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OPTIMAL ESTIMATION AND CONTROL OF DYNAMIC SYSTEMS WITH TIME-VARYING ACTUATION DELAYS

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Abstract: This paper provides the solution of optimal filtering problems for the broad class of continuous linear systems with discrete and continuous measurements with time-varying sampling intervals and time-varying measurement delays caused by communication network, human in the loop and a priori unknown triggering of data acquisition. Using the duality principle, the dual control problem with delays in actuation is then solved. The solution of filtering and control problems is obtained using the integral model of measurements in the form of Ito-Volterra integrals with discontinuous measures.

Keywords: network systems, actuation delays, controllers, optimal control, optimal filtering, time delay systems

1. INTRODUCTION AND MOTIVATION

The importance of the posed problem is most easily demonstrated by the example of the networked control systems. The decentralized and distributed control networks are becoming the reality in a variety of application areas. The practical interest in systems with communication network as part of the information path for data acquisition and control is driven by the need for integration and coordination of multiple spatially remote processes. It has been recognized for a long time that network in the loop presents some unusual challenges. For example, the nondeterministic nature of network traffic leads to time-varying delays in delivery of streaming measurements (treated in this paper as continuous [1]). Discrete (or packet) measurements are also delayed by a priori unknown and varying time. Furthermore, the loss of the data during network delivery will effectively result in the time-varying sampling rate even if the sampling at the remote site is uniform and deterministic. From the control perspective, based on delayed information, the remote controller calculates commands (setpoints), which, because of the network properties, will be delayed in their arrival to the local controllers. Therefore, from the remote controller "perspective," this looks as delayed actuation of generated commands, and these delays are random and a priori unknown. Further examples of systems with time-varying and a priori unknown delays in discrete and continuous measurements and controls include systems with human in the loop making sampling and control decisions, and systems in which measurement and control are triggered by exogenous events.

Despite an extensive research effort, the optimal filtering and control problem for continuous linear systems with discrete and continuous delayed measurements and its dual optimal control problem remain unsolved in its most general formulation. The existing methods of controlling and estimating continuous systems based on delayed discrete and continuous measurements, though often ingenious, are heuristic in nature, and do not provide a uniform theoretical framework for the general problem

[1] In reality, streaming measurements may represent discrete measurements sampled at a relatively high rate.

with an arbitrary number of time-varying sampling rates and time-varying delays in both discrete and continuous measurements. The fundamental reason for theoretical difficulties lies in the fact that discrete measurements in continuous systems imply that the optimal filter is driven by discontinuous inputs, which requires that basic questions of what is understood as a solution and how to find it must be addressed. It is not surprising that the state estimates with discontinuous measurements are themselves discontinuous. In fact, a discontinuous "update" of the state estimation is a routine practice to handle discrete and infrequent measurements based on heuristic algorithms. The commonly utilized strategy, used without rigorous justification or proof of optimality, is to discretize a continuous system in some convenient way so that the discrete Kalman filter is directly applicable. Such an approach fails in the case of time-varying and unknown sampling rates and delays. Similar observations are true for the control problem with continuous and impulsive (discrete) controls with and without delayed actuation.

Our approach to the solution of the filtering and control problems with delayed discrete and continuous measurements and time-varying sampling rates is based on the integral Volterra description of deterministic linear systems and the integral Ito-Volterra description of stochastic linear systems, and the mathematical theory of optimal control of systems with discontinuities. This approach is referred to as an *integral* approach, which is applicable to systems with discontinuities in measurements, controls and states, – increasingly important theoretical and practical problems – and systems with time-varying and a priori unknown delays.

The paper has the following organization. The Ito-Volterra description is introduced, first, for continuous systems, followed by the systems with discontinuities. The optimal filtering results are then presented for these systems and their generalization on the case of systems with time-varying and a priori unknown time delays. After introducing the duality principle, the solution of the dual control problem with delays in actuation is given.

2. ITO-VOLTERRA DESCRIPTION OF OBSERVATION PROCESS

Let (Ω, F, P) be a complete probability space with an increasing right-continuous family of σ-algebras $F_t, t \geq 0$, and let $(W_1(t), F_t, t \geq 0)$ and $(W_2(t), F_t, t \geq 0)$ be independent Wiener processes. The partially observed F_t-measurable random process $(x(t), y(t))$ is described using a differential equation for the dynamic system state and an Ito-Volterra equation for the observation process:

$$dx(t) = (a_0(t) + a(t)x(t))dt + b(t)dW_1(t) \qquad (1)$$

$$y(t) = \int_0^t A_0(t,s) + A(t,s)x(s)ds + \int_0^t B(t,s)dW_2(s) \quad (2)$$

where $x(t) \in R^n$ is the state vector, and $y(t) \in R^m$ is a vector of measurements[2] *integrated* over the time interval $[0,t]$. The vector-valued function $a_0(s)$ describes the effect of system inputs (controls and disturbances). Matrix functions $a(s)$ and $b(s)$ and vector-function $a_0(s)$ are smooth functions of s. Functions $A_0(t,s)$, $A(t,s)$, and $B(t,s)$ are continuous in t and s. Both t and s are independent (time) variables, and can be used, among other things, to assign a variable number of time-varying delays in both states and measurements to adequately describe the particular application at hand. It is also assumed that $A(t,s)$ is a nonzero matrix and $B(t,s)B^T(t,s)$ is a positive definite matrix. All coefficients in (1)–(2) are deterministic functions of appropriate dimensions.

The estimation problem is to find the estimate of the system state $x(t)$ based on the observation process $Y(t) = \{y(s), 0 \leq s \leq t\}$, which minimizes the Euclidean 2-norm

$$J = E[(x(t) - \hat{x}(t))^T (x(t) - \hat{x}(t))]$$

at each time moment t. In other words, our objective is to find the conditional expectation

$$m(t) = \hat{x}(t) = E(x(t) \mid F_t^Y).$$

As usual, the matrix function

$$P(t) = E[(x(t) - m(t))(x(t) - m(t))^T \mid F_t^Y]$$

is the estimate variance.

Our formulation is, in fact, the Kalman filtering problem for the integral Ito-Volterra observation process. The standard state space formulation is recovered by making all functional parameters in (2) dependent on s only.

Next, the integral model is introduced for systems with discontinuities in measurements.

3. OPTIMAL FILTERING FOR SYSTEMS WITH BOUNDED DISCONTINUITIES

Consider a nondecreasing vector-valued function of bounded variation: $g(t) = (g_1(t), \ldots, g_m(t)) \in R^m$. In essence, $g(t)$ is an arbitrary function, and it is only required that it remains bounded on each finite subinterval of its definition, and $g(t_1) \leq g(t_2)$ if $t_1 \leq t_2$. Continuity of $g(t)$ is not required. In fact, any bounded variation function (including vector-valued case) can be written as

$$g(t) = \{g_k^c(t) + \sum_{i=1}^{N} \Delta g_{ki} \chi(t - t_{ki}),\ k = 1 \ldots m\} \quad (3)$$

[2] In integral formulation, $y(t)$ is a vector of integrated measurements. The vector of the differential measurements, $dy(t)$, is the vector of the actual physical measurements (see (Åström, 1970) for discussion, in particular pp. 82–85). Note, however, that this interpretation may change during, as is the case with equation (8), where $y(t)$ is treated as the vector of actual measurements.

where $g_k^c(t)$ is a continuous nondecreasing function and the second term describes bounded jumps in k-th components of $g(t)$ at times t_{ki}, and where χ is the Heaviside unit step function and Δg_{ki} is the size of the jump.

The discontinuous measure g can be used to describe discontinuities in states and measurements. The Ito-Volterra model with discontinuous measure in the observation equation generated by a bounded variation function has the following form:

$$y_k(t) = \int_0^t (A_{0k}(t,s) + (A_k(t,s)x(s))dg_k(s)$$

$$\int_0^t B_k(t,s)dW_{2k}\left(g_k(s)\right), \quad k = 1 \ldots m \qquad (4)$$

where the notation is analogous to the one used in (2), k identifies k-th component of the measurement vector, and g_k is the k-th component of g, equation (3).

The measurement model given by equation (4) is in the Stieltjes integral form. Obviously, if $g(t) = t$, the description of the system is reduced to the initial model (1)–(2). If $g^c \equiv 0$, the observation model given by equation (4) describes the case of a continuous system with discrete measurements. The general case of equation (3) describes the dynamic system with an arbitrary combination of discrete and continuous measurements.

The general result for the optimal filtering of integral systems with measurement discontinuities is given by the following theorem.

Theorem 1. The optimal in Kalman sense estimate $m(t)$ of the states of system (1) based on discontinuous integral measurements (4) satisfies the filter equation

$$m(t) = \int_0^t (a_0(s) + a(s)m(s))ds + \int_0^t P(s-)$$

$$\times[I + A^T(t,s)\left(B(t,s)B^T(t,s)\right)^{-1}A(t,s)P(s-)\Delta g(s)]^{-1}$$

$$\times A^T(t,s)\left(B(t,s)B^T(t,s)\right)^{-1}$$

$$\times[dy(s) - (A_0(t,s) + A(t,s)m(s-))]dg(s), \qquad (5)$$

where $\Delta g(s)$ is a jump of $g(s)$ at s, and the standard notation for the value of the function at discontinuity is used, and the variance $P(t)$ satisfies the integral Riccati-like equation

$$P(t) = \int_0^t [a(s)P(s) + P(s)a^T(s) + b(s)b^T(s)]ds \qquad (6)$$

$$-\int_0^t P(s-)[I + A^T(t,s)\left(B(t,s)B^T(t,s)\right)^{-1}A(t,s)P(s-)$$

$$\times\Delta g(s)]^{-1}A^T(t,s)\left(B(t,s)B^T(t,s)\right)^{-1}A(t,s)P(s-)dg(s)$$

where multiplication by m-dimensional measure $dg(s)$ is understood in a componentwise sense.

If the bounded variation function $g(s)$ has continuous and discontinuous components, then the corresponding observation process $y(t) = \{y_k(t)\}$ also has continuous and discontinuous components, and can be written as

$$y_k(t) = y_k^c(t) + y_k^d(t).$$

Physically, the continuous component of y corresponds to the integral of continuous measurements with discontinuous (discrete) measurements superimposed over them. The discontinuity in y leads to discontinuity in estimate $m(t)$ and the variance function $P(t)$. At the point of discontinuity t_{ki} when a new discrete measurement becomes available in k-th measurement channel, the optimal value of $m(t)$ and $P(t)$ can be directly calculated from (5)–(6), where, with reference to equation (3), $\Delta g(t_i) = \{\Delta g_{ki}\}$, and typically $\Delta g_{ki} = 1$.

If only discrete measurements are present, then between the time of $t = t_i$ of the last available measurement in any of the measurement channels and the next measurement, the estimate of the state is given by the following integral equation:

$$\hat{x}(t) = m(t) = m(t_i+) + \int_{t_i+}^t (a_0(s) + a(s)m(s))ds,$$

and the variance of the estimation process is found from

$$P(t) = P(t_i+) + \int_{t_i+}^t [a(s)P(s) + P(s)a^T(s) + b(s)b^T(s)]ds.$$

The state estimate and variance at the time of arrival of the new discrete measurement are equal

$$m(t_i+) = m(t_i-) + \Delta m(t_i),$$

$$P(t_i+) = P(t_i-) + \Delta P(t_i),$$

where the jumps are *explicitly* given as

$$\Delta m(t_i) = K(t_i)\left\{dy(t_i) - [A_0(t_i) + A(t_i)m(t_i-)]\right\},$$

$$\Delta P(t_i) = -K(t_i)A(t_i)P(t_i-),$$

where the shorthand notation $A(t_i) = A(t_i, t_i)$, $A_0(t_i) = A_0(t_i, t_i)$ and $B(t_i) = B(t_i, t_i)$ was used, and

$$K(t_i) = P(t_i-)(I + A^T(t_i)\left[B(t_i)B^T(t_i)\right]^{-1}$$

$$\times A(t_i)P(t_i-))^{-1}A^T(t_i)\left[B(t_i)B^T(t_i)\right]^{-1}.$$

The second limiting case (only continuous measurements are present) is obtained by setting $g(t) = t$. The resulting filter for the system in the differential form are obtained from equations (5)–(6), and is equivalent to the traditional Kalman filter. However, if the observation process is described in the integral form (2), the optimal filter for continuous systems with continuous measurements is not reducible to the Kalman filter, and is given by Theorem 1.

In summary, the general case of the Ito-Volterra system differential measure $dg(s)$ allows us to formally describe a dynamic system with any combination of continuous and discrete measurements with time-varying sampling intervals. The application of Theorem 1 then provides the solution of the optimal filtering problem for this broadest class of continuous systems with continuous and discrete measurements.

4. FILTERING FOR SYSTEMS WITH TIME DELAYS

Let us consider how time delays in discrete and continuous measurements can be handled within the framework of the integral approach, which appears to be substantively different from infinite-dimensional (Diekmann *et al.*, 1995; Kolmanovskii and Shaikhet, 1996), algebraic (Marshall *et al.*, 1992) and functional differential systems approach (Kolmanovskii and Myshkis, 1992; Hale and Lunel, 1991; Gorecki *et al.*, 1989) to systems with time delay. In addition to giving an alternative approach, which is important in itself, the advantages of the integral formulation is in its consistent theoretical foundation, and its ability to address systems with discontinuities of different origin, as discussed earlier.

An immediate observation is that when both continuous and discrete time-delayed measurements are present, one has a problem with discontinuous observations considered above without time delays. Now define a bounded variation function

$$g(t,s) = \sum_{i=1}^{N} \Delta g_i \chi(s - [t - d_i(t)]), \qquad (7)$$

where s is an independent time variable, and t is treated as a parameter; $d_i(t)$ is the i-th time delay at time t. After substitution of (7) into the integral measurement model with discontinuous measure, the equation (4) takes the form

$$y(t) = \sum_i A_0(t, t - d_i(t)) + \sum_i A(t, t - d_i(t))x(t - d_i(t)) + \sum_i B(t, t - d_i(t))dW_2(t), \qquad (8)$$

where y is interpreted as a vector of differential continuous measurement, taking into account that for the white Gaussian noise $dW_2(t)$, $dW_2(t - d_i(t)) = dW_2(t)$, and assuming that $\Delta g_i = 1$. Equation (8) is the model of the measurement system, describing continuous measurements $y(t)$ as a linear function of states delayed by different and time-varying delays $d_i(t)$. It is in the form that allows the direct application of Theorem 1, giving the expression for the optimal filter with an arbitrary combination of delayed continuous measurements.

The model (8) can also be used to describe the discrete measurement system with time-varying delays. If $t = t_j$ is set, where t_j's are discrete time instants when discrete measurements become available (*a priori* knowledge of the arrival time and the corresponding time delay of a discrete measurement is not required) and vector y is treated as an integral of discrete measurements, then (8) describes the case of a continuous system with delayed discrete measurements. Delays and sampling rates in different channels can be different and time varying. Theorem 1 is again directly applicable, yielding an optimal filter for the continuous system with delayed discrete measurements.

The case of an arbitrary combination of delayed continuous and discrete measurements is obtained when different components of vector y describe continuous differential measurements, and discrete integral measurements. Theorem 1 is still applicable giving an optimal filter for this most general case.

5. DUAL CONTROL PROBLEM

Using the duality principle for integral systems (Basin and Valadez Guzman, 2000), the optimal state estimation problem for system with discrete and continuous measurements, equations (1) and (4), is dual to the optimal control problem of finding control u, which minimize the quadratic cost function

$$J = \frac{1}{2}[x(T)]^T \Psi^{-1} [x(T)] + \frac{1}{2}\int_{t_0}^{T} u^T(t,s)B^T(s)B(s)u(t,s)dg(s) + \frac{1}{2}\int_{t_0}^{T} x^T(s)b^T(s)b(s)x(s)ds],$$

subject to the following Ito-Volterra system with states z and disturbances a_0^T:

$$z(t) = \int_{t_0}^{t} (a_0^T(s) - a^T(s)z(s))ds + \int_{t_0}^{t} A^T(t,s)u(t,s)dg(s),$$

and where $P(t_0) = \Psi > 0$. Following from the solution of the dual filtering problem, the optimal control is given by

$$u^*(t,s) = (B^T(t,s)B(t,s))^{-1}A^T(t,s)P(s)z(s), \qquad (9)$$

with $P(s)$ satisfies the integral Riccati equation

$$P(t) = P(t_0) - \int_{t_0}^{T} [a(s)P(s) + P(s)a^T(s) + b^T(s)b(s)]ds + \int_{t_0}^{T} [P(s)A^T(t,s)(B^T(s)B(s))^{-1}A(t,s)P(s)]dg(s), \qquad (10)$$

with the terminal condition $P(T) = \Psi^{-1}$. The jumps in z and P at the points of discontinuity of $g(t)$ have the following explicit form:

$$\Delta z(t) = A^T(t)(B^T B)^{-1}A(t)P(t-)\Delta g(t), \qquad (11)$$

$$\Delta P(t) = -P(t-)[I + A^T(t)(B^T(t)B(t))^{-1}$$

$$\times A(t)P(t-)\Delta g(t)]^{-1}A^T(t)(B^T(t)B(t))^{-1}$$
$$A(t)P(t-)\Delta g(t). \qquad (12)$$

The assumptions that should be made about the particular form of the discontinuous function g correspond to different practical cases. If both continuous and discontinuous control commands are present, then point of discontinuity of g will specify the instances when discontinuous controls are applied. The assumption of the known g would then correspond to the case when the time of application of discontinuous controls is externally specified. If g is not known but can be manipulated, then the selection of points of discontinuity can be considered as part of the controller design, i.e. $\min_{u,g} J$, and it appears that $\min_{u,g} J = \min_g J(u^*(g))$, where u^* is determined in this section. Finally, if points of discontinuity of g are a priori unknown and determined by exogenous events, then some estimates of the instances of discontinuity are required to apply the developed results.

6. DELAYED ACTUATION

Introduction of time-varying delays in actuation can be accomplished similarly to the way time-varying delays have been introduced into the measurements. The developed theory is directly applicable if actuation is delayed in an arbitrary and time-varying fashion, as long as the time delays are actually known. If delays are not known, as in the case of the networked control system, then measurement delays – a priori unknown, but available once the time-labelled measurement arrives – can be used as an estimation of the actuation delays. The performance degradation and stability bounds for uncertain actuation delays can be analyzed using minimax methods.

The optimal discontinuous control equations (9)-(12) enable us to consider control functions with multiple variable delays for state space dynamic systems (1). The corresponding state equation is given by

$$dz(t) = (a_0(s) + a(s)z(s))dt + \qquad (13)$$
$$(\sum_{t_i} H(t,t_i)u(t_i))dt, \quad z(t_0) = z_0.$$

Here $u(t_i) \in R^p$ is delayed control applied at a time t_i but affecting the system state at another moment t. The sum $\sum_{t_i} H(t,t_i)u(t_i)$ means that the system state at t is regulated by a combination of delayed controls applied at times $t_1, t_2, t_3, \ldots$, thus allowing fusion of control actions. The matrices $H(t,t_i) \in R^{n\times p}$ are gain matrices between the delayed control $u(t_i)$ and the system state $z(t)$.

The quadratic cost function J to be minimized for every moment t between t_0 and T is defined as follows

$$J = \frac{1}{2}\left[z(T) - z_0\right]^T \Psi^{-1}\left[z(T) - z_0\right] \qquad (14)$$
$$+\frac{1}{2}\int_{t_0}^{T} u^T(s)R(t,s)u(s)dg(s)$$
$$+\frac{1}{2}\int_{t_0}^{T} z^T(s)Q(s)z(s)ds,$$

where z_0 is a given vector, Ψ, R, Q are positive (nonnegative) definite symmetric matrices, $T > t_0$ is a certain time moment.

The key point to obtain the optimal regulator in this case is to assume the control distribution function $g(s)$ being a linear combination of the Heaviside functions $\sum_{t_i}\chi(s-t_i)$ with unit jumps at the time moments t_i when the delayed controls $u(t_i)$ affecting the system state $z(t)$ are applied (see (13)). In doing so, the term including delayed control can be represented as integral with discontinuous measure

$$\sum_{t_i} H(t,t_i)u(t_i)) = \int_{t_0}^{t} H(t,s)u(s)d(\sum_{t_i}\chi(s-t_i)),$$

where $\chi(s-t_i)$ is Heaviside function (unit jump) applied at a point t_i. Integrating now the equation (13) with t and expressing the delayed control term in the new form yields

$$z(t) = z(t_0) + \int_{t_0}^{t}(a_0(s) + a(s)z(s))ds +$$
$$\int_{t_0}^{t}\int_{t_0}^{t} H(t,s)u(s)d(\sum_{t_i}\chi(s-t_i))dt.$$

Changing integration order in the delayed control term and denoting $h(t,s) = \int_{t_0}^{t} H(t,s)dt$ implies

$$z(t) = z(t_0) + \int_{t_0}^{t}(a_0(s) + a(s)z(s))ds +$$
$$\int_{t_0}^{t} h(t,s)u(s)d(\sum_{t_i}\chi(t-t_i)).$$

This yields the following result based on the formulas (9)–(12).

The optimal control law for the system state (13) that minimizes the criterion (14) for every moment t should be given by

$$u^* = R^{-1}(t,s)h^T(t,s)P(s)z(s), \qquad (15)$$

where $P(t)$ satisfies the integral equation

$$P(t) = \int_{t_0}^{t}[-a^T(s)P(s) - P(s)a(s) + Q(s)]ds \qquad (16)$$
$$-\sum_{t_i\le t}[P(t_i-)[I + (h^T(t,t_i)(R(t,t_i))^{-1}h(t,t_i)P(t_i-)]^{-1}$$
$$\times h^T(t,t_i)(R(t,t_i))^{-1}h(t,t_i)P(t_i-),$$

with the terminal condition $P(T) = \Psi^{-1}$. The corresponding optimal trajectory satisfies the equation

$$z(t) = z(t_0) + \int_{t_0}^{t} (a_0(s) + a(s)z(s))ds + \tag{17}$$
$$\sum_{t_i \le t} h^T(t, t_i) R^{-1}(t, t_i) h(t, t_i) P(t_i-) z(t_i-).$$

Let us note that the function $f(t, s)$, the optimal trajectory $z(t)$, and the optimal control law u^* should be obtained as continuous functions, if the initial problem statement is in the form (13). Also note that the equations (16) and (17) must be solved anew beginning from the initial time t_0 until the current moment t, for every t where one would like to know a value of the optimal trajectory $z(t)$. It is well explainable: a differential equation with multiple variable delays is actually an infinite-dimensional system of differential equations, so the optimal control solution should have the same property, i.e., be obtained as a solution of an integral equation corresponding to every current moment t.

7. CONCLUSIONS

The combination of the Ito-Volterra formalism and the results on the necessary conditions for optimality of control systems with impulsive discontinuities (Orlov and Basin, 1995; Miller, 1996) allowed us to solve a broad range of optimal filtering problems for the continuous linear systems with discrete and continuous measurements, including the case of time-delayed measurements. The optimal control problem with continuous and discontinuous controls is dual to the corresponding filtering problems. An important difference between filtering and control problems is that optimal estimates can be obtained without prior knowledge of the measurement delays, while the control problem requires the knowledge of the actuation delays to guarantee stability and optimality.

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