-Verlag.

Krstic, M. and H.H. Wang (2000). Stability of extremum seeking feedback for general dynamic systems. Automatica **4**, 595–601.

Krstic, M., I. Kanellakopoulos and P. Kokotovic (1995). Nonlinear and Adaptive Control Design. Wiley and Sons. New York.

Landau, Y.D. (1979). Adaptive Control. Marcel Dekker. New York.

Leblanc, M. (1922). Sur l'*é*lectrification des chemins de fer au moyen de courants alternatifs de fr*é*quence *é*lev*é*e. Revue G*é*n*é*rale de l'Electricit*é*.

Morari, M., G. Stephanopoulos and Y. Arkun (1980). Studies in the synthesis of control structures for chemical processes. part i: Formulation of the problem. process decomposition and the classifica-tion of the control task. analysis of the optimizing control structures. AIChE Journal.

Narendra, K. S. and A. M. Annaswamy (1989). Stable Adaptive System. Prentice-Hall. Englewood Cliffs, NJ.

Ortega, J. M. and W. C. Rheinboldt (1970). Iterative solution of nonlinear equations in several variables. Academic Press. New York and London.

Skogestad, S. (2000). Plantwide control: the search for the self-optimizing control structure. Journal of Proc. Control **10**, 487–507.

Sternby, J. (1980). Extremum control systems: An area for adaptive control?. Preprints of the Joint American Control Conference, San Francisco, CA.

Vasu, G. (1957). Experiments with optimizing controls applied to rapid control of engine presses with high amplitude noise signals. Transactions of the ASME pp. 481–488.

Wang, H., M. Krstic and G. Bastin (1999). Optimizing bioreactors by extremum seeking. Int. Journal Adaptive Control and Signal Processing **13**, 651–669.

Wang, H., S. Yeung and M. Krstic (1998). Experimental application of extremum seeking on an axial-flow compressor. Proc. American Control Conference, Philadelphia pp. 1989–1993.

Zhang, T. and M. Guay (2001). Adaptive parameter estimation for microbial growth kinetics. accepted for publication in AIChE J.

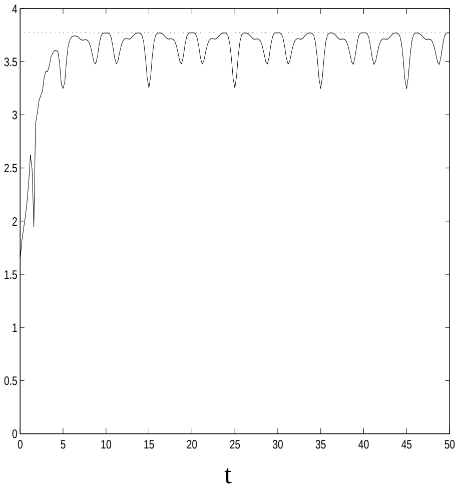

Fig. 1. Production rate y ("—") and its maximum y^* ("- -")

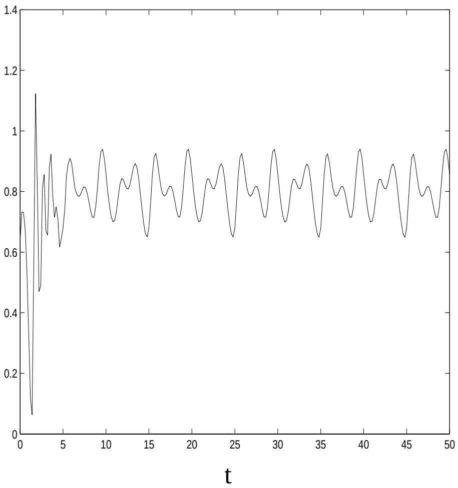

Fig. 2. Control input $u(t)$

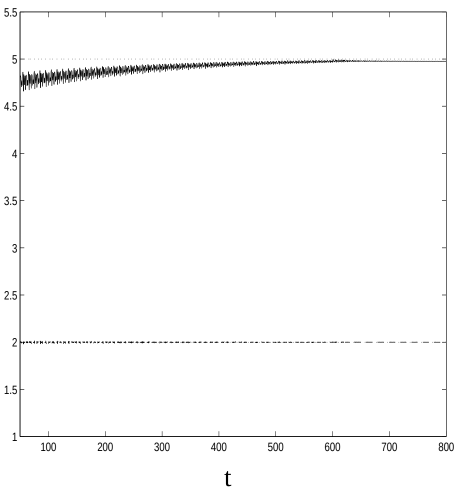

Fig. 3. Parameter θ_μ ("...") and its estimate $\hat{\theta}_\mu$ ("—") Parameter θ_k ("...") and its estimate $\hat{\theta}_k$ ("- -")