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PID AUTO-TUNING BASED ON A SECOND POINT OF FREQUENCY RESPONSE

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Abstract: This paper summarizes a new method for the auto-tuning of PID controllers, and extends it to take into account all the information that the process identification can give. The identification is based on two characteristic points of open-loop frequency response: the ultimate and the crossover frequencies. A 4-parameter model can be obtained and used for controller design based on the ITAE index. The main contribution is that this method makes use of all the information provided by the identification step, suppresses the need for a separation between integrating and non-integrating processes, an can address a wider set of plants. *Copyright © 2002 IFAC*

Keywords: Autotuners, Adaptive control, Process models, Process identification, Closed-loop identification, PID Control, Integral performance indices.

1. INTRODUCTION

Most of the controllers used in industry are of PID type. The performance of many of them is quite poor due to, among other factors, inadequate tuning of the controller parameters (Åström and Hägglund, 1995). Hence, auto-tuning is a very desirable feature and almost every industrial PID controller provides it nowadays. There are many different auto-tuning methods, several of them compared in (Hang and Sin, 1991). In spite of that, the problem is not solved, because the existing methods fail in some usual cases, such as integrating processes. This paper uses a method that improves the performance and works properly with a wider class of industrial processes.

In a way similar to other adaptive control techniques, an auto-tuning method consists of two steps, 1) process identification and 2) controller design. The most extended identification method is based on frequency response analysis by means of a relay feedback (Åström and Hägglund, 1984). Early frequency response methods obtained only the ultimate point characteristics (Ziegler and Nichols, 1942) and relied on a first-order-plus-dead-time (FOPDT) model; but it is interesting to note that the <u>two</u> obtained measurements cannot determine <u>three</u> parameters. More recent methods complete the information with the static process gain (Zhuang and Atherton, 1993; Åström and Hägglund, 1995). The most significant idea is the recognition of the fact that ultimate point information is not rich enough for obtaining a good model of the process, provided the great variety of industrial plants: the trend is to use some more information. Several points of the process frequency response are explored in (Leva, 1993), but finally only one is used in the controller design. Different methods have been proposed for the identification of the crossover point (Schei, 1994; Åström, 1996; Pecharromán and Pagola, 1999). The enhanced information available may result in an improved model such as a second-order-plus-dead-time (SOPDT) model: <u>four</u> measurements can determine <u>four</u> parameters.

This paper assumes the use of a new identification technique presented by Pecharromán and Pagola (1999). By substituting an *Amplitude Dependent Gain* for the relay in the experiments of Åström and Hägglund (1984) and Schei (1994), both identifications (of the ultimate point and the crossover point) are performed with better accuracy.

The next step is to use efficiently the additional information: Åström and Hägglund (1995) and Pecharromán and Pagola (2000) make use of test batches of process models. In this paper, the design based on a SOPDT model of the process is explored. This model can accommodate the results given by almost all the plants included in the test batch shown in the Appendix A, that intends to represent the industrial practice. It accommodates also FOPDT plants and others, blurring the distinction between integrating and non-integrating processes. The model remains implicit in the controller design step; but it can be obtained for user assessment.

The designed controller uses a modified version of the basic PID algorithm, incorporating a two-degree-of-freedom structure. Its tuning is obtained by optimization of the ITAE performance index, subject to a damping constraint given by the *maximum sensitivity function*. This is performed in two steps, allowing simultaneous quasi-optimization of responses to both load disturbance and setpoint steps.

The results are compared with those obtained by other methods, for two models that give significantly different identification results than those included in the test batch. The performance is very good.

The paper is organized as follows. Section 2 gives a summary of the identification procedure and the obtained data. Section 3 presents a normalized SOPDT model, and makes a comparison with the data that can be provided by this model and those in the test batch. Section 4 shows how to obtain the parameters of a 'true' (i.e., not normalized) SOPDT model. Section 5 specifies the PID structure that is used and explains the controller tuning method, giving the parameters of the controller as a function of the identification data. Section 6 compares the controllers for some plants not included in the test batch. Conclusions are given in section 7. Appendix A contains the test batch and Appendix B details a method of analysis of the SOPDT model.

2. PROCESS IDENTIFICATION

The identification procedure consists of two steps, summarily described below. More details can be obtained in (Pecharromán and Pagola, 1999).

2.1 Identification of the ultimate point

The relay feedback technique is very well known and has been used successfully on industrial controllers. Pecharromán and Pagola (1999) have proposed to use instead the *Amplitude Dependent Gain* (ADG); this non-linear block does not need to make use of hysteresis with noisy measurements and leads to a better accuracy. The frequency for an open-loop phase of –180º is identified, together with the plant amplitude at this frequency. Since Ziegler and Nichols (1942), it is customary to give the ultimate gain K_u of the proportional control and the oscillation period T_u. For the plant $P(s)$,

$$P(j\omega_u) = -A_u \qquad K_u = \frac{1}{A_u} \qquad T_u = \frac{2\pi}{\omega_u} \tag{1}$$

2.2 Identification of the crossover point

Schei (1994) and Åström (1996) have proposed a method based on the relay feedback for obtaining the crossover point of a system. As before, better accuracy is obtained by using ADG instead of the relay.

Since for many of the plants in the test batch a crossover point does not exist with proportional control, a PI control $C(s)$ is used instead, based on the ultimate point identification data:

$$C(s) = K_p \frac{1 + T_i s}{T_i s} \qquad K_p = \alpha K_u \qquad T_i = \beta T_u \tag{2}$$

The tuning in Ziegler and Nichols (1942) would be α=0.5, β=1/1.2. In this paper α=0.3 and β=1.6 are used (to obtain more damping) but this is not critical.

A limit cycle oscillation is generated directly with the modified Schei (1994) scheme, in a non-iterative way. The frequency ω_0 for an open-loop gain of 1 (0 dB) is identified, together with the amplitude A_0 and phase φ_0 of the plant at this frequency. For the plant $P(s)$ with control $C(s)$,

$$\mathrm{mod}\left[C(j\omega_0)P(j\omega_0)\right] = 1 \quad P(j\omega_0) = A_0 e^{j\varphi_0} \tag{3}$$

2.3 Summary of data and normalization

From the previously described procedure four parameters are obtained: ω_u, A_u, ω_0, φ_0. It is convenient to normalize the frequency and amplitude obtained in the second step of the identification, in terms of those obtained in the first:

$$\omega_{0u} = \omega_0 / \omega_u \qquad A_{0u} = A_0 / A_u \tag{4}$$

From the previous expressions (1) to (4), it can be shown that A_{0u} is a function of ω_{0u} for any plant: it does not offer new information,

$$A_{0u} = \frac{1}{\alpha \ \mathrm{mod}\left[1 - \dfrac{1}{2\pi\beta\omega_{0u}} j\right]} \tag{5}$$

3. TEST BATCH AND SOPDT MODEL

The necessity for using a complete test batch of process models for developing and testing auto-tuning methods is more and more accepted each day: most of the methods are developed for one kind of processes and fail when they are applied to a different one. A test batch is given in Appendix A. Additional plant models will be included in the sequel. It must be noted that all the models have their own normalization: gain and one time constant (if any) are 1.

Figure 1 shows the normalized results of the identification for the eleven models in the test batch; some are superposed, and are not readily apparent. It is to be noted the similarity between all non-integrating processes (on the right hand), and integrating processes (on the left hand). In previous studies, the authors have seized the simplification that this offers: ω_{0u} and φ_0 are highly correlated for the plants in the test batch, and give essentially the same information. Hence, the design can use only one parameter, provided that a distinction is made between non-integrating and integrating processes. But there is no apparent reason for the gap in between: there must exist plants with intermediate behaviour. Furthermore, simple FOPDT processes are not included.

By assuming that P_1 and P_6 are representative of both types of plants, a SOPDT parameterization that covers both can be found, namely:

$$P_{a1}(s) = \frac{e^{-Ds}}{(1+s)(s+a)} \qquad 0 \le a \le 1 \tag{6}$$

P_1 is obtained for a =1 and P_6 is obtained for a =0. It will be shown later that this kind of model covers all the intermediate space between the extremes. But it does not reach over P_1 (higher ω_{0u} for a given φ_0), where different plants in the test batch lie. In fact, when trying to match the data of plants like P_2 or P_4 to this model, a =1 was always found. So, another SOPDT parameterization is proposed instead (save for a different normalization, it is the same for $\zeta \ge 1$):

$$P_{a2}(s) = \frac{e^{-Ds}}{1+2\varsigma\, s+s^2} \qquad 0.65 \le \varsigma \le \infty \tag{7}$$

Figure 2 shows the normalized results of the identification for this model, see Appendix B. It is to be noted that almost all plants in the test batch are covered, excepting P_7, under P_6 (lower ω_{0u} for a given φ_0). Also, that an intermediate space between non-integrating and integrating processes is covered.

FOPDT model P_{a3} is got as the limit case when ζ goes to infinity for low values of ω_{0u} (P_6 is the corresponding limit for high values of ω_{0u}, and the transition point can be modeled as P_5). The extreme for low φ_0 could be modeled as a pure delay P_{a4}.

$$P_{a3}(s) = \frac{e^{-Ds}}{1+s} \qquad\qquad P_{a4}(s) = e^{-s} \tag{8}$$

In the sequel the model in (7) will be assumed, although in the mentioned limits it could be preferable to choose particular models, which can have more physical sense.

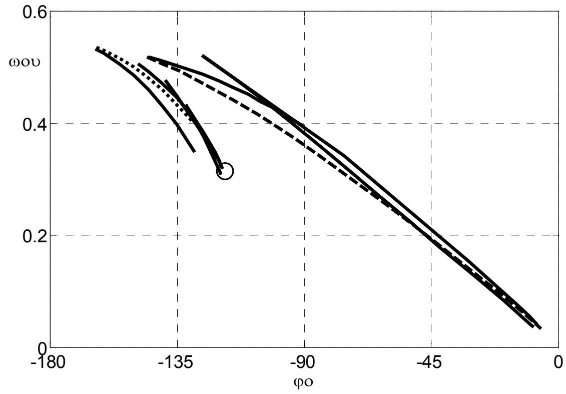

Fig. 1. Relationships between normalized parameters φ_0 and ω_{0u} for the test batch of process models. P_1 (---); near it, non-integrating processes P_2 to P_4. P_6 (…); near it, integrating processes P_7 to P_{11} and P_5 (O).

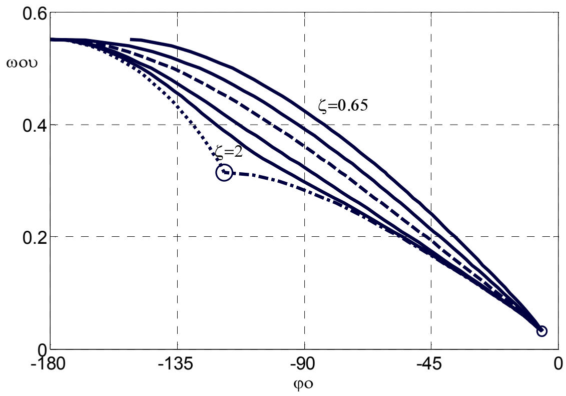

Fig. 2. Relationships between normalized parameters φ_0 and ω_{0u} for different values of ζ , for the normalized SOPDT model (7). P_1 (---). P_6 (…). P_{a3} (-.-). P_5 (O). P_{a4} (o).

4. 'TRUE' SOPDT MODEL

The identification data are used now to obtain a SOPDT model. This is not needed for the design stage, since the final control settings will be given as a direct function of the identification data. Nevertheless, this model may be useful for user assessment. What is needed is to undo the normalization, to get a plant model with correct time scale and gain. This model is:

$$P_T(s) = K\,\frac{e^{-D_t s}}{1+2\varsigma Ts+T^2 s^2} \tag{9}$$

First, one must be aware of the distinction between the properties of the models in (7) and (9): the normalized model will have different oscillation frequency ω_{um} and amplitude A_{um}. The first step is to obtain, for the model in (7) as a function of φ_0 and ω_{0u}: ζ, ω_{um}, $D\omega_{um}$, A_{um}. See Appendix B.

The time constants and gain in model (9) are in proportion. Therefore the time constants and the gain in the 'true' model can be given in terms of the data of the first identification step.

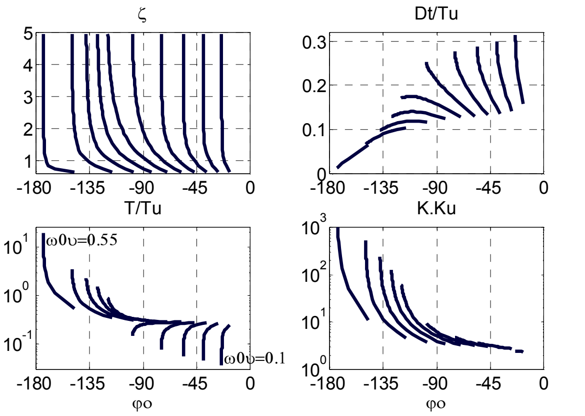

Fig. 3. 'True' SOPDT model parameters as a function of φ_0 for different values of ω_{0u}.

$$
\begin{aligned}
T &= \omega_{um} / \omega_u \\
D_t &= D\omega_{um} / \omega_u \\
K &= A_u / A_{um}
\end{aligned}
\tag{10}
$$

Figure 3 shows, as a function of φ_0 and for different values of ω_{0u}, the four parameters of the 'true' SOPDT model (9).

5. CONTROLLER DESIGN

5.1 Structure

PID controllers have been used for decades. During this time, many modifications of the original algorithm have been presented in the literature. Some of them have been proved very useful and have been incorporated to the industrial practice (Åström and Hägglund, 1995). Making use of these improvements, the controller is assumed to have a two-degree-of-freedom structure: the command signal $u(s)$ is obtained from setpoint $r(s)$ and measurement $y(s)$ separately. The derivative action (if any) is filtered and applied only to $y(s)$. Setpoint weighting is used to avoid excessive overshoot for setpoint steps.

$$
u(s) = K_p \left(b\, r(s) - \left[1 + \frac{T_d s}{1 + 0.1 T_d s} \right] y(s) + \right. \tag{11}
$$

$$
\left. + \frac{1}{T_i\, s} \left[r(s) - y(s) \right] \right)
$$

5.2 Optimization

Among the many design procedures available in the literature, the ITAE criterion has been chosen, because of its reliability. The optimization index is given by:

$$
ITAE = \int_0^{\infty} t \left| e(t) \right| dt \tag{12}
$$

Most of the auto-tuning methods require the operator to decide between optimizing the response to setpoint or load disturbance changes, depending on which one is more important in that particular system. In the proposed method, the PID structure adopted makes it possible the quasi-optimization of both simultaneously. The design has two steps:

a) Optimization of the load disturbance response with the ITAE criterion (12). This optimization gives good results in most cases, but sometimes the resulting system had not enough damping. Hence, the optimization is subject to the constraint $M_s \leq 2$, M_s being the maximum of the sensitivity function,

$$
M_s = \max_{0 \le \omega < \infty} \left| \frac{1}{1 + C(j\omega) P(j\omega)} \right| \tag{13}
$$

It is important to notice that the parameter b, related to the setpoint weighting, does not affect the response to load disturbance changes. Hence, this step gives the optimum value of the rest of the PID parameters: K_p, T_i and T_d.

b) Optimization of the response to setpoint changes with the ITAE criterion, given the values of K_p, T_i and T_d obtained in the previous step. Parameter b is obtained at this step. This is not the absolute optimum set of parameters for the response to setpoint changes, but it is close to it.

More details about the design procedure can be obtained in (Pecharromán and Pagola, 2000). The design procedure has now been applied to the SOPDT plant model in equation (7). Figures 4 and 5 show the PI and PID control normalized parameters, as a function of the identification data, φ_0 and ω_{0u}. For comparison, previous results obtained by assuming a high correlation between φ_0 and ω_{0u} are given. These were obtained for all the plants in the test batch, followed by an averaging process, separately for integrating and non-integrating processes.

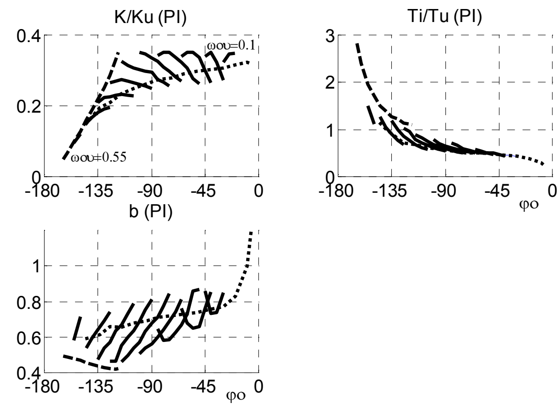

Fig. 4. PI control parameters as a function of φ_0 for different values of ω_{0u}. Integrating plants (--). Non-integrating plants (...).

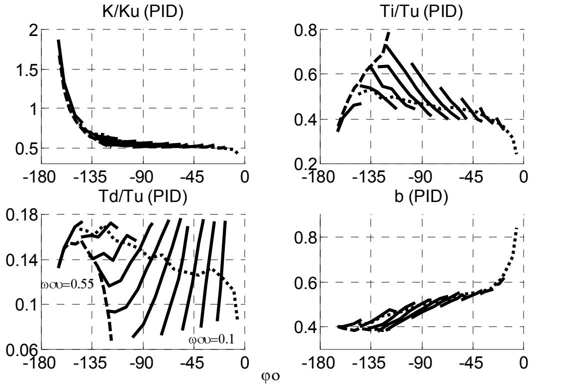

Fig. 5. PID control parameters as a function of φ_0 for different values of ω_{0u}. Integrating plants (--). Non-integrating plants (...).

6. EXAMPLES

The proposed method has been tested for the processes in the test batch and the additional ones. The performance is very good. It is always better than the one obtained applying the method presented in Åström and Hägglund (1995) and the well-known (Ziegler and Nichols, 1942). Comparing with the method previously proposed by the authors in (Pecharromán and Pagola, 2000) (hereafter PP method), the differences are less significant in most cases. Looking at figures 2 and 4, main differences can be expected for plants that give φ_0 around –100º. In this zone the correlation between φ_0 and ω_{0u} is not so high and the value of the control parameters is consequently affected. Hence, two examples of PI control are going to be analysed in this zone.

6.1 Example 1: SOPDT

Section 2 has shown that the SOPDT model shown in equation (7) covers the intermediate space between non-integrating and integrating processes. This example analyses this model with D =0.3 and $\zeta=\sqrt{2}$, that yields ω_{0u} =0.4 and φ_0=-115º.

Figure 6 shows the performance with this process and PI control. Z-N method gives poorly damped responses. Åström method gives good responses but clearly better ones are obtained with the other two. The method proposed in this paper is the best one. PP method control parameters are obtained with the corresponding table for PI control and non-integrating processes. Improvement over PP method is appreciated in the undershoot of the response to load disturbance changes and the overshoot of the response to setpoint changes.

Trying to quantify these differences, the decrements of the ITAE index (from the PP method to the proposed one) in the response to load disturbance and setpoint changes are about 15% and 25% respectively.

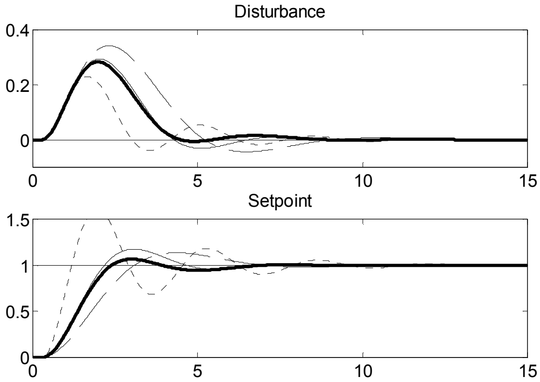

Fig. 6. Responses to setpoint and load disturbance changes for example 1 with PI control. Proposed (thick). PP method (—). Åström (--). Z-N (...).

6.2 Example 2: FOPDT

FOPDT model is used in most of the auto-tuning methods presented in the literature. It has significantly different characteristics compared to the non-integrating processes, as is shown in figure 2. This example analyses this model with D =0.16 that yields ω_{0u} =0.3 and φ_0=-100º. This is the area where more differences between PP and proposed method could be expected, as was previously commented.

Figure 7 shows the performance with this process and PI control. Z-N method gives surprisingly good results in the response to load disturbance changes. Anyway, the proposed method gives even better results, at least in terms of the ITAE index. It also improves the performance of the Z-N method in the response to setpoint changes and the Åström and PP methods in both responses.

The decrements of the ITAE index (from the PP method to the proposed one) in the response to load disturbance and setpoint changes are about 50% and 65% respectively.

As expected, Example 2 yields greater differences with the PP method than Example 1.

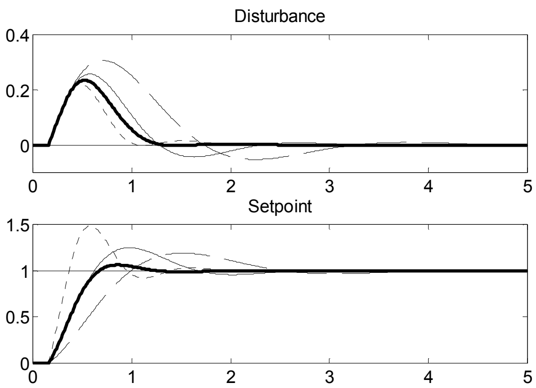

Fig. 7. Responses to setpoint and load disturbance changes for example 2 with PI control. Proposed (thick). PP method (—). Åström (--). Z-N (...).

7. CONCLUSIONS

In previous publications, the authors have presented a complete auto-tuning method, including process identification and controller design. This method outperforms the very well-known Ziegler-Nichols method and also more recent developments, such as those proposed in (Åström and Hägglund, 1995). Much of the improvement was based on a better process identification and the use of a test batch for controller design. In this paper controller design takes even fuller advantage of this enhanced information about the process. Plants that are identifiable but not included in the test batch are shown to be better controlled by the new method.

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APPENDIX A

The test batch contains 11 plants. It includes the test batch used by Åström and Hägglund (1995) and also some other plants used in relevant papers.

$$P_1(s) = \frac{e^{-Ds}}{(1+s)^2} \qquad 0.1 < D < 10$$

$$P_2(s) = \frac{1}{(1+s)^n} \qquad 3 < n < 20$$

$$P_3(s) = \frac{(1-\alpha\, s)}{(1+s)^3} \qquad 0 < \alpha < 2$$

$$P_4(s) = \frac{1}{(1+s)\,(1+\alpha\, s)\,(1+\alpha^2 s)\,(1+\alpha^3 s)} \qquad 0.2 < \alpha < 0.7$$

$$P_5(s) = \frac{e^{-s}}{s}$$

$$P_6(s) = \frac{e^{-Ds}}{s\,(1+s)} \qquad 0.05 < D < 0.8$$

$$P_7(s) = \frac{(1-\alpha\, s)}{s\,(1+s)} \qquad 0.05 < \alpha < 0.75$$

$$P_8(s) = \frac{e^{-Ds}}{s\,(1+s)^2} \qquad 0.1 < D < 10$$

$$P_9(s) = \frac{1}{s\,(1+s)^n} \qquad 3 < n < 20$$

$$P_{10}(s) = \frac{1-\alpha\, s}{s\,(1+s)^3} \qquad 0 < \alpha < 2$$

$$P_{11}(s) = \frac{1}{s\,(1+s)\,(1+\alpha\, s)\,(1+\alpha^2 s)\,(1+\alpha^3 s)} \qquad 0.2 < \alpha < 0.7$$

APPENDIX B

For the normalized model in (7) all the needed properties can be obtained as a function of ω_{0u} and ς, with no need of solving for gain margin and phase margin.

First, use (5) to obtain A_{0u}. Then, by noting the relationship between amplitudes at two frequencies in the model (7),

$$A_{0u}^2 = \frac{\left(1-\omega_{um}^2\right)^2 + \left(2\varsigma\omega_{um}\right)^2}{\left(1-\omega_{0m}^2\right)^2 + \left(2\varsigma\omega_{0m}\right)^2}$$

a biquadrate equation in ω_{um} results:

$$(1 - A_{0u}^2\,\omega_{0u}^4)\omega_{um}^4 + (4\varsigma^2 - 2)(1 - A_{0u}^2\,\omega_{0u}^2)\omega_{um}^2 + (1 - A_{0u}^2) = 0$$

Once this equation is solved, the rest is straightforward:

$$1/A_{um} = \mathrm{mod}\left(1 - \omega_{um}^2 + 2\varsigma\omega_{um}\, j\right)$$

$$D\omega_{um} = \pi - \arg\left(1 - \omega_{um}^2 + 2\varsigma\omega_{um}\, j\right)$$

$$\omega_{0m} = \omega_{0u}\omega_{um}$$

$$\varphi_0 = -D\omega_{0m} - \arg\left(1 - \omega_{0m}^2 + 2\varsigma\omega\omega_{0m}\, j\right)$$