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OPTIMAL LOCATION OF DISCRETE SCANNING SENSORS FOR PARAMETER ESTIMATION OF DISTRIBUTED SYSTEMS

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Abstract: We investigate possibilities of choosing an activation policy of discrete scanning sensors in such a way as to maximize the accuracy of parameter estimation of a distributed system defined in a given multidimensional domain. A general functional defined on the Fisher information matrix is used as the design criterion. The setting examined here corresponds to situations where one has many sensors and activates only some of them during a given time interval, or alternatively, has several sensors which are mobile. The proposed approach, which has been suggested by Fedorov's idea of directly constrained design measures, consists in imposing constraints on the sensor density in a given spatial domain. As a result, an extremely fast iterative procedure is obtained whose each step reduces to replacing less informative sensor locations with points which furnish more informaton about the parameters. The performance of the proposed algorithm is evaluated by numerical experiments.

Keywords: Distributed-Parameter Systems, Parameter Identification, Sensors, Optimum Experiment Design

1. INTRODUCTION

The problem of estimating spatially-varying parameters in partial differential equations (PDE's) from noisy data arises in many areas of science and engineering, e.g. in the study of air pollution, meteorology, groundwater resources management or emerging smart materials. Since measurements at certain points may yield more information about the parameters than the measurements at other points, the accuracy of parameter estimates depends on the number and locations of sensing devices. As the number of sensors is generally governed by economic considerations, it is desirable to locate the given number of measurement sensors at points that lead to best parameter estimates.

The sensor location problem was attacked from various angles, but mainly in the context of stationary (or motionless) sensor placement (for reviews, we refer the reader to (Uciński, 1999; Uciński, 2000*a*; Uciński, 2000*c*; Kubrusly and Malebranche, 1985)). On the other hand, the optimal measurement problem for spatially movable or scanning sensors seems to be very attractive from the viewpoint of the degree of optimality. So far, this question has been addressed only in (Rafajłowicz, 1986; Uciński, 2000*c*; Uciński, 2000*b*; Uciński, 2001; Uciński and Korbicz, 2001; Uciński, 1999) where some constructive solution methods have been discussed for design of moving sensor trajectories. In some situations, however, the observation system comprises multiple sensors whose positions are already specified and it is desired to activate only a subset of them

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during a given time interval while the other sensors remain dormant (Demetriou, 2000). A reason for not using all the available sensors could be the reduction of the observation system complexity and the cost of operation and maintenance (van de Wal and de Jager, 2001). Such a scanning strategy of taking measurements can be also interpreted in terms of several sensors which are mobile. To the best of the authors' knowledge, the problem has received no attention yet (though some trials have been conducted in a related context of state estimation, see e.g. (Nakano and Sagara, 1988)), and therefore the aim of this work is to outline an approach to fill this gap. Our basic idea is to extend the idea of replication-free designs which have emerged relatively late in the context of spatial statistics (see the monograph (Müller, 1998)).

2. OPTIMAL MEASUREMENT PROBLEM

We consider a distributed parameter system (DPS) defined on a connected open spatial domain $\Omega \subset \mathbb{R}^d$, whose state at a spatial point $x \in \Omega$ and time instant $t \in T = [0, t_f]$, $t_f < \infty$, is an s-dimensional vector $y(x, t; \theta)$. Here θ represents an unknown constant parameter vector which must be estimated using observations of the system.

In what follows, we form an arbitrary partition on the time interval T by choosing points $0 < t_1 < t_2 < \cdots < t_K = t_f$ defining subintervals $T_k = [t_{k-1}, t_k]$, $k = 1, \ldots, K$. We then consider N moving sensors which possibly will be changing their locations at the beginning of every time subinterval but will be remaining stationary for the duration of each of the subintervals. In other words, the measurement process can be formally represented as

$$z^j(t) = h(y(x_k^j, t; \theta), t) + \varepsilon^j(t), \quad t \in T_k \tag{1}$$

for $j = 1, \ldots, N$ and $k = 1, \ldots, K$, where $h(\,\cdot\,, \cdot\,)$ is a given function, $z^j(t)$ is an r-dimensional output, $x_k^j \in X$ stands for the location of the j-th sensor on the subinterval T_k, X signifies the part of Ω where the measurements can be made, and $\varepsilon^j(\,\cdot\,)$ denotes the measurement noise. It is customary to assume that the noise is zero-mean, Gaussian, uncorrelated in both time and space (Fedorov and Hackl, 1997; Kubrusly and Malebranche, 1985).

Sensor positions which guarantee the best accuracy of the least-squares estimates of θ are then found by choosing x_k^j, $j = 1, \ldots, N$, $k = 1, \ldots, K$ so as to minimize some scalar measure of performance Ψ defined on the *average Fisher Information Matrix* (FIM) given by (Rafajłowicz, 1986)

$$M = \frac{1}{N} \sum_{k=1}^{K} \sum_{j=1}^{N} \Upsilon_k(x_k^j), \tag{2}$$

where

$$\Upsilon_k(x) = \frac{1}{t_f} \int_{T_k} F^{\mathrm{T}}(x,t) F(x,t)\, \mathrm{d}t, \tag{3}$$

$$F(x,t) = \frac{\partial h(y(x,t;\theta),t)}{\partial y} \frac{\partial y(x,t;\theta)}{\partial \theta}\Big|_{\theta=\theta^0}, \tag{4}$$

θ^0 being a prior estimate to the unknown parameter vector θ (Uciński, 2000*b*). Such a formulation is generally accepted in optimum experimental design for DPS's, since the inverse of the FIM constitutes, up to a constant multiplier, the Cramér-Rao lower bound on the covariance matrix of any unbiased estimator of θ (Walter and Pronzato, 1997).

As for Ψ, various choices exist for such a function (Walter and Pronzato, 1997; Fedorov and Hackl, 1997), including e.g. the following:

- the D-optimality (determinant) criterion:

$$\Psi(M) = -\log\det M, \tag{5}$$

- the A-optimality (trace) criterion:

$$\Psi(M) = \operatorname{trace} M^{-1}. \tag{6}$$

The assumption of independent measurements made by different sensors implies that we admit of replicated measurements, i.e. some values x_k^j may appear several times in the optimal solution (this is an unavoidable consequence of independent measurements). Consequently, it is sensible to distinguish only the components of the sequence $x_k^1, \ldots, x_k^N$ which are different and, if there are $\ell(k)$ such components, to relabel them as $x_k^1, \ldots, x_k^{\ell(k)}$ while introducing $r_k^1, \ldots, r_k^{\ell(k)}$ as the corresponding numbers of replications. The redefined x_k^i's are said to be the *design* or *support* points. The collection of variables

$$\xi_k^N = \begin{Bmatrix} x_k^1, & x_k^2, & \ldots, & x_k^{\ell(k)} \\ p_k^1, & p_k^2, & \ldots, & p_k^{\ell(k)} \end{Bmatrix}, \tag{7}$$

where $p_k^i = r_k^i / N$, $N = \sum_{i=1}^{\ell(k)} r_k^i$, is called the *exact design* of the experiment on the subinterval T_k. The proportion p_k^i of observations performed at x_k^i can be considered as the percentage of experimental effort spent at that point.

On account of the above remarks, we rewrite the FIM in the form

$$M = \sum_{k=1}^{K} \sum_{i=1}^{\ell(k)} p_k^i \Upsilon_k(x_k^i). \tag{8}$$

Here the p_k^i's are rational numbers, since both r_k^i's and N are integers. Removing this constraint by assuming that they can be any real numbers of the interval $[0, 1]$ such that $\sum_{i=1}^{\ell(k)} p_k^i = 1$, we may think of the designs as discrete probability distributions on X. But if so, we may attempt to take one more step to widen the class of admissible designs a bit further, i.e. to all probability measures ξ_k over X which are absolutely continuous

with respect to the Lebesgue measure and satisfy by definition the condition

$$\int_X \xi_k(\mathrm{d}x) = 1, \quad k = 1, \ldots, K. \tag{9}$$

Such an extension of the design concept allows us to replace (8) by

$$M(\xi) = \sum_{k=1}^{K} \int_X \Upsilon_k(x)\, \xi_k(\mathrm{d}x), \tag{10}$$

where

$$\xi = (\xi_1, \ldots, \xi_K) \tag{11}$$

and the integration in (9) and (10) is to be understood in the Stieltjes-Lebesgue sense. This leads to the so-called *continuous* designs which constitute the basis of the modern theory of optimal experiments (Fedorov and Hackl, 1997; Walter and Pronzato, 1997). It turns out that, in spite of its slightly abstract form, such an approach drastically simplifies the design.

Then we may redefine an optimal design as a solution to the optimization problem

$$\xi^\star = \arg\min_{\xi \in \Xi} \Psi[M(\xi)], \tag{12}$$

where Ξ denotes the set of all designs of the form (11).

In the remainder of this paper we shall make the following assumptions:

(A1) X is compact,
(A2) $h(\,\cdot\,,\,\cdot\,)$ and $F(\,\cdot\,,\,\cdot\,)$ are continuous,
(A3) Ψ is convex,
(A4) If $M_1 \leq M_2$, then $\Psi(M_1) \geq \Psi(M_2)$,
(A5) There exists a finite real q such that

$$\big\{\xi : \Psi[M(\xi)] \leq q < \infty\big\} = \Xi(q) \neq \emptyset,$$

(A6) For any $\xi \in \Xi(q)$ and $\bar{\xi} \in \Xi$, we have

$$\begin{aligned} \Psi[M(\xi) + \lambda(M(\bar{\xi}) - M(\xi))] &= \Psi[M(\xi)] \\ &\quad + \lambda \sum_{k=1}^{K} \int_X \psi_k(x, \xi)\, \bar{\xi}(\mathrm{d}x) \\ &\quad + o(\lambda; \xi, \bar{\xi}), \end{aligned} \tag{13}$$

where $\lim_{\lambda \downarrow 0} o(\lambda; \xi, \bar{\xi})/\lambda = 0$.

As regards the notation in (A4), we adopt that of the Loewner ordering of symmetric matrices, i.e. $M_1 \leq M_2$ iff $M_2 - M_1$ is non-negative definite. Note that (A6) simply amounts to the existence of the directional derivative whose form must be on one hand specific, but on the other hand, for most practical criteria such a condition is not particularly restrictive.

In fact, requiring Ψ to be differentiable with respect to individual elements of its matrix argument, we obtain

$$\psi_k(x, \xi) = \operatorname{trace}\Big[\overset{\circ}{\Psi}(\xi)\Upsilon_k(x)\Big] \tag{14}$$

$$- \frac{1}{K} \operatorname{trace}\Big[\overset{\circ}{\Psi}(\xi)M(\xi)\Big], \tag{15}$$

where

$$\overset{\circ}{\Psi}(\xi) = \left.\frac{\partial \Psi(M)}{\partial M}\right|_{M=M(\xi)}.$$

For particular criteria we have e.g.:

- the D-optimality criterion: $\overset{\circ}{\Psi}(\xi) = -M^{-1}(\xi)$,
- the A-optimality criterion: $\overset{\circ}{\Psi}(\xi) = -M^{-2}(\xi)$.

3. PROBLEM REFORMULATION AND OPTIMALITY CONDITIONS

The assumption of independent observations is advantageous from a theoretical point of view, but it can hardly be justified when in an optimal solution several sensors are to take measurements near one another (this phenomenon is called *sensor clusterization*). Indeed, in the spatial data collection schemes there is usually no possibility of replicated measurements, i.e. different sensors cannot take measurements at one point without influencing one another. Anyway, several sensors situated in the close vicinity of one another usually do not give more information than a single sensor.

In order to avoid such clustered sensor configurations, we implement the idea of operating on the density of sensors (i.e. the number of sensors per unit area), rather than on the sensors' locations, which is justified for a sufficiently large total number of sensors N. In contrast to the designs discussed in Section 2, however, we impose the crucial restriction that the density of sensor allocation must not exceed some prescribed level. This amounts to the condition

$$\xi_k(\mathrm{d}x) \leq \omega(\mathrm{d}x), \quad k = 1, \ldots, K, \tag{16}$$

where $\omega(\mathrm{d}x)$ signifies the maximal possible 'number' of sensors per $\mathrm{d}x$ (Fedorov and Hackl, 1997) such that

$$\int_X \omega(\mathrm{d}x) \geq 1. \tag{17}$$

Consequently, we are faced with the following optimization problem: Find

$$\xi^\star = \arg\min_{\xi \in \Xi} \Psi[M(\xi)] \tag{18}$$

subject to

$$\xi_k(\mathrm{d}x) \leq \omega(\mathrm{d}x), \quad k = 1, \ldots, K. \tag{19}$$

The design $\xi^\star$ above is then said to be a (Ψ, ω)-*optimal design* on the analogy of the definition introduced in (Fedorov and Hackl, 1997) in the context of directly constrained design measures.

Apart from Assumptions (A1)–(A6), a proper mathematical formulation calls for the following proviso:

(A7) $\omega(\mathrm{d}x)$ is atomless, i.e. for any $\Delta X \subset X$ there exists a $\Delta X' \subset \Delta X$ such that

$$\int_{\Delta X'} \omega(\mathrm{d}x) < \int_{\Delta X} \omega(\mathrm{d}x). \tag{20}$$

In what follows, we write $\bar{\Xi} \subset \Xi$ for the collection of all the design measures (11) which satisfy the requirement

$$\xi_k(\Delta X) = \begin{cases} \omega(\Delta X) & \text{for } \Delta X \subset \operatorname{supp} \xi_k, \\ 0 & \text{for } \Delta X \subset X \setminus \operatorname{supp} \xi_k, \end{cases} \tag{21}$$

$k = 1, \ldots, K$. Given a design ξ, we will say that the function $\psi_k(\,\cdot\,, \xi)$ defined by (14) separates sets X_1 and X_2 with respect to $\omega(\mathrm{d}x)$ if for any two sets $\Delta X_1 \subset X_1$ and $\Delta X_2 \subset X_2$ with equal non-zero measures we have

$$\int_{\Delta X_1} \psi_k(x, \xi)\, \omega(\mathrm{d}x) \leq \int_{\Delta X_2} \psi_k(x, \xi)\, \omega(\mathrm{d}x). \tag{22}$$

We can now formulate the main result which provides a characterization of (Ψ, ω)-optimal designs.

Theorem 1. Let Assumptions (A1)–(A7) hold. Then:

(i) There exists an optimal design $\xi^\star \in \bar{\Xi}$, and
(ii) A necessary and sufficient condition for $\xi^\star \in \bar{\Xi}$ to be (Ψ, ω)-optimal is that $\psi_k(\,\cdot\,, \xi^\star)$ separates $X_k^\star = \operatorname{supp} \xi_k^\star$ and its complement $X \setminus X_k^\star$ with respect to the measure $\omega(\mathrm{d}x)$, $k = 1, \ldots, K$.

This constitutes a fairly straightforward generalization of Theorem 4.3.1 of (Fedorov and Hackl, 1997, p. 63), also see (Cook and Fedorov, 1995), and the main ideas of the proof given therein are retained here.

4. SCANNING POLICY

From a practical point of view, Theorem 1 means that at all the support points of an optimal design component $\xi_k^\star$ the mapping $\psi_k(\,\cdot\,, \xi^\star)$ should be less than anywhere else, i.e. preferably $\operatorname{supp} \xi_k^\star$ should coincide with minimum points of $\psi_k(\,\cdot\,, \xi^\star)$, which amounts to allocating observations to the points at which we know least of all about the system response.

If we were able to construct a design with this property, then it would be qualified as an optimal design. This conclusion forms a basis for numerical algorithms of constructing solutions to the problem under consideration.

As regards the interpretation of the resultant optimal designs (provided that we are in a position to calculate at least their approximations), one possibility is to partition X into subdomains ΔX_i of relatively small areas and then, on the subinterval T_k, to allocate to each of them the number

$$N_k^\star(\Delta X_i) = \left\lceil N \int_{\Delta X_i} \xi_k^\star(\mathrm{d}x) \right\rceil \tag{23}$$

of sensors whose positions may coincide with nodes of some uniform grid (here $\lceil \zeta \rceil$ is the smallest integer greater than or equal to ζ). This grid can consist e.g. of points at which scanning sensors may be located, which will be exploited in what follows.

Clearly, unless the considered design problem is quite simple, we must employ a numerical algorithm to make the outlined conceptions useful. Since $\xi_k^\star(\mathrm{d}x)$ should be non-zero in the areas where $\psi_k(\,\cdot\,, \xi^\star)$ takes on a smaller value, the central idea is to move some measure from areas with higher values of $\psi_k(\,\cdot\,, \xi^n)$ to those with smaller values, as we expect that such a procedure will improve ξ^n. This is embodied by the iterative algorithm presented below:

Algorithm for an optimal scanning policy

Step 1. Guess an initial design $\xi^0 \in \bar{\Xi}$. Set $n = 0$.

Step 2. For $k = 1, \ldots, K$ separately set $X_1^n(k) = \operatorname{supp} \xi_k^n$ and $X_2^n(k) = X \setminus X_1^n(k)$. Determine

$$x_1^n(k) = \arg \max_{x \in X_1^n(k)} \psi_k(x, \xi^n),$$
$$x_2^n(k) = \arg \min_{x \in X_2^n(k)} \psi_k(x, \xi^n).$$

If $\psi_k(x_1^n(k), \xi^n) > \psi_k(x_2^n(k), \xi^n) + \eta$, where $\eta \ll 1$, then find two sets $S_1^n(k) \subset X_1^n(k)$ and $S_2^n(k) \subset X_2^n(k)$ such that $x_1^n(k) \in S_1^n(k)$, $x_2^n(k) \in S_2^n(k)$ and

$$\int_{S_1^n(k)} \omega(\mathrm{d}x) = \int_{S_2^n(k)} \omega(\mathrm{d}x) = \alpha_n$$

(i.e. the measures of $S_1^n(k)$ and $S_2^n(k)$ must be identical) for some $\alpha_n > 0$. Otherwise, set $S_1^n(k) = S_2^n(k) = \emptyset$. If $\psi_k(x_1^n(k), \xi^n) < \psi_k(x_2^n(k), \xi^n) + \eta$ for all $k = 1, \ldots, K$, then STOP.

Step 3. Construct ξ^{n+1} such that

$$\begin{aligned} \operatorname{supp} \xi_k^{n+1} &= X_1^{n+1}(k) \\ &= (X_1^n(k) \setminus S_1^n(k)) \cup S_2^n(k). \end{aligned}$$

for $k = 1, \ldots, K$. Increment n and to go Step 2.

Convergence is guaranteed if the sequence $\{\alpha_n\}_{n=0}^\infty$ satisfies the conditions

$$\lim_{n \to \infty} \alpha_n = 0, \quad \sum_{n=0}^{\infty} \alpha_n = \infty, \tag{24}$$

which is established in much the same way as in (Fedorov, 1989).

Within the framework of sensor placement, we usually have $\omega(\mathrm{d}x) = \varrho(x)\mathrm{d}x$, where ϱ is a density function. But in this situation we may restrict our attention to constant ϱ's (indeed, in any case we can perform an appropriate change of coordinates). Moreover, while implementing the algorithm on a computer, all integrals are replaced by sums over some regular grid elements. Analogously, the sets X, $X_1^n(k)$, $X_2^n(k)$, $S_1^n(k)$ and $S_2^n(k)$ then simply consist of grid elements (or potential sensor locations). Consequently, the above iterative procedure may be considered as an exchange-type algorithm with the additional constraint that every grid element must not contain more than one supporting point and the weights of all supporting points are equal to $1/N$. In practice, α_n is usually fixed and, what is more, one-point exchanges are most often adopted, i.e. $S_1^n(k) = \{x_1^n(k)\}$ and $S_2^n(k) = \{x_2^n(k)\}$, which substantially simplifies implementation. Let us note, however, that convergence to an optimal design is assured only for decreasing α_n's and hence some oscillations in $\Psi[M(\xi^n)]$ may sometimes be observed. A denser spatial grid usually constitutes a remedy for this predicament (Müller, 1998).

5. ILLUSTRATIVE EXAMPLE

As an example, consider a vibrating T-shaped membrane shown in Fig. 1. The membrane is fixed on the top and bottom boundaries, and is free elsewhere. The amplitude $y(x,t)$ of the transverse vibrations over a given time interval $T = [0, 10]$ is described by the hyperbolic equation

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \mathrm{div}\big(\gamma(x)\nabla y(x,t)\big) + 20\exp\big(-50[x_2 - (0.2t-1)]^2\big) \quad \text{in } \Omega, \quad (25)$$

subject to the boundary and initial conditions

$$\begin{cases} y(x,t) = 0 & \text{on } \{\Gamma_1 \cup \Gamma_2\} \times T, \\ \dfrac{\partial y(x,t)}{\partial n} = 0 & \text{on } \{\Gamma_3 \cup \Gamma_4 \cup \Gamma_5 \cup \Gamma_6\} \times T, \\ y(x,0) = 0 & \text{in } \Omega, \\ \dfrac{\partial y(x,0)}{\partial t} = 0 & \text{in } \Omega, \end{cases} \quad (26)$$

where $\partial y/\partial n$ means the partial derivative of y with respect to the outward normal of Γ. The coefficient of transverse elasticity has distributed form

$$\gamma(x) = \theta_1 + \theta_2 x_1^2 + \theta_3 x_2, \quad (27)$$

where parameter values $\theta_1 = 100.0$, $\theta_2 = 5.0$ and $\theta_3 = 25.5$ were assumed to be nominal and known prior to the experiment. Our purpose is to construct a D-optimal scanning strategy for determining most accurate estimates of the true parameters θ_1, θ_2 and θ_3 when applying $N = 40$ sensors and the partition of T defined by the switching points $t_k = k/2$, $k = 0, \ldots, 20$. The resulting optimal solution is shown in Fig. 2, where open circles indicate the actual sensor positions.

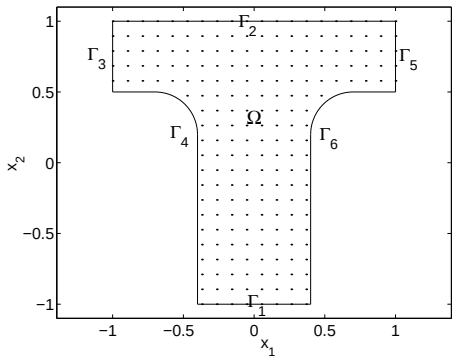

Fig. 1. Membrane and potential sites where the sensors can be placed.

The initial design was generated by randomly selecting its support points. A simple one-point correction algorithm was employed ($\eta = 10^{-2}$) which produced the solution after only 53 iterations, practically within 10 seconds on a low-cost PC (Pentium II, 300 Mhz, using the Lahey/Fujitsu Fortran 95 compiler v.5.6).

As regards the forcing term in our model, it approximates the action of a line source whose support is constantly oriented along the x_1-axis and moves with constant speed from the bottom to the top boundary of Ω. This is reflected by the consecutive configurations of scanning sensors which also advance upwards.

6. CONCLUSIONS

In this work, we have proposed a computationally attractive approach to the optimal placement problem of scanning sensors in parameter estimation of distributed systems. The results extend some ideas employed for constructing replication-free designs and proposed by Fedorov (Fedorov and Hackl, 1997; Cook and Fedorov, 1995; Fedorov, 1989) who restricted his attention, however, solely to static systems. Accordingly, much more efficient scanning measurement policies can be determined compared with the stationary sensor strategies which have been considered in the literature so far. In spite of its somewhat abstract assumptions, the resulting algorithm of exchange type is very easy to implement.

Bear in mind, however, that the delinated approach should in principle be used if the number of sensors is relatively high. If this is not the case, we can resort to standard discrete optimization routines which ensure that the constraints on the design measure and region are satisfied.

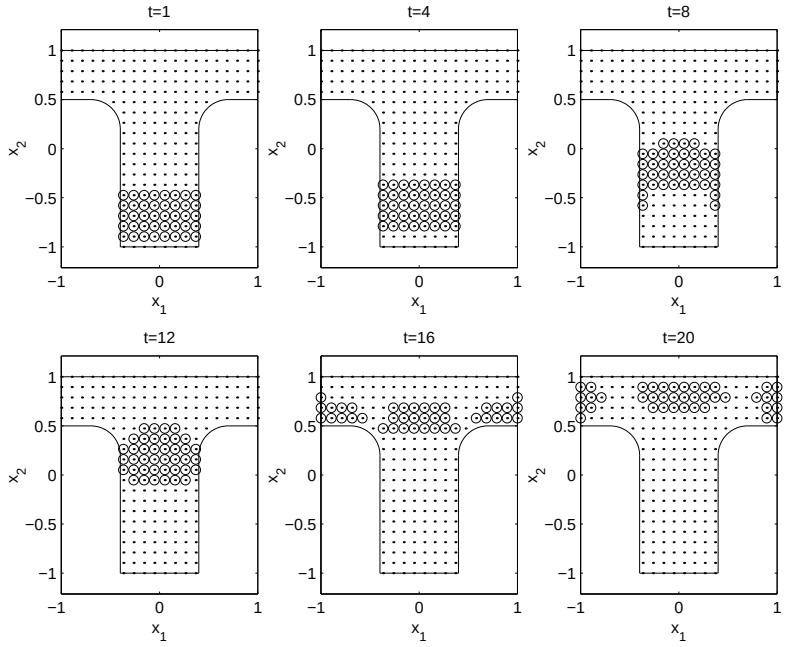

Fig. 2. Consecutive sensor configurations for the D-optimality criterion.

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