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ROBUST NONLINEAR CONTROL DESIGN OF DISTRIBUTED PROCESS SYSTEMS WITH INPUT CONSTRAINTS

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Abstract:
This paper presents a control design approach for nonlinear distributed process systems. The approach is developed on the framework of a thermodynamic formalism that exploits convexity of exergy-like functions and dissipation to derive passivity conditions for process systems. In our case, however, the convex function candidate is part of the controller design problem and will be selected as that which reduces/minimizes non-dissipative effects. On this framework, control implementation issues such as finite number of inputs, outputs and input saturation, will be discussed. In this regard, criteria for appropriate sensor/actuator placement and stability preservation under constrained inputs are given.

Keywords:
Distributed process systems, convex extensions, nonlinear control design, sensor/actuator placement, input constraints

1. INTRODUCTION

In this paper we consider the problem of stabilization of nonlinear distributed process systems through finite control actions subject to saturation. These systems play a central role in chemical and material processing industries as many of their operations involve convection diffusion and reaction phenomena. Interesting examples include, to name a few, catalytic reactors, chemical vapor deposition units, crystallization or thermal processing. The control of distributed process systems has received considerable attention from the control community over the last years. The recent review by (Christofides, 2001) witnesses these efforts and main developments.

A widely accepted approach to controlling distributed process systems relies on a state-space-like representation of the original infinite dimensional system through projection of the partial differential equations on appropriate basis function sets. Finite differences, finite elements and spectral decomposition schemes are the most common examples. This structure is then employed to address different control related aspects such as observer and controller design or actuators/sensors placement -see for instance (Christofides and Daoutidis, 1997) and (Antoniades and Chrsitofides, 2000).

A different, although complementary, approach is the one recently proposed by (Alonso and Ydstie, 2001) to develop passive stabilizing controls for distributed process systems. This approach sets its roots on the second law of thermodynamics and passivity, as it is understood in systems theory (Sepulchre *et al.*, 1997). The second law in the exergy form gives convexity which in turns provides a general answer to the question of finding Lyapunov function candidates to assess system's evolution. Passivity concepts link inputs to outputs while preserving the infinite dimensional structure of the system. These two concepts served as the basis to state general guidelines to design stabilizing high gain decentralized controllers (Alonso *et al.*, 2000).

This thermodynamic formalism will be the underlying framework on which we will address the design of nonlinear stabilizing controllers. Issues related with control implementation will also be considered. In particular, attention will be paid to the selection of appropriate sensor/actuator placements and stability preservation under input constraints (Lin and Sontag, 1991). There is however a main added ingredient: in the approach we present, the convex function candidate is not *a priori* selected but constructed as part of the control design problem. In fact, this function will be chosen as that which *reduces/minimizes* non-dissipative effects. It will also impose the class of nonlinear dependence between the field and the actuators and consequently will determine the structure of the control law.

The paper is organized as follows: In Section 2 we provide a brief description of the class of systems under consideration. The main results on stabilization of nonlinear distributed process systems in the presence of input constraints are contained in Section 3. Finally, in Section 4, the different control synthesis aspects will be illustrated on a case study that involves diffusion and reaction.

2. DISTRIBUTED PROCESS SYSTEMS: DESCRIPTION AND PROPERTIES

The class of systems we are considering in this paper is described by sets of partial differential equations (PDEs) of the form:

$$u_t = -\mathbf{v}\nabla u + k\Delta u + f(u) + p \qquad (1)$$

where $u(\mathbf{x}, t)$ and $p(\mathbf{x}, t)$ are vector functions with elements defined in time and space on a compact domain $\mathcal{V}$ with smooth boundary $\mathcal{B}$. u is known as the *field* and relates to densities of component mole numbers and energy (Alonso *et al.*, 2000). p includes as elements functions that can be manipulated through feedback control. $\mathbf{v}$ is a stationary velocity field that directs convective transfer and k a diffusion coefficient. The effect of chemical reactions on the conservation laws for mole numbers and energy is included in the nonlinear term $f(u)$. Eqn (1) can be interpreted as an infinite dimensional system on a Hilbert space equipped with inner product and norm:

$$< g, h >_{\mathcal{V}} = \int_{\mathcal{V}} g h d\mathbf{x} \ , \ \|g\|_2^2 = < g, g >_{\mathcal{V}}$$

we complete system's description with boundary conditions of the form:

$$a_0 u + a_1 \frac{du}{d\mathbf{n}} = b_0 \qquad (2)$$

with $\mathbf{n}$ being a unit vector normal to the surface $\mathcal{B}$ and pointing outwards, while a_0, a_1 and b_0 are appropriate functions that account for first order, second order or mixed boundary conditions. The dissipative nature of systems of the form (1) allows the representation of the solution u in terms of an infinite series expansion (Alonso and Ydstie, 2001) of the form:

$$u = \sum_{j=1}^{\infty} c_j(t)\varphi_j(\mathbf{x}) \qquad (3)$$

where φ_j are orthonormal functions (eigenfunctions) satisfying the eigenvalue problem:

$$\Delta\varphi_j = -\lambda_j\varphi_j \qquad (4)$$

with boundary conditions (2). The eigenspectrum $\sigma(\Delta) = \{\lambda_j\}_{j=1}^{\infty}$ consists of an ordered set of positive numbers such that $\lambda_i < \lambda_j$ for $i < j$. This property allows the definition of a small positive parameter $\varepsilon < 1$ which partitions the dynamic evolution of the c coefficients in (3), into a slow and a fast time scales t and $\tau = t/\varepsilon$, respectively. The explicit form of the dynamics can be obtained by Galerkin projection -see for instance (Christofides and Daoutidis, 1997)- of the original equation (1) on the set of eigenfunctions $\{\varphi_j\}$ so that:

$$\dot{c}_s = A_s c_s + w_s(c_s, c_f) + \pi_s \qquad (5)$$

$$\frac{dc_f}{d\tau} = \varepsilon A_f c_f + \varepsilon(w_f(c_s, c_f) + \pi_f) \qquad (6)$$

where $c^T = (c_s^T, c_f^T)$ and the elements of the vector functions $w = (w_s^T, w_f^T)^T$ and $\pi = (\pi_s^T, \pi_f^T)^T$ are of the form:

$$\omega_j = < \varphi_j, f(u) >_{\mathcal{V}}$$
$$\pi_j = < \varphi_j, p >_{\mathcal{V}}$$

In the limit when $\varepsilon \to 0$, fast modes relax so that $\frac{dc_f}{d\tau} \to 0$. If in addition $c_f \to 0$, the solution $u(\mathbf{x}, t)$ can be approximated by a finite number of slow modes $c_s \in R^{n_s}$ in (5) so that $u \to u_s$ with $u_s = \sum_{j=1}^{n_s} c_j(t)\varphi_j(\mathbf{x})$. We will refer to u_s as the *slow solution*. This notion will be employed later on to design stabilizing finite dimensional controllers.

3. ROBUST STABILIZING CONTROL OF DISTRIBUTED PROCESS SYSTEMS

This section contains the main theoretical ingredients required to design robust stabilizing nonlinear controllers for convection-diffusion-reaction systems. First we address the stabilization problem in the more general -distributed- form and then discuss implementation aspects related with finite numbers of inputs, outputs and control saturations.

Lemma 1. *Let $b(u)$ be a convex function in u, $A = d_u b$ and $u, A \in H^{1,2}(V, R^n)$ (Alonso and Ydstie, 2001) so that:*

$$u = \sum_{j=1}^{\infty} c_j(t)\varphi_j(x)$$
$$A = \sum_{j=1}^{\infty} a_j(t)\varphi_j(x) \tag{7}$$

with φ_j satisfying (4). Then the following inequalities hold

1. $< A, A >_{\mathcal{V}} \geq \delta^2 < u, u >_{\mathcal{V}}$

2. $< A, \Delta u >_{\mathcal{V}} \leq -\delta\lambda_1 < u, u >_{\mathcal{V}}$

with δ being the smallest eigenvalue of the b-Hessian over all possible u and λ_1 the principal eigenvalue of the Laplacian operator Δ.

Proof:

The first part of the statement is a direct consequence of convexity of b. Since b is convex, there exists a one-to-one map between A and u such that $A = Q(u)u$ with $Q(u)$ being a positive definite matrix of the form $Q(u) = \int_0^1 H(\alpha u)d\alpha$, and H the Hessian of b. Then, for any arbitrary vector y we have that $y^T Q y \geq \delta y^T y > 0$ and therefore $< A, A >_{\mathcal{V}} \geq \delta^2 < u, u >_{\mathcal{V}}$. To prove statement 2 we note that:

$$< A, u >_{\mathcal{V}} \geq \delta < u, u >_{\mathcal{V}} \tag{8}$$

Using (7) and the fact that the eigenfunctions $\{\varphi_j\}_{j=1}^{\infty}$ are orthonormal and satisfy (4) we also have that:

$$< u, u >_{\mathcal{V}} = \sum_{j=1}^{\infty} c_j^2$$
$$< A, u >_{\mathcal{V}} = \sum_{j=1}^{\infty} a_j c_j$$

which combined with inequality (8) leads to $\sum_{j=1}^{\infty}(a_j c_j - \delta c_j^2) \geq 0$. In order for this inequality to hold for any $u \in H^{1,2}(V, R^n)$, $a_j c_j \geq \delta c_j^2$ for every j and the statement is verified through the following set of implications:

$$- < A, \Delta u >_{\mathcal{V}} = \sum_{j=1}^{\infty} \lambda_j a_j c_j \geq \delta \sum_{j=1}^{\infty} \lambda_j c_j^2$$
$$- < A, \Delta u >_{\mathcal{V}} \geq \delta\lambda_1 \sum_{j=1}^{\infty} c_j^2$$
$$< A, \Delta u >_{\mathcal{V}} \leq -\delta\lambda_1 < u, u >_{\mathcal{V}}$$

Proposition 1. *Consider the system (1), and a reference u^* with stationary boundary conditions (2) satisfying:*

$$-\mathbf{v}\nabla u^* + k\Delta u^* + f(u^*) + p^* = 0 \tag{9}$$

Let $\overline{u} = u - u^*$ and assume that there exists a convex positive definite function $b(\overline{u})$ such that $b(0) = 0$ and

$$\beta \geq \frac{\overline{A}^T\,[f(u) - f(u^*)]}{\overline{A}^T\overline{A}} > 0 \tag{10}$$

with $\overline{A} = d_{\overline{u}} b$ and β a positive constant. Then, under the control law $p - p^* = -\omega\overline{A}$ with $\omega \geq \beta$ the reference u^* will be globally asymptotically stable

Proof:

Combining (1) with (9) we obtain an equivalent system in deviation form:

$$\overline{u}_t = -\mathbf{v}\nabla\overline{u} + k\Delta\overline{u} + f(u) - f(u^*) + \overline{p} \tag{11}$$

Computing the time derivative of b along (11) and integrating the resulting expression over the domain $\mathcal{V}$ leads to:

$$B_t = < d_{\overline{u}} b, \overline{u}_t >_{\mathcal{V}} \leq k < \overline{A}, \Delta\overline{u} >_{\mathcal{V}}$$
$$+ < \overline{A}, [f(u) - f(u^*)] >_{\mathcal{V}} + < \overline{A}, \overline{p} >_{\mathcal{V}} \tag{12}$$

where the following relation for the convective contribution was employed (Alonso *et al.*, 2000):

$$< \mathbf{v}\overline{A}, \nabla\overline{u} >_{\mathcal{V}} = \int_{\mathcal{B}} b\mathbf{v}\mathbf{n}d\mathcal{B} \geq 0$$

Using Lemma 1 and (10), inequality (12) becomes:

$$B_t \leq -k\delta\lambda_1 < \overline{u}, \overline{u} >_{\mathcal{V}} + \beta < \overline{A}, \overline{A} >_{\mathcal{V}} + < \overline{A}, \overline{p} >_{\mathcal{V}} \tag{13}$$

Substituting the expression for the control law in (13) we obtain

$$B_t \leq -\mu < \overline{u}, \overline{u} >_{\mathcal{V}} \tag{14}$$

with $\mu = k\delta\lambda_1 + (\omega - \beta)\delta^2 > 0$. The result then follows through standard Lyapunov stability arguments (see for instance (Khalil, 1996)). $\square$

Note that asymptotic stabilization can be ensured locally with gains ω lower than β. This can be shown from convexity of b, by noting that for any solution bounded as $\|\overline{u}\|_2 \leq \gamma$, there exists a positive number such that $< \overline{A}, \overline{A} >_{\mathcal{V}} \leq \eta(\gamma) < \overline{u}, \overline{u} >_{\mathcal{V}}$. Thus

$$B_t \leq -(k\delta\lambda_1 + \omega\delta^2 - \beta\eta) < \overline{u}, \overline{u} >_{\mathcal{V}}$$

and uniform asymptotic stability can be concluded for $\omega > (\beta\eta^2 - k\delta\lambda_1)/\delta^2$.

3.1 *Implementation Aspects: Inputs, Outputs and Constraints*

The stabilizing control law presented in Proposition 1 requires complete measurement of the field or at least its efficient reconstruction from observation schemes. In addition, actions must be distributed along the domain. In this section, we relax these conditions in order to comply with practical implementation restrictions, namely finite number of sensors and actuators as well as input constraints. In what follows, we will assume that measurements and actuators are only available at a finite, and usually reduced, number of locations. The effect of control saturations on the stability of the closed loop system will be considered as well.

Let us consider system (1) with appropriate boundary conditions and a set of eigenfunctions $\{\phi_j\}_{j=1}^{\infty}$. This set will be interpreted as the n-discrete version of the set $\{\varphi_j\}_{j=1}^{\infty}$ along the spatial coordinates $\mathbf{x}$ in (7), so that $\phi_j \in R^n$ and $\phi_j^T \phi_k = \delta_{jk}$ for every j and k, with δ_{jk} being the Kronecker delta. Lets also define the operator $P_m \in R^{m \times n}$ as that which projects any ϕ_j on m (locations) of the n discrete coordinates, so that if $\Phi_s = \{\phi_j\}_{j=1}^{n_s}$ is the set of eigenfunctions associated with the n_s slow modes and $u_s = \sum_{j=1}^{n_s} \phi_j c_{sj}$ we have:

$$P_m u_s = B_s^T c_s \;\; with \;\; B_s^T = P_m \Phi_s$$

$$< u_s, u_s >_{\mathcal{V}} = c_s^T B_s B_s^T c_s$$

The following result states conditions under which stabilization is enforced through finite control subject to saturation.

Proposition 2. *Consider system (1) and the reference u^* as in Proposition 1. Let m be a number of sensors and actuators placed on the domain $\mathcal{V}$ at locations such that:*

$$y^T B_s B_s^T y \geq \underline{\alpha} y^T y > 0 \tag{15}$$

for arbitrary n_s-dimensional y vectors. Also, let $\overline{A}_m(\overline{u}_m) \in R^m$ be a vector of measurements obtained at the m-locations. Then for every $\left\|\overline{A}\right\|_2^2 \leq \gamma^2$, there exists a control law $\overline{p}_m = -\omega_m \overline{A}_m$ with $-\psi \leq (\overline{p}_m)_j \leq \psi$ for every j actuator location with

$$\omega_m > \frac{\beta}{\underline{\alpha}} \tag{16}$$

$$\psi \geq \gamma\sqrt{\beta\omega_m} \tag{17}$$

such that the reference u^* is asymptotically stable.

Proof:

From convexity of b, we have that $< \overline{u}, \overline{u} >_{\mathcal{V}} \geq \eta^{-1}(\gamma) < \overline{A}, \overline{A} >_{\mathcal{V}}$ for some positive number η dependent of $\overline{u}$ so that (13) becomes:

$$\begin{array}{l} B_t \leq -k\delta\lambda_1\eta^{-1} < \overline{A}, \overline{A} >_{\mathcal{V}} \\ +\beta < \overline{A}, \overline{A} >_{\mathcal{V}} + < \overline{A}, \overline{p}_m >_{\mathcal{V}} \end{array} \tag{18}$$

On the other hand, the time scale separation property discussed in Section 2 ensures the existence of a small positive ε such that $u \rightarrow u_s$ for large τ. Therefore, in the t-time scale, the following holds:

$$\begin{array}{r} - < \overline{A}, \overline{p}_m >_{\mathcal{V}} \rightarrow \omega_m a_s^T B_s B_s^T a_s \\ \left\|\overline{A}\right\|_2^2 = < \overline{A}, \overline{A} >_{\mathcal{V}} \rightarrow a_s^T a_s \end{array}$$

Let us re-write the control law in the t-scale (Section 2) as $\overline{p}_j \rightarrow -\omega_m b_j^T a_s$, where b_j^T represents the j-row of B_s^T, and construct a saturation function $v = (\, v_1 \; \cdots \; v_m \,)^T$ with elements:

$$v_j = \left\{ \begin{array}{ccc} -\omega_m b_j^T a_s & if & -\chi < b_j^T a_s < \chi \\ \psi & if & b_j^T a_s \leq -\chi \\ -\psi & if & b_j^T a_s \geq \chi \end{array} \right| \tag{19}$$

and $\chi = \psi/\omega_m$. Two cases will be considered:

- $-\chi < b_j^T a_s < \chi$ for every $j = 1, ..., m$.

In this case inequality (18) becomes:

$$B_t \leq -k\delta\lambda_1\eta^{-1} a_s^T a_s - (\omega_m\underline{\alpha} - \beta) a_s^T a_s$$

where use of lower bound (15) has been made. Consequently, for $\omega_m > \beta/\underline{\alpha}$, we have that $B_t \leq -\mu \left\|\overline{A}\right\|_2^2$ with $\mu = k\delta\lambda_1\eta^{-1} + (\omega_m\underline{\alpha} - \beta) > 0$ and asymptotic stability follows as in Proposition 1.

- Either $b_k^T a_s \leq -\chi$ or $b_k^T a_s \geq \chi$ for k actuators with $1 \leq k \leq m$.

From the definition (19), $(b_j^T a_s)v_j \le 0$ for $j = 1, ...m$. In particular, each of the k actuators under saturation satisfies that $(b_k^T a_s)v_k \le -\psi\chi$. Thus:

$$\sum_{j=1}^{m}(b_j^T a_s)v_j \le \sum_{j=k}(b_j^T a_s)v_j \le -\frac{\psi^2}{\omega_m}$$

Since $< \overline{A}, \overline{p}_m >_{\mathcal{V}} \to \sum_{j=1}^{m}(b_j^T a_s)v_j$, the control term in (18) is bounded as $< \overline{A}, \overline{p}_m >_{\mathcal{V}} \le -\psi^2/\omega_m$. Combining this bound with (17) and (18) we obtain:

$$< \overline{A}, \overline{p}_m >_{\mathcal{V}} \le -\frac{\psi^2}{\omega_m} \le -\gamma^2\beta$$

$$< \overline{A}, \overline{p}_m >_{\mathcal{V}} \le -\beta \left\|\overline{A}\right\|_2^2$$

$$B_t \le -k\delta\lambda_1\eta^{-1} \left\|\overline{A}\right\|_2^2$$

and the result follows.

Note that condition (15) implicitly restricts the number of possible locations for sensors and actuators to those that satisfy that inequality. In fact, this inequality could be used as a criterion to select placements so to maximize the minimum eigenvalue $\underline{\alpha}$. It must be also pointed out that the application of conditions (16) and (17) for control design will depend on the pre-defined control objectives. Among others, two relevant scenarios can be foreseen:

- Given a set of admissible perturbations (bounded by γ) and allowed actions (ψ) find the minimum number of sensor/actuators and their location so to ensure stability.
- Given a number of sensors/actuators and allowed actions find the set of perturbations for which stability can be preserved.

4. EXAMPLE: A NONLINEAR DIFFUSION-REACTION PROCESS

The results presented so far are employed in this section to set up a stabilizing control synthesis scheme for nonlinear distributed process systems. The example we select involves a one dimensional diffusion process where a zero order, exothermic reaction $A \to B$ is taking place (Antoniades and Chrsitofides, 2000). Reactant A is assumed to be available, for all times, everywhere on the domain. The heat produced by reaction is released through a cooling medium (at temperature T_c) in contact with the domain along its length ($L = 1$). Such a system, although structurally simple, can display instability phenomena as a consequence of the trade-off between heat dissipation, due to diffusion and transfer to the cooling medium, and heat

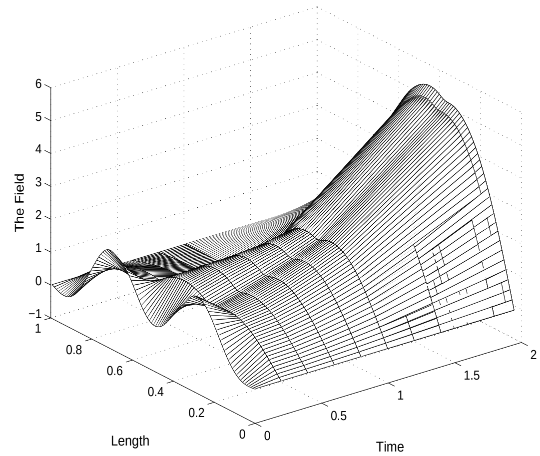

Fig. 1. Runaway behaviour under constrained control for a perturbation $\gamma = 1.2$, $\omega = 25$ and actuator bound $\psi = 12$

produced by reaction. In this way, the objective is to stabilize the temperature distribution T at a certain stationary reference (T^*, T_c^*) by measuring and acting at a small number of locations on the domain, with actuators subject to saturation. For convenience, we define the following dimensionless temperature u for the field:

$$u = \frac{T - T^*}{T^*}$$

with reference $u^* = 0$. In these variable, the energy balance becomes:

$$u_t = \varepsilon\frac{\partial^2 u}{\partial x^2} - \beta_U(u - p) + \theta\left[exp\left(\frac{\nu u}{1+u}\right) - 1\right]$$

with initial and boundary conditions $u(0, x) = u_0$, $u(t, 0) = u(t, 1) = 0$. $p(T_c; T_c^*)$ is the control function related to the temperature of the cooling medium. The following parameter values will be employed in the simulation experiments: $\varepsilon = \pi^{-2}$, $\beta_T = 50$, $\beta_U = 2$, $\nu = 4$ and $\theta = \beta_T e^{-\nu}$. Note that the field u is defined over the interval $]-1, \infty)$ since T in the exponential (Arrhenius type) term must be absolute temperature.

The first step for the control synthesis involves the construction of a convex function satisfying conditions of Proposition 1. In particular, b must be such that (10) holds for every u. This inequality suggests the construction of b candidates with nonlinear terms matching -through A- the nonlinearity of the process so to minimize the upper bound β. In our case, we choose b to be of the form:

$$b(u) = \frac{1}{2}u^2 - 1 - \sigma u + exp(\sigma u) \qquad (20)$$

where σ is a positive design parameter. This function satisfies Lemma 1 with:

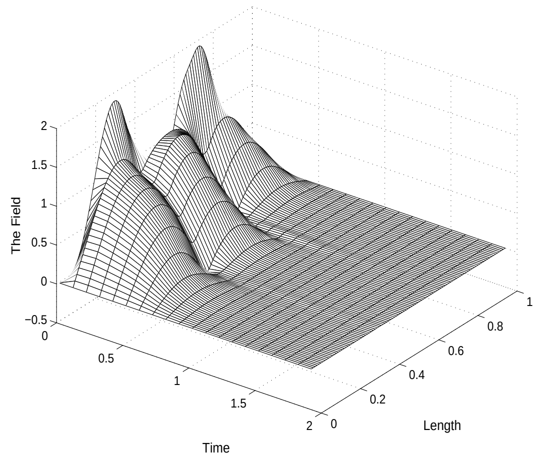

Fig. 2. Evolution of the field under stabilizing constrained control for $\gamma = 1.2$, $\omega = 25$ and actuator bound $\psi = 15$

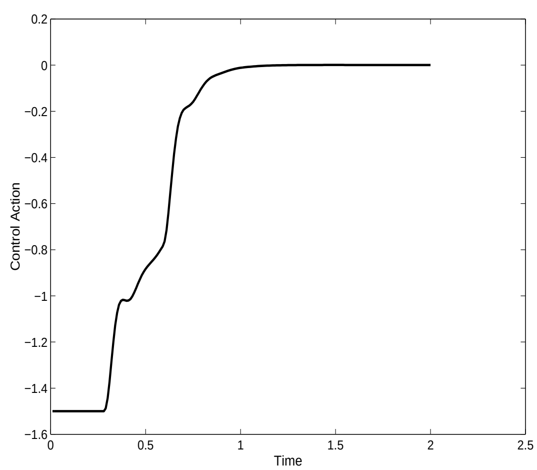

Fig. 3. Stabilizing control action defined as $\int_0^1 p_m(x,t)dx$. $\gamma = 1.2$, $\omega = 25$ and actuator bound $\psi = 15$

$$\delta = \min_{u \in]-1,\infty)} \left[1 + \sigma \left(\frac{-1 + exp(\sigma u)}{u} \right) \right]$$

The control law then becomes $p = -\omega A$, with:

$$A = \frac{db}{du} = u + \sigma \left[-1 + exp(\sigma u) \right]$$

It is pointed out that this approach can be easily connected with the thermodynamic theory developed by (Alonso and Ydstie, 2001). In that formalism b would correspond to a convex extension and A to the space dual to the field u. In this way, control design can be interpreted as nonlinearly transforming the field so to reduce (minimize) non-dissipative effects.

In a second step, we make use of Proposition 2 to comply with implementation aspects. As an example, we consider the stabilization problem under a given set of bounded perturbations with a given input-output arrangement. Let $\gamma = 1.2$ be the upper bound for all admissible perturbations $\|\overline{A}\|_2^2 \leq \gamma^2$ with $\sigma = 0.25$ in (20) and $\beta = 6$ (10). The input/output set up consists of 10 equally spaced sensors and actuators placed at lengths $(0.34 - 0.37)$ and $(0.66 - 0.71)$. For this arrangement the lower bound in (15) is $\underline{\alpha} = 0.1172$. With this information, we select a gain ($\omega_m = 25$) which according to Proposition 1 should be enough to ensure stabilization in the absence of saturation effects. However, bounded control actions will induce instability as it can be seen from Figure 1. Nevertheless, stability can be recovered by extending the region of available actions to limits satisfying (17). In our case stability is ensured for bounds $\psi \geq 15$. This situation is illustrated in Figure 2 (evolution of the field). Figure 3 represents the evolution of the control action described as $\int_0^1 p_m(x,t)dx$.

5. CONCLUSIONS

A stabilizing controller design approach for distributed process systems has been developed by exploiting and extending concepts from convex forms and dissipation. The resulting nonlinear control schemes are derived so to reduce/minimize non-dissipative effects and can be implemented through finite (and usually small) number of sensors and actuators subject to input constraints.

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