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ON $\mathcal{H}_\infty$ CONTROLLERS WITH INTEGRAL ACTION: AN EXPERIMENTAL EVALUATION

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Abstract: This paper presents an experimental evaluation of two design methods for including integral action in $\mathcal{H}_\infty$ controllers. There are two basic possibilities for doing that: (i) the specification of an integral weighting of the regulation error; or (ii) the direct inclusion of an integrator in series with the plant control input. The GS/T mixed sensitivity scheme has been employed, in order to avoid the generalized plant transfer matrix inversion. Alternative (i) is performed via an approximation that takes the generalized plant poles out of the imaginary axis (namely, the pole at origin). Such approximation is necessary to allow the Riccati equation to have a solution. An approximation procedure is proposed here, based on a linear mapping of the generalized plant model into another model close to the original one, and the inverse (approximate) mapping of the resulting controller. Alternative (ii) is implemented via an LMI (Linear Matrix Inequalities) algorithm. The experimental data suggests that the integral weighting specification leads to better results, possibly due to the lower order of the resulting controller.

Keywords: H-infinity control, integral action, process control, control applications.

1. INTRODUCTION

Most practical control systems must have an integral action, either to track a constant reference without steady state error or to reject constant offset disturbances (Mellichamp and Seborg, 1989; Shinskey, 1988; Skogestad and Postlethwaite, 1996; Morari and Zafiriou, 1989). In the context of the $\mathcal{H}_\infty$ robust control, however, the insertion of the integral action is not straightforward, as discussed in references (Safonov, 1987), (Liu and Mita, 1992), (Mita *et al.*, 1993), (Meinsma, 1995) and (Qi and Tsuji, 1996). This is so because the $\mathcal{H}_\infty$ control design algorithms do not have, in their primary form, any means for constraining the resulting controller to have one or more poles in the origin of the *s*-plane. There are two basic ways of introducing integral action in a $\mathcal{H}_\infty$ controller:

(1) Introducing poles at the origin in the weighting filters related to the system error. An integrator is expected to appear in the controller in order to account for the error requirement.
(2) Adding integrators in series with the plant. The $\mathcal{H}_\infty$ controller is calculated for the augmented plant.

Alternative (2) above has the drawback of producing a controller of higher order than it should be necessary, since, for each additional integrator, the controller (which has the same order of the generalized plant) will have increased degree. This problem is studied and solved in (Meinsma, 1995).

Alternative (1) presents some numerical difficulties, since the pole at the origin is not observable nor controllable from the viewpoint of the closed-loop signals. This precludes the application of conventional

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$\mathcal{H}_\infty$ control design techniques. Two approaches for solving this issue are:

- Slightly generalize the $\mathcal{H}_\infty$ control synthesis machinery, in order to allow weightings with poles or zeros in the imaginary axis (Liu and Mita, 1992; Mita *et al.*, 1993).
- Replace the exact integrator ($1/s$) by an approximation of the type $\frac{1}{s+\varepsilon}$, as suggested in (Skogestad and Postlethwaite, 1996). In (Zhou *et al.*, 1996), this approximation is used in the weighting function related with the error signal, and the integral action is accomplished by replacing an approximate integrator that appears in the controller by the exact integrator.

In this paper, an alternative approach is proposed, in which the generalized plant is mapped into another representation via a coordinate shifting such that the poles of the weighting filters are strictly inside the open left complex plane. The $\mathcal{H}_\infty$ design is then performed in this new representation. Finally, using the inverse transformation, the system is brought back to the original representation, with an open-loop pole exactly located at the origin of the complex plane. To guarantee that at least one controller pole is at the origin, the inverse mapping may sometimes be slightly different from the direct mapping, thus the method is denominated here as the *Shift with Approximate Inverse* (SAI). This approximation may be seen as a simplified version of the bilinear transformation that was suggested in (Safonov, 1987) and (Chiang and Safonov, 1992), in the context of $\mathcal{H}_\infty$-optimal control design.

Two schemes for designing $\mathcal{H}_\infty$ controllers with integral action are compared in the paper. The first one (Alternative 1) uses the *central controller* equations (Doyle *et al.*, 1989), with approximation by the SAI. For brevity, this will be named the *SAI procedure*. The second one (Alternative 2) is based on an LMI *(Linear Matrix Inequalities)* algorithm (Scherer *et al.*, 1997). For brevity, this will be named the *PAI procedure*, standing for *Plant Augmentation with Integrator*.

The controllers derived from these two schemes were implemented in a pilot-scale plant named *interacting tank system* (ITS) (Miranda, 2000; Miranda and Jota, 2000), whose schematic diagram is shown in figure 1.

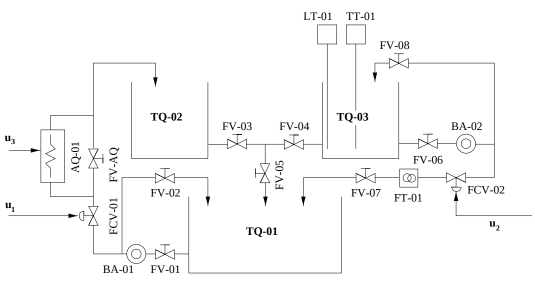

Fig. 1. Schematic Diagram of ITS

Conclusions are taken from typical experimental data records, that were found to be representative of the data set, among more than 300 experiments on the pilot-scale plant. Although guaranteeing the $\mathcal{H}_\infty$ norm bound, the resulting controllers were found to have quite different behaviors.

2. INTERACTING TANK SYSTEM MODEL

The ITS, shown schematically in figure 1, is composed of a $700\,l$ reservoir (TQ-01) and two $300\,l$ passively interconnected tanks (TQ-02 and TQ-03). The coupling between these two tanks can be adjusted manually by means of flow valves FV-03 and FV-04. The basic operation of the system consists of pumping the liquid fluid from the reservoir (TQ-01) directly into the second tank (TQ-02). From TQ-02, the fluid flows naturally to the product tank (TQ-03); the liquid is then pumped back to the reservoir by BA-02. The simultaneous control of level and flow-rate in TQ-03 is accomplished by *equal-percentage* pneumatic valves, FCV-01 and FCV-02. There are two manipulated variables, u_1 and u_2, which are the control signals for the valves FCV-01 and FCV-02 (not the actual valve positions, nor the associated flow rates), and two outputs: y_1, the level, and y_2, the flow-rate at TQ-03 outlet. The ITS has been built with actual industrial sensors and actuators. The data acquisition system has been built to imitate a real process control system. A Programmable Logic Controller is used to interface the plant to a microcomputer, where the control algorithms actually run. All the signal transmission is accomplished via current loops of 4 to 20 mA. A more detailed description of the system and its identification procedure can be found in references (Miranda, 2000) and (Miranda and Jota, 2000).

The nominal model that has been identified is given by the model on equation (1).

$$G(s) = \begin{bmatrix} \frac{2.8\times 10^{-4}}{den} & \frac{4.7\times 10^{-3}s + 1.4\times 10^{-4}}{den} \\ \frac{2.46\times 10^{-6}}{den} & \frac{-0.9(2.9\times 10^{-6}+s)(0.06+s)}{den} \end{bmatrix} \quad (1)$$

where

$$den = s^2 + 0.06s + 1.4\times 10^{-6}$$

The objective of the controller is to track the level and output flow profile.

3. MIXED SENSITIVITY DESIGN

The $\mathcal{H}_\infty$ control design involves the choice of weightings for different system input and output signals, as shown in figure 2, for a specific scheme, namely the GS/T. In this figure, there are two input signal vectors (v and r) and two output signal vectors (z_1 and z_2), which are used to define the cost function for the design. The transfer matrix from an input vector to an output vector is called a *channel* (see, for example, (Scherer *et al.*, 1997)). When two or more channels, which are interpreted as *sensitivity functions* (Skogestad and Postlethwaite, 1996), are used in the cost function definition, the design is called a *mixed sensitivity problem*.

There are many different configurations, e.g. S/KS, S/KS/T, S/T, GS/T, for the mixed sensitivity problem, depending on the choice of the sensitivity function (see for instance (Skogestad and Postlethwaite, 1996) or (Christen *et al.*, 1997; Christen and Geering, 1997; Christen, 1996)). In most of them, the resulting controller turns out to approximately invert the plant transfer matrix (Sefton and Glover, 1990). The GS/T scheme is one exception (Christen and Geering, 1997) and for this reason has been employed here.

The GS/T mixed sensitivity scheme is shown in figure 2, where there are three weighting filters, W_v, W_2 and W_r, a controller K and a plant G. The GS/T mixed sensitivity problem is defined as:

$$\text{Find } K(s) \text{ that gives } ||H||_\infty < \gamma, \tag{2}$$

with:

$$H = \begin{bmatrix} -W_2T_iW_v & W_2KS_oW_r \\ S_oGW_v & T_oW_r \end{bmatrix}, \tag{3}$$

and

$$\begin{aligned} S_i &= (\mathbb{I}+KG)^{-1} \\ S_o &= (\mathbb{I}+GK)^{-1} \\ T_i &= KGS_i \\ T_o &= GK(\mathbb{I}+GK)^{-1} \end{aligned} \tag{4}$$

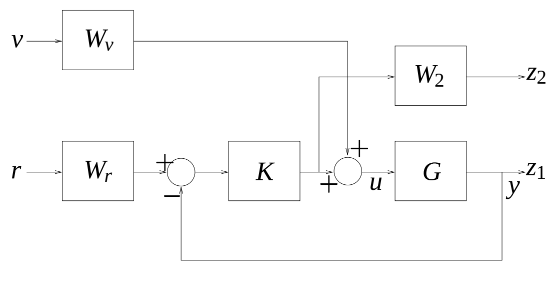

Fig. 2. Schematic diagram of the GS/T mixed sensitivity design.

The integral action is introduced, in GS/T scheme, by the specification of poles at the origin in weighting filter W_v. A rough explanation of the role of such poles can be stated as follows: in order to avoid an infinite weighting in the low frequencies ($\omega \approx 0$) in the block (S_oGW_v), the resulting $\mathcal{H}_\infty$ controller must have a pole at the origin, that gives rise to a zero at the origin in the sensitivity function S_o. The opposite effect in block ($-W_2T_iW_v$) does not occur, provided that the weighting filter W_2 weights the zero frequency with a zero weight. This explains why a pole at the origin of the s-plane, in filter W_v, leads to an integrator in the controller transfer function.

4. SHIFT WITH APPROXIMATE INVERSE

Consider the generalized system matrix $P(s)$ of the GS/T mixed-sensitivity problem given by diagram of figure 2. This generalized matrix is such that:

$$H(s) = \mathcal{F}_l(P(s), K(s)) \tag{5}$$

in which the operator $\mathcal{F}_l(\cdot,\cdot)$ stands for the lower linear fractional transform.

There are imaginary axis uncontrollable poles due to the specification of weighting filter W_v (in fact, poles at the origin), which appear in P. Consider the following transformation that maps $P(s)$ into a new transfer matrix $\hat{P}(w)$ in variable w:

$$w = s + \delta. \tag{6}$$

With the system represented in the w-plane, it is possible to find a controller (also on the w-plane), such that:

$$\begin{aligned} \hat{H}(w) &= \mathcal{F}_l(\hat{P}(w), \hat{K}(w)) \\ ||\hat{H}(w)||_\infty &< \gamma \end{aligned} \tag{7}$$

and for which the closed loop system is internally stable. After the design (on the w-plane), the controller is brought back to the original s-plane.

It is required that the direct loop, $L(s) = G(s)K(s)$ had at least one pole and no zeros at the origin. If the smallest real part of the stable controller poles (in absolute value) is equal to the value of the displacement, the inverse displacement is simply $s = w - \delta$, where δ is the value of the original displacement. However, in some mixed sensitivity schemes, the resulting controller can have poles with value smaller than δ. In these cases, in order to guarantee that the controller has at least one pole at the origin, and no unstable poles, the inverse displacement should not be greater than the smallest absolute value of the real part of a controller pole, instead of the original displacement δ. As a general rule, the inverse displacement should be:

$$\tilde{\delta} = \min_{\lambda}(|\Re[\lambda_i(\hat{K})]|) \tag{8}$$

where λ_i is the set of eigenvalues of $\hat{K}(w)$, the calculated controller.

Let $\bar{K}(s)$ be the inverse-transformed controller that comes from $\hat{K}(w)$. In order to guarantee the validity of the condition:

$$||\mathcal{F}_l(P(s), \bar{K}(s))||_\infty \approx ||\mathcal{F}_l(\hat{P}(w), \hat{K}(w))||_\infty \tag{9}$$

the shift δ should be small. Condition (9) must be verified *a posteriori*. As a rule of thumb, the condition:

$$\delta < \min_{\lambda}(|\Re[\lambda_i(P(s))]|) \qquad (10)$$

was found to be sufficient.

4.1 *Controller Computation*

The selection of the weighting functions was made based on the shape of the desired sensitivity function and some trial and error procedure. For the particular problem considered here, the weighting functions were chosen to be:

$$W_v = \begin{bmatrix} \dfrac{0.5s + 2\times 10^{-2}}{s} & 0 \\ 0 & \dfrac{0.5s + 2\times 10^{-1}}{s} \end{bmatrix}$$
$$W_r = \frac{10s+0.1}{s+0.1}\mathbb{I}_{2\times 2} \qquad (11)$$
$$W_2 = \frac{s+1\times 10^{-4}}{s+10}\mathbb{I}_{2\times 2}$$

The poles of the original generalized system matrix, $P(s)$, are:

$$\{0; 0; -1.67\times 10^{-5}; -7.15\times 10^{-2}; \\ -0.1; -0.1; -10; -10\}.$$

$P(s)$ has two poles at the origin. In this specific case, the value δ (the shifting parameter) should be less than 1.67×10^{-5}.

In order to investigate the effect of this shift, several design experiments were tried, for displacements varying from 1×10^{-7} to 1×10^{-5}. Figure 3 presents the difference $\delta - \tilde{\delta}$ between the displacement δ and the smaller pole of the resulting controller $K(w)$, $\tilde{\delta}$, as function of δ.

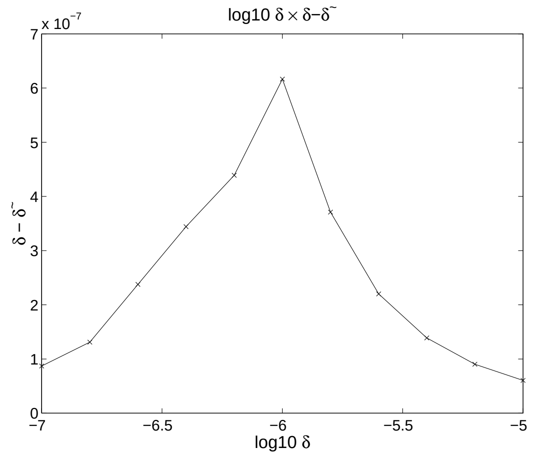

Fig. 3. *Relation between δ and $\delta - \tilde{\delta}$ for the design problem under consideration.*

For this particular example, there is a significant difference between the smaller pole of $K(w)$ and the displacement originally employed. In this case, the inverse shift cannot be equal to the original one, but equal to the smallest real part of the poles of the transformed controller.

4.2 *Experimental Results*

The controller has been implemented in a PLC and a microcomputer to run on the real pilot-scale plant. The level profile of a typical run is shown in figure 4. Comments on this result are presented in section 6.

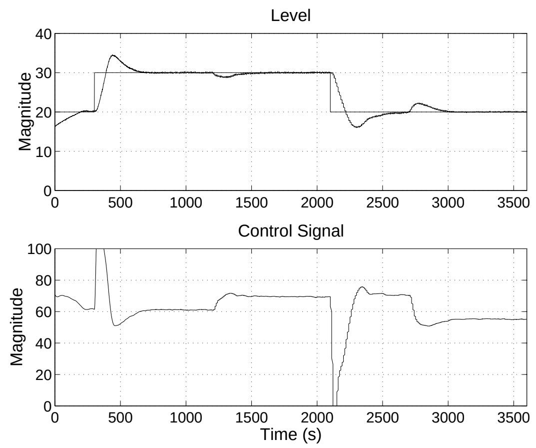

Fig. 4. Experimental results: time response of the fluid level in tank TQ-03 for the SAI design. The fluid level is presented as a percentage of the tank capacity, and the control action is presented as a percentage of the maximum range of valve FCV-01.

5. PLANT AUGMENTATION WITH INTEGRATORS

Another approach to tackle the integrator problem is based on the principle of plant augmentation, with the insertion of integrators in the forward path (PAI scheme). The diagram related to this procedure is shown in figure 5. It should be noted that, although

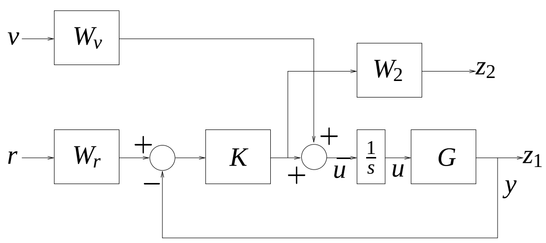

Fig. 5. Schematic diagram of the S/KS mixed sensitivity design with integrators inserted in control inputs (PAI scheme).

the actual plant input is u, the control input, for the controller design algorithm, is the signal $\bar{u}$.

The design algorithm that was employed is based on an LMI formulation (Scherer *et al.*, 1997). The LMI solutions admit poles on the imaginary axis (Gahinet and Apkarian, 1994; Skogestad and Postlethwaite, 1996). The augmented plant is defined by:

$$\tilde{P} = P(s)K_{int}(s) \tag{12}$$

where K_{int} is a matrix with compatible dimension, given as:

$$K_{int} = diag\{\frac{1}{s}, \frac{1}{s}, \cdots, \frac{1}{s}\}.$$

The actual controller to be used in the original plant is:

$$K = K_{int}K_{\infty}$$

where K_{∞} is the solution to the $\mathcal{H}_{\infty}$ problem for the $\tilde{P}$ system.

5.1 *Experimental Results*

The weights for the PAI solution were chosen to be:

$$W_v = \begin{bmatrix} \frac{0.5s + 1\times10^{-2}}{s+1.0\times10^{-6}} & 0 \\ 0 & \frac{0.5s+2\times10^{-1}}{s+1.0\times10^{-6}} \end{bmatrix}$$

$$W_r = \frac{10s+0.1}{s+0.1}\mathbb{I}_{2\times2} \tag{13}$$

$$W_2 = \begin{bmatrix} \frac{s+1\times10^{-4}}{s+1} & 0 \\ 0 & \frac{s+1\times10^{-5}}{s+1} \end{bmatrix}$$

The controller is then applied to the real pilot-scale plant. The level profile of a typical run is shown in figure 6.

6. EXPERIMENTAL DATA DISCUSSION

About 300 experiments, following procedures described here, have been accomplished in this study. In the experiments, there is a set-point change at time 300*s* in the fluid level control. The controller is expected to lead the TQ-03 tank level from the original level of 20% to the new level of 30% of the tank capacity. At the time 1200*s*, a set-point change in the flow-rate control, of 10% of the variable range, is applied. This action causes a disturbance in the fluid level variable. The controller is expected to reject this disturbance, keeping the tank level as constant as possible. At time 2100*s*, the level set-point is brought back to 20%. At time 2700*s* a new flow rate set point change is applied, back to its original value. Figures 4 and 6 show typical results that are representative

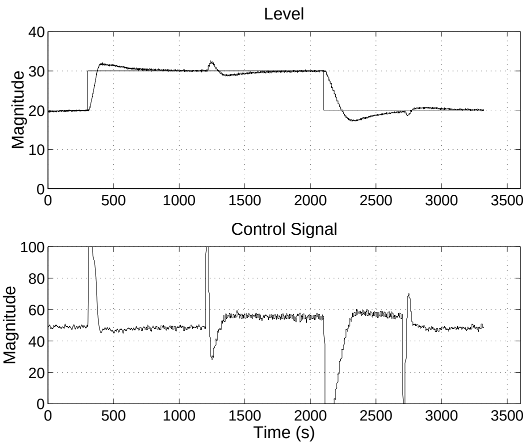

Fig. 6. Experimental results: time response of the fluid level in tank TQ-03 for the PAI design. The fluid level is presented as a percentage of the tank capacity, and the control action is presented as a percentage of the maximum range of valve FCV-01.

of several different runs. Only the level behavior is discussed here, because the other controlled variable, the flow rate, is somewhat less interesting, following closely the predefined profile in all cases.

It should be noted that the design scheme is basically the same for both cases (PAI and SAI). The resulting controller is of order 8 in the case of the SAI procedure, and of order 10 in the case of the PAI procedure.

The main difference between the resulting level behaviors of the system in closed-loop with the two controllers appears in the control signal that drives valve FCV-01. Clearly, the control signal is composed of higher frequencies in the PAI scheme. The peaks of the control signal become, in this case, greater and of smaller duration than in the case of SAI scheme. The controlled variable (level) has on the other hand smaller peaks in the PAI scheme, for the reference signal changes, but not for disturbance signal rejection.

As a final comment, it must be noted that the experimental data presented here is a subset of a larger study, that involved the selection of the weighting filters and of the mixed sensitivity structure, with more than 300 experiments. The complete analysis of such data records is presented in (Miranda, 2000).

7. CONCLUSION

The GS/T mixed sensitivity structure seems to be suitable for the purpose of including integral action in $\mathcal{H}_{\infty}$ controllers.

The data that has been obtained in a prototype pilot plant suggests that the inclusion of integral action in $\mathcal{H}_{\infty}$ control should be performed via any method based on the specification of a pole in the origin in any weighting filter that weights the regulation error. This seems to be preferable when compared with the option for a direct plant augmentation procedure for an integrator inclusion. This conclusion comes not only from the fact that the plant augmentation leads to higher order controllers that could be difficult to implement (this can be the case if the plant order is high): in addition to this reasoning, the data set that was collected shows a better performance of the weighting filter modification approach.

The weighting filter modification, in turn, makes necessary some kind of approximation in order to define a problem of controller computation without any generalized plant pole in the imaginary axis. The scheme of shift with approximate inverse that was proposed here has shown to be an interesting alternative to perform that approximation, leading to good results.

As a regularity that can be inferred from the data set of about 300 experiments, it can be stated that:

- *The SAI-procedure based design with GS/T mixed sensitivity structure leads to the smaller order controllers, among all alternatives that were investigated.*
- *Lower order controllers lead to smaller passbands for the transfer functions from the disturbances to the control signal.*
- *There is not any significant difference among the results of the different controllers under the viewpoint of the controlled variables.*

This leads to the final conclusion: the integral action should be included in process control via the GS/T mixed-sensitivity scheme, with some approximation procedure like the SAI-procedure being employed for controller computation.

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