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QUASI-DEADBEAT MINIMAX FILTERS FOR DETERMINISTIC CONTINUOUS-TIME STA TE SPACE SIGNAL MODELS

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Abstract: In this paper, a quasi-deadbeat propert y is introduced and a quasi-deadbeat minimax filter (QMMF) is proposed for contin uous-time state space signal models. Linearity, quasi-deadbeat property, FIR structure, and independence of the initial state information will be required in advance, in addition to a performance index of the worst case gain between the disturbance and the current estimation error. The proposed QMMF is obtained by directly minimizing a performance index with the quasi-deadbeat constraint. The proposed QMMF is represen ted first in a standard FIR form and then in an iterative form. The QMMF will be shown to be used also for the IIR structure.

Keywords: State estimation, Quasi-Deadbeat minimax filter (QMMF), FIR structure, quasi-deadbeat property.

1. INTRODUCTION

F or the frequency domain based filter design, linear phase filters are often preferred because the original signal can be processed without distortion. This is equiv alen t to the exact estimation, i.e, zero estimation error, in the time domain based filter design. The exact estimation filters are often called the deadbeat filters. F or signal models with disturbances it is too strong that thedeadbeat propert y holds even under every case of disturbances. Thus, the existing deadbeat filters have been obtained for some class of disturbances, particularly for zero disturbance(Valcher, 1999) (Suh and Choi, 1999). If there exist undesirable disturbances, the deadeat property does not hold and estimation error could be very large. That is, the deadbeat filter can be sensitiv e to exterior disturbances and can not guarantee some performances.

Meanwhile there are other class of filters such as worst case filters, where worst case estimation error due to some w orst case element of possible disturbances is minimized. They are minimax filters for stochastic systems (Verdu and Poor, 1984) (Darragh and Looze, 1984) (Poor and Loose, 1981) (Krener, 1980) and H_∞ filters for deterministic systems (Li and F u, 1997) (Nagpal and Khargonekar, 1991) (Fu and de Souza, 1992). In particular, an H_∞ filter is designed such that the H_∞ norm, which reflects the w orst case estimation error, is minimized. An optimal solution of H_∞ filters is difficult to implement. Th us, suboptimal solutions are often obtained in order to guarantee some upper bound of the error. In this case, there is still no systematic guide to find a reasonable bound. F or some real disturbances, the estimation error could be unnecessarily large since it is designed for w orst case element of disturbances.

Therefore it will be good if deadbeat property holds for some class of disturbances, particularly for zero disturbance and at the same time worst case estimation error is minimized for some worst case element of disturbances. This approach is suggested in this paper. The above filters will be called quasi-deadbeat filters since the filters have deadbeat property for zero disturbance but may not have such property if there are undesirable disturbances.

F or most existing state estimation filters including abo ve filters, initial state information is often assumed known even if the initial state is also a state to be estimated. This is not reasonable. Therefore the initial state information is assumed completely unknown in this paper. The suggested filters will be obtained independently of the initial state information.

Filters can be of the infinite impulse response (IIR) type or the finite impulse response (FIR) type. In this paper, the focus will be on FIR filters, while the IIR filter will be summarized at the end of each section. The linear FIR filter, independent of the initial state information can be represented by

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)y(\sigma)d\sigma + \int_{t-T}^{t} L(t-\sigma)u(\sigma)d\sigma \quad (1)$$

at the present time t for some gains $H(\cdot)$ and $L(\cdot)$. The IIR filter has a similar form to (1) with $t-T$ replaced by the initial time t_0. For IIR and FIR types, the initial state means $x(t_0)$ and $x(t-T)$, respectively.

The suggested filter (1) with either a FIR or an IIR structure will not have a state term and the filter gain $H(\cdot)$ and $L(\cdot)$ will be independent of the initial state information. It is noted that standard Kalman filters have an initial state term and the gain $H(\cdot)$ also depends on the initial state information.

FIR filters make use of finite measurements and inputs on the most recent time interval $[t-T, t]$, called the receding horizon, or the window, to avoid long calculation times that arise, as is often the case of IIR structures, from large data sets as time increases. It has been generally accepted that the FIR structure in filters is more robust to temporary modeling uncertainties and numerical errors than the IIR structure.

Measure of worst case estimation error can be defined differently from existing ones if necessary. In this paper, a new measure of worst case estimation error will be suggested, which in fact results in very interesting solutions. Among linear FIR filters with quasi-deadbeat property, the following new optimal criterion will be suggested:

$$\min_{H(\cdot)L(\cdot)} \max_{w(\cdot)\neq 0} \frac{[x(t)-\hat{x}(t|t)]^T[x(t)-\hat{x}(t|t)]}{\int_{t-T}^{t} w^T(\tau)w(\tau)d\tau}. \quad (2)$$

These optimal filters will be called the quasi-deadbeat minimax filter(QMMF) with FIR structure. In (Han *et al.*, 2001*a*), the deterministic discrete system is considered based on the minimax optimal criterion, which only requires the algebraic manipulation. However, there is no result for the corresponding problem in case of continuous-time systems since an entailed functional optimization problem is difficult to solve. This paper proposes a new minimax optimal criterion (2) under the assumption of the unknown initial state and the requirement of the quasi-deadbeat property for continuous-time systems.

It is noted that the criterion (2) differs from the existing criterion for H_∞ problems for IIR filters as

$$\inf_{H(\cdot)L(\cdot)} \sup_{w(\cdot)\neq 0} \frac{\int_{t_0}^{t}[x(\tau)-\hat{x}(\tau)]^T[x(\tau)-\hat{x}(\tau)]d\tau}{\int_{t_0}^{t} w^T(\tau)w(\tau)d\tau} \quad (3)$$

where t_0 is the initial time. Note that the numerator of (2) considers only the current estimation error compared with (3) using the distributed terms in the numerator. Since the current estimation error is an issue, it is reasonable to take only the current estimation error in the numerator of the cost function instead of the accumulated estimation error including the previous estimation errors. H_∞ problems are difficult to solve and this problem can be solved by considering an upper bound on the H_∞ norm, which yields solutions of a differential game. However, the proposed QMMF will be shown to provide an optimal solution explicitly, even with the quasi-deadbeat property. It is also shown that the approach for the FIR structure can be extended to the IIR structure.

The QMMF is both quasi-deadbeat and optimal *by design* for the proposed cost. The '*by design*' means that the quasi-deadbeat property and optimality are built into the QMMF during its design. While only the algebraic manipulations are necessary for discrete-time systems, in this work, the calculus of variation is mainly used for continuous-time systems. The proposed QMMF will be represented in both a standard batch form and an iterative form. It will be shown that the QMMF for deterministic systems is similar in form to the existing receding horizon unbiased FIR filter (RHUFF) for stochastic systems (Han *et al.*, 2001*b*)(Kwon *et al.*, n.d.).

This paper is organized as follows. In Section 2, the QMMF for continuous-time state space models is proposed in a standard batch form. In Section 3, iterative forms will be obtained and the comparison with the existing RHUFF and the Kalman filter are shown. Finally, conclusions are stated in Section 4.

2. QUASI-DEADBEAT MINIMAX FILTERS

Consider a linear continuous-time state space model with control input:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad (4)$$

$$y(t) = Cx(t) + Dw(t) \quad (5)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^l$, $y(t) \in \Re^q$, and $w(t) \in \Re^p$ are the state, the input, the measurement, and the disturbance, respectively. $GD^T = O$ and $DD^T = I$ are satisfied to decouple the system disturbance and the measurement disturbance.

The systems (4) and (5) will be represented on the most recent time interval $[t-T, t]$, called the horizon. The current state $x(t)$ is given by a solution of (4) as follows:

$$x(t) = e^{A(t-\sigma)}x(\sigma) + \int_{\sigma}^{t} e^{A(t-\tau)}Bu(\tau)d\tau + \int_{\sigma}^{t} e^{A(t-\tau)}Gw(\tau)d\tau, \quad t-T \le \sigma \le t. \quad (6)$$

Then, $x(\sigma)$ in (6) is written as

$$x(\sigma) = e^{A(\sigma-t)}x(t) - \int_{\sigma}^{t} e^{A(\sigma-\tau)}Bu(\tau)d\tau - \int_{\sigma}^{t} e^{A(\sigma-\tau)}Gw(\tau)d\tau.$$

Therefore, on the horizon $[t-T, t]$, the finite measurements and inputs can be expressed in terms of the state $x(t)$ at the current time t as follows:

$$\begin{aligned} y(\sigma) &= Cx(\sigma) + Dw(\sigma) \\ &= C\left[e^{A(\sigma-t)}x(t) - \int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau \right. \\ &\quad \left. - \int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau\right] + Dw(\sigma). \end{aligned} \tag{7}$$

The output $y(\sigma)$ and the integral term including input $u(\tau)$ are assumed to be known. Hence, known and unknown parts can be separated as

$$\begin{aligned} y(\sigma) + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau &= C\left[e^{A(\sigma-t)}x(t) \right. \\ &\quad \left. - \int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau\right] + Dw(\sigma). \end{aligned}$$

Using known variables, the QMMF for the current state $x(t)$ can be expressed as a linear functional of the finite measurements and inputs on the horizon $[t-T, t]$ as follows:

$$\begin{aligned} \hat{x}(t|t) &\triangleq \int_{t-T}^t H(t-\sigma)\left[y(\sigma) + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau\right]d\sigma \\ &= \int_{t-T}^t H(t-\sigma)y(\sigma)d\sigma + \int_{t-T}^t L(t-\sigma)u(\sigma)d\sigma \end{aligned} \tag{8}$$

where

$$L(t-\sigma) = \int_{t-T}^\sigma H(t-\tau)Ce^{A(\tau-\sigma)}Bd\tau. \tag{9}$$

Note that $H(t-\sigma)$ and $L(t-\sigma)$ are gain matrices of a linear filter. It is noted that the filter defined in (8) is an FIR structure without any *a priori* statistical information on the horizon initial state $x(t-T)$. The gain matrix $H(t-\sigma)$ will be designed such that $\hat{x}(t|t)$ is an deadbeat estimation filter of the current state $x(t)$ as

$$\begin{aligned} \hat{x}(t|t) &= \int_{t-T}^t H(t-\sigma)\left[y(\sigma) + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau\right]d\sigma \\ &= \int_{t-T}^t H(t-\sigma)\left[Ce^{A(\sigma-t)}x(t) \right. \\ &\quad \left. - C\int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau + w(\sigma)\right]d\sigma \end{aligned}$$

If there is no disturbance,

$$\hat{x}(t|t) = \int_{t-T}^t H(t-\sigma)Ce^{A(\sigma-t)}x(t)d\sigma.$$

In order for $x(\hat{t}|t) = x(t)$, the following constraint on $H(t-\sigma)$ is required:

$$\int_{t-T}^t H(t-\sigma)Ce^{A(\sigma-t)}d\sigma \;=\; I \tag{10}$$

which will be called the quasi-deadbeat constraint. It is noted that constraint (10) must hold regardless of the information on the horizon initial state $x(t-T)$ on the horizon $[t-T, t]$. This constraint may be too strict, but surprisingly, we were able to obtain the solution.

The objective now is to obtain the best gain matrix $H_B(t-\sigma)$, subject to the quasi-deadbeat constraint (10), based on the following criterion:

$$\begin{aligned} H_B(t-\sigma) \;&=\; \arg\min_{H(t-\sigma)}\max_{w(\cdot)\neq 0} \\ &\left\{\frac{\left[x(t)-\hat{x}(t|t)\right]^T\left[x(t)-\hat{x}(t|t)\right]}{\int_{t-T}^t w^T(\tau)Fw(\tau)d\tau}\right\}. \end{aligned} \tag{11}$$

To solve the above state estimation problem with quasi-deadbeat constraint, an optimization problem with constraints will be introduced. It will be shown that the constraints consist of an algebraic equation and a differential equation.

Replacing $y(\sigma)$ with the right side of (7), we have the estimate as follows:

$$\begin{aligned} &\hat{x}(t|t) \\ &= \int_{t-T}^t H(t-\sigma)\left[y(\sigma) + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau\right]d\sigma \\ &= \int_{t-T}^t H(t-\sigma)\left[Cx(\sigma) + Dw(\sigma) \right. \\ &\quad \left. + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau\right]d\sigma \\ &= \int_{t-T}^t H(t-\sigma)\left[C\{e^{A(\sigma-t)}x(t) - \int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau \right. \\ &\quad - \int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau\} + Dw(\sigma) \\ &\quad \left. + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau\right]d\sigma. \end{aligned} \tag{12}$$

Using the quasi-deadbeat constraint (10) and rearranging the terms, the error between the real current state and the estimate can be expressed as

$$\begin{aligned} x(t) - \hat{x}(t|t) = \int_{t-T}^t H(t-\sigma)\{C\int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau \\ - Dw(\sigma)\}d\sigma. \end{aligned} \tag{13}$$

In solving for $H(t-\sigma)$, it will be convenient to define $H(t-\sigma)$ consisting of the row vector $h_i^T(t-\sigma)$ for $1 \le i \le n$ as

$$H(t-\sigma) \;\triangleq\; \begin{bmatrix} h_1^T(t-\sigma) \\ h_2^T(t-\sigma) \\ h_3^T(t-\sigma) \\ \vdots \\ h_n^T(t-\sigma) \end{bmatrix}. \tag{14}$$

Then, the error of the i-th state $x_i(t)$ can now be expressed in terms of the vector components of $H(t-\sigma)$ as follows:

$$
\begin{aligned}
&\{x_i(t) - \hat{x}_i(t|t)\}^2 \\
&= [\int_{t-T}^{t} h_i(t-\sigma)\{C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Gw(\tau)d\tau \\
&- Dw(\sigma)\}d\sigma]^2 \\
&= [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau \\
&- h_i(t-\sigma)D\}w(\sigma)d\sigma]^2. \qquad (15)
\end{aligned}
$$

Though as state previously, the QMMF is required to be deadbeat with zero disturbance, the QMMF can be deadbeat if the integral kernel and the $w(\cdot)$ in (15) are orthogonal each other. This shows that the proposed QMMF can be deadbeat for all $w(\cdot)$ belong to the orthogonal space with respect to the integral kernel.

By the Causchy-Schwartz inequality, the following relation is obtained from (15):

$$
\begin{aligned}
\{x_i(t) - \hat{x}_i(t|t)\}^2 \quad &\le [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau \\
&- h_i(t-\sigma)D\}^2 d\sigma]\int_{t-T}^{t} w^T(\tau)w(\tau)d\tau.
\end{aligned}
$$

Thus

$$
\begin{aligned}
\frac{\{x_i(t) - \hat{x}_i(t|t)\}^2}{\int_{t-T}^{t} w^T(\tau)w(\tau)d\tau} \quad &\le [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau \\
&- h_i(t-\sigma)D\}^2 d\sigma].
\end{aligned}
$$

Note that an equality is satisfied for some $w(\tau)$ which is linearly dependent on an error. So,

$$
\begin{aligned}
\max_{w(\cdot)\neq 0} \frac{\{x_i(t) - \hat{x}_i(t|t)\}^2}{\int_{t-T}^{t} w^T(\tau)w(\tau)d\tau} &= [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau \\
&- h_i(t-\sigma)D\}^2 d\sigma]. \qquad (16)
\end{aligned}
$$

The right side of (16) can be represented as

$$
\begin{aligned}
&\int_{t-T}^{t}\int_{t-T}^{\sigma}\int_{t-T}^{\sigma} h_i^T(t-\tau)Ce^{A(\tau-\sigma)}G \\
&G^T e^{A^T(n-\sigma)}C^T h_i(t-n)d\tau dn d\sigma \\
&+ \int_{t-T}^{t} h_i^T(t-\sigma)h_i(t-\sigma)d\sigma. \qquad (17)
\end{aligned}
$$

Introducing another variable, we obtain the somewhat simplified form requiring only a minimization operation as follows:

$$
\operatorname*{Min}_{h_i(t-\sigma),f(\sigma)} \int_{t-T}^{t} f_i^T(\sigma)GG^T f_i(\sigma)d\sigma + \int_{t-T}^{t} h_i^T(t-\sigma)h_i(t-\sigma)d\sigma
$$

$$
\text{subject to} \quad \dot{f}_i^T(\sigma) = h_i^T(t-\sigma)C - f_i^T(\sigma)A \quad \text{and} \quad f_i^T(t) = e_i^T
$$

where $f_i(\sigma) = \int_{t-T}^{\sigma} e^{A^T(\tau-\sigma)}C^T h_i(t-\tau)d\tau$, $f_i(t-T) = 0$, and e_i is the ith unit vector such that $e_i = [0,\cdots,0,1,0,\cdots,0]^T$ with the nonzero element in the ith position. To solve the above optimization problem, the key idea is introduced. To extremize the integral

$$
I = \int_{t_1}^{t_2} f(x_1, x_2, \cdots, \dot{x_1}, \dot{x_2}, \cdots, t)dt \qquad (18)
$$

with respect to the continuously differentiable functions $x_1, x_2, \cdots$ which achieve the prescribed values $t = t_1$ and $t = t_2$, and satisfy the given equation $G(x_1, x_2, \cdots, \dot{x_1}, \dot{x_2}, \cdots, t) = 0$, the following differential equations must be satisfied

$$
\frac{\partial F}{\partial x_i} - \frac{d}{dt}(\frac{\partial F}{\partial \dot{x_i}}) = 0 \quad \text{for i} = 1, 2, \cdots \qquad (19)
$$

where $F(x_1, x_2, \cdots, \dot{x_1}, \dot{x_2}, \cdots, t) = f(x_1, x_2, \cdots, \dot{x_1}, \dot{x_2}, \cdots, t) + \lambda(t)G(x_1, x_2, \cdots, \dot{x_1}, \dot{x_2}, \cdots, t)$ for function $\lambda(t)$ which is determined to extremize I.

Define the following notation:

$$
\begin{aligned}
F(\sigma) \triangleq \; &f_i^T(\sigma)GG^T f_i(\sigma) + h_i^T(t-\sigma)h_i(t-\sigma) \\
&+ \lambda_i^T(\dot{f}_i(\sigma) - C^T h_i(t-\sigma) + A^T f_i(\sigma)).
\end{aligned}
$$

We need to calculate the following value:

$$
\frac{F}{\partial f_i} = \; 2GG^T f_i(\sigma) + A\lambda_i(\sigma) \qquad (20)
$$

$$
\frac{F}{\partial \dot{f}_i} = \lambda_i(\sigma) \qquad (21)
$$

$$
\frac{F}{\partial h_i} = 2h_i(t-\sigma) - C\lambda_i(\sigma). \qquad (22)
$$

From equation (19), we can obtain $f_i(\sigma)$ in a Hamiltonian matrix form

$$
\begin{aligned}
\frac{d}{d\sigma}\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma) \end{bmatrix} &= \begin{bmatrix} -A^T & \frac{1}{2}C^T C \\ 2GG^T & A \end{bmatrix}\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma) \end{bmatrix} \\
&\triangleq \mathbf{H}\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma) \end{bmatrix}.
\end{aligned}
$$

From (22), $h_i(t-\sigma)$ is of the form

$$
\begin{aligned}
h_i(t-\sigma) &= \frac{1}{2}C\lambda_i(\sigma) \\
&= \frac{1}{2}C\begin{bmatrix} 0 & I \end{bmatrix} e^{H(\sigma-t+T)}\begin{bmatrix} f_i(t-T) \\ \lambda_i(t-T) \end{bmatrix}.
\end{aligned}
$$

Using $f_i(t-T) = 0$ and $e^{\mathbf{H}(\sigma-t+T)}$ defined by

$$
e^{\mathbf{H}(\sigma-t+T)} = \begin{bmatrix} X(\sigma-t+T) & Y(\sigma-t+T) \\ Z(\sigma-t+T) & W(\sigma-t+T) \end{bmatrix}, \qquad (23)
$$

$h_i(t-\sigma)$ can be expressed as

$$
h_i(t-\sigma) = \frac{1}{2}CW(\sigma-t+T)\lambda_i(t-T).
$$

Using the quasi-deadbeat condition

$$
\int_{t-T}^{t} \frac{1}{2}\lambda_i^T(t-T)W^T(\sigma-t+T)C^T Ce^{A(\sigma-t)}d\sigma = e_i^T,
$$

we have

$$
\lambda_i^T(t-T) = e_i^T\left[\int_{t-T}^{t} \frac{1}{2}W^T(\sigma-t+T)C^T Ce^{A(\sigma-t)}d\sigma\right]^{-1}
$$

where the inverse exists as follows. By tedious calculation using (23), $\int_{t-T}^{t} \frac{1}{2}W^T(\sigma-t+T)C^T Ce^{A(\sigma-t)}d\sigma$

can be replaced by $Y^T(T)$. If (A, C) is observable, $Y(T)$ is guaranteed to be nonsingular as follows. $e^{H(\sigma-t+T)}$ are similar to the block upper triangular matrix

$$e^{\mathbf{H}(\sigma-t+T)} = \begin{bmatrix} I & O \\ S & I \end{bmatrix} exp\Bigg\{\begin{bmatrix} -(A-\mathbf{G}C)^T & \frac{1}{2}C^TC \\ O & A-\mathbf{G}C \end{bmatrix}$$
$$(\sigma-t+T)\Bigg\}\begin{bmatrix} I & O \\ -S & I \end{bmatrix}$$

where S and $\mathbf{G}$ satisfy $O = SA^T + AS + 2GG^T - \frac{1}{2}SC^TCS$ and $\mathbf{G} = \frac{1}{2}SC^T$, which are guaranteed to exist for (A, C) observability. Using the inverse Laplace transform and the convolution, $Y(T)$ can be obtained as follows:

$$Y(T) = e^{-(A-GC)^T T}\int_0^T \frac{1}{2}e^{(A-GC)^T\tau}C^TCe^{(A-GC)\tau}d\tau.$$

If (A, C) is observable, $(A - GC, C)$ is also observable. Thus, $Y(T)$, consisting of an exponential matrix and the observability Gramian, is nonsingular.

Theorem 1. The QMMF for the observable system (4) and (5) can be expressed as

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)y(\sigma)d\sigma + \int_{t-T}^{t} L(t-\sigma)u(\sigma)d\sigma \quad (24)$$

where $H(t-\sigma)$ and $L(t-\sigma)$ are as follows:

$$H(t-\sigma) = \left[\int_{t-T}^{t} W^T(\tau-t+T)C^TCe^{A(\tau-t)}d\tau\right]^{-1}$$
$$W^T(\sigma-t+T)C^T \quad (25)$$

and

$$L(t-\sigma) = \int_{t-T}^{\sigma} H(t-\tau)Ce^{A(\tau-\sigma)}Bd\tau \quad (26)$$

where $W(\tau - t + T)$ is given by (23). ∎

It is surprising that there exists a closed form solution (25) and (26) even under the strong condition (10) and that the gain $H(\cdot)$ is independent of the initial state information. While most of filters including H_∞ filter and Kalman filter encounter singular problems for zero disturbance, the proposed QMMF still holds even under zero disturbance.

Remark 2. In case of zero disturbance, the filter gain of the QMMF is reduced to the following form:

$$H(t-\sigma) = \left[\int_{t-T}^{t} e^{A^T(\tau-t+T)}C^TCe^{A(\tau-t)}d\tau\right]^{-1}$$
$$e^{A^T(\sigma-t+T)}C^T \quad (27)$$

Remark (1) shows that the QMMF is well defined under zero disturbance and the inverse is guaranteed for the observability of $(A.C)$.
It is noted that, using similar procedures, the batch form of IIR filters requiring no *a priori* initial state information can be given by (24), (25), and (26) with $t - T$ replaced by t_0. In the next section, an iterative form for a batch form (25) and (26) will be shown.

3. ITERATIVE FORM

Consider $S(\sigma - t + T)$ defined by

$$S(\sigma-t+T) \triangleq W^{-T}(\sigma-t+T)$$
$$\left[\int_{t-T}^{\sigma} W^T(\tau-t+T)C^TCe^{A(\tau-\sigma)}d\tau\right] \quad (28)$$

for $t - T \le \sigma \le t$. From the definition of $W(\cdot)$ and $Y(\cdot)$, $W(\cdot)$ and $Y(\cdot)$ satisfy the following differential equations with respect to σ:

$$\frac{\partial W(\sigma-t+T)}{\partial\sigma} = 2GG^TY(\sigma-t+T) + AW(\sigma-t+T) \quad (29)$$
$$\frac{\partial Y(\sigma-t+T)}{\partial\sigma} = -A^TY(\sigma-t+T) + \frac{1}{2}C^TCW(\sigma-t+T). \quad (30)$$

Then, differentiating $S(\sigma - t + T)$ with respect to σ and substituting (28) and (30) into the result, we have the derivative of $S(\sigma - t + T)$ as follows:

$$\frac{\partial S(\sigma-t+T)}{\partial\sigma} = -S(\sigma-t+T)GG^TS(\sigma-t+T) - A^TS(\sigma-t+T) + C^TC - S(\sigma-t+T)A. \quad (31)$$

If we define $\hat{\eta}(\sigma|t) \triangleq S(\sigma - t + T)\hat{x}(\sigma|t)$, i.e.,

$$\hat{\eta}(\sigma|t) = \int_{t-T}^{\sigma} W^{-T}(\sigma-t+T)W^T(s-t+T)C^T$$
$$\left[y(s) + C\int_s^{\sigma} e^{A(s-\tau)}Bu(\tau)d\tau\right]ds \quad (32)$$

where

$$\hat{x}(\sigma|t) = \left[\int_{t-T}^{\sigma} W^T(\tau-t+T)C^TCe^{A(\tau-\sigma)}d\tau\right]^{-1}$$
$$\int_{t-T}^{\sigma} W^T(s-t+T)C^T\Bigg[y(s) + C\int_s^{\sigma} e^{A(s-\tau)}Bu(\tau)d\tau\Bigg]ds, \quad (33)$$

we obtain another recursive equation:

$$\frac{\partial\hat{\eta}(\sigma|t)}{\partial\sigma} = -[A^T + S(\sigma-t+T)GG^T]\hat{\eta}(\sigma|t) + C^Ty(\sigma) + S(\sigma-t+T)Bu(\sigma).$$

Theorem 3. Assume that $\{A, C\}$ is observable and $T > 0$. The QMMF $\hat{x}(t|t)$ for continuous-time state space models (4), (5) is given on the horizon $[t - T, t]$ as follows:

$$\hat{x}(t|t) = S^{-1}(T)\hat{\eta}(t|t) \quad (34)$$

where $\hat{\eta}(t|t)$ and $S(T)$ are obtained as follows:

$$\frac{dS(\tau)}{d\tau} = -S(\tau)A - A^T S(\tau) - S(\tau)GG^T S(\tau) + C^T C$$
$$\frac{\partial\hat{\eta}(\tau+t-T|t)}{\partial\tau} = -[A^T + S(\tau)GG^T]\hat{\eta}(\tau+t-T|t) + C^T y(\tau+t-T) + S(\tau)Bu(\tau+t-T)$$

where $0 \leq \tau \leq T$, $S(0) = 0$, and $\hat{\eta}(t-T|t) = 0$. ∎

It is surprising to observe that the QMMF with an iterative form is the same as the RHUFF with unknow horizon initial state in (Han *et al.*, 2001*b*), where the covariances of the system noise and the measurement noise are taken as unit matrices.

Remark 4. The iterative form of IIR filters can be represented as $\hat{x}(t) = S^{-1}(t)\hat{\eta}(t)$ for $t > t_0$ where $\dot{S}(t) = -S(t)GG^T S(t) - A^T S(t) + C^T C - S(t)A$ and $\dot{\eta}(t) = -[A^T + S(t)GG^T]\hat{\eta}(t) + C^T y(t) + S(t)Bu(t)$ with $S(t_0) = 0$ and $\hat{\eta}(t_0) = 0$. This form is different from the standard Kalman filter which is also an IIR filter. It can be called a Kalman filter requiring no initial information. In these cases, the batch form needs more computation than the iterative form.

4. CONCLUSION

In this paper, a quasi-deadbeat property is introduced and a quasi-deadbeat minimax filter(QMMF) with FIR structure is proposed for deterministic continuous-time state space signal models, which is also extended to IIR structure. The proposed QMMF is linear with the most recent finite measurements and inputs, does not require *a priori* information about the horizon initial state, and has the quasi-deadbeat property. Even though deadbeat property of the QMMF is considered only for zero disturbance, the QMMF can be shown to be deadbeat for a certain class of nonzero disturbances. It is surprising in that a closed form solution exists even with the quasi-deadbeat condition. The proposed QMMF is first represented in a standard batch form and then an iterative form that has computational advantages. It is shown that the QMMF for deterministic systems is similar in form to the existing RHUFF for stochastic systems with unit covariance matrices of both the system noise and the measurement noise. Furthermore, due to the FIR structure, the QMMF is believed to be robust against temporary modeling uncertainties or numerical errors, while other minimax filters and H_∞ filters with an IIR structure may show poor robustness in these cases. Extension to the IIR type is also suggested and is summarized at the end of each section. It is shown that the IIR filter derived from QMMF for deterministic systems is similar in form to the RHUFF (Han *et al.*, 2001*b*) for stochastic systems. Although the cost function is somewhat similar to the H_∞ problem, the solution is quite different from that of the H_∞ problem. The proposed QMMF will be very useful for real plants that are usually modeled in continuous-time state space. In addition, the proposed QMMF with FIR structure can substitute the commonly used H_∞ and deadbeat filters.

5. REFERENCES

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