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DESIGN MODIFICATIONS FOR IMPROVED CONTROLLABILITY OF INTEGRATED PLANTS – BUFFER DESIGN

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Abstract: As chemical plants are becoming more and more tightly integrated, with extensive material and energy recycling, the need for improving their dynamic properties through process design modifications is increasing. However, this is not a trivial task since process integration usually also introduces a relatively complex relationship between properties of the individual units and the overall plant. In this paper, with designing buffer tanks for disturbance attenuation in tightly integrated plants as an example, we show that the structural location where the design modification is made in an integrated plant is a crucial decision with respect to their disturbance attenuation properties. A simple model based tool is derived which can be used to determine the optimal location and minimum buffer size for a given level of disturbance attenuation in an integrated plant. *Copyright ©2002 IFAC*

Keywords: design modification, buffer tank, controllability, disturbance attenuation, recycle, feedback

1. INTRODUCTION

Material and energy recycling is becoming a standard part of most chemical plants. The recycling usually imposes a positive feedback effect, thereby increasing the disturbance sensitivity at low frequencies as compared to the process without recycle (Gilliland *et al.*, 1964; Denn and Lavie, 1982). Ideally, a control system should be used to attenuate the effect of disturbances to an acceptable level. Morud and Skogestad (1994) note that the choice of controlled variables has a significant influence on the plant behavior. Yi and Luyben (1997), illustrate this interdependency through some case studies. However, recycling can in many cases also introduce fundamental control limitations, such as non-minimum phase behavior and input constraints (Jacobsen, 1999; Cui, 2000). This implies a less effective feedback control system and hence possibly unacceptable controllability. If the controllability can not be improved by modifying the control structure, then it is necessary to modify the process design to improve the dynamic properties.

In this paper we consider design modifications for a given level of disturbance attenuation in integrated plants. One simple and traditional design modification with the aim of reducing the disturbance sensitivity is the addition of buffer tanks. Skogestad and Postlethwaite (1996) and Faanes and Skogestad (2000) have developed systematic tools, based on linear controllability analysis, for buffer design in traditional cascaded plants, i.e. units connected in series and parallel. Here, optimal design of buffers in strongly integrated plants with respect to minimum required buffer size is considered in particular, as an example for process design modification in integrated plants.

While the effect of a buffer in a cascaded system is quite straightforward to derive, the presence of material and energy recycling makes the relationship between the properties of process units, including buffers, and the overall plant much more complex. We will here show how linear systems analysis may serve to relate unit properties to the overall plant controllability, and hence support design modifications at the process unit level with the aim of achieving certain dynamic properties of the overall system. We start the paper by briefly reviewing some principles and results on controllability analysis and buffer design in cascaded processes.

2. BUFFER DESIGN FOR IMPROVED CONTROLLABILITY OF CASCADED PROCESSES

2.1 Controllability Analysis

With controllability is here understood *input-output controllability*, i.e. the ability to achieve acceptable control performance using feedback control (Skogestad, 1996). We restrict ourselves to consider linearized models only and employ frequency domain analysis. The linear model is written

$$y(s) = g(s)u(s) + g_d(s)d(s) \tag{1}$$

Here y is the output to be controlled, d is the disturbance and u is the control (manipulated) input. We assume all signals to be scalar, but extensions to multivariable systems is quite straightforward. In order to simplify the analysis, we assume the system has been scaled such that $|d| = 1$ corresponds to the maximum expected disturbance, $|u| = 1$ to the maximum allowed control input while acceptable performance corresponds to keeping $|y| \leq 1$.

Define ω_d as the frequency where the scaled disturbance gain is unity, i.e.

$$|g_d(j\omega_d)| = 1 \tag{2}$$

and assume that this frequency is unique. For $\omega < \omega_d$, we then have $|y|/|d| > 1$ and hence the disturbance sensitivity needs to be reduced in this frequency range. This can be achieved either through feedback control, through a modification of the process design, e.g. by adding buffers, or a combination of the two. Usually, feedback control is the least expensive solution and a modification of the process design should therefore only be considered when acceptable performance can not be achieved through feedback control alone.

Applying the feedback control law $u(s) = -c(s)y(s)$, the system is described by

$$y(s) = S(s)g_d(s)d(s) \tag{3}$$

where

$$S(s) = \frac{1}{1 + g(s)c(s)} \tag{4}$$

is the closed-loop sensitivity function. Acceptable disturbance sensitivity then corresponds to $|Sg_d(j\omega)| < 1, \forall\omega$. Define the bandwidth ω_B as the frequency for which $|S(j\omega_B)| = 1$ and $|S(j\omega)| < 1, \omega < \omega_B$. Then, for acceptable disturbance rejection, we get the bandwidth requirement $\omega_B > \omega_d$, i.e. the control must be effective at least up to the frequency ω_d.

Unfortunately, a plant always has fundamental limitations which restrict the highest bandwidth ω_B that the feedback control system can achieve, even with the best possible controller. Fundamental limitations include time delays θ and right half plane (RHP) zeros z. In addition, the phase lag of a plant imposes a limitation when low order controllers, such as PID-controllers, are employed. Assume the plant model can be written on the form

$$g(s) = \frac{ke^{-\theta s}(s - z)}{(\tau_1 s + 1)(\tau_2 s + 1) \cdots (\tau_n s + 1)} \tag{5}$$

where $\tau_1 > \tau_2 > ... > \tau_n$. Then the upper bound on the bandwidth ω_B is approximately determined as

$$\omega_B < \omega_{B*} = 1/\theta_e \tag{6}$$

where θ_e is the effective delay defined as (Skogestad, 1999)

$$\theta_e = \theta + 1/z + \tau_2/2 + \sum_{i \geq 3} \tau_i \tag{7}$$

In addition, constraints on the control input u imposes a limitation. In particular, with $|y| = |d| = 1$ we require $|u| = |g^{-1}|(|g_d| - 1) \leq 1$. Thus, effective feedback control can only be achieved at frequencies for which (Skogestad and Postlethwaite, 1996)

$$|g| > |g_d| - 1 \tag{8}$$

For frequencies where this is not satisfied, acceptable disturbance attenuation can not be achieved using feedback control alone. The smallest frequency for which (8) is not satisfied is denoted ω_{Bu}.

If $\omega_B = min(\omega_{B*}, \omega_{Bu})$ is smaller than ω_d, then acceptable disturbance sensitivity can not be achieved using feedback control alone, and some modification of the process design is required in order to reduce the disturbance sensitivity of the process in the frequency range $\omega \in [\omega_B, \omega_d]$. A simple process modification, which is commonly employed in the process industries, is the addition of buffer tanks.

2.2 Required Buffer Size

For quality (e.g. composition and temperature) disturbances, the transfer function of a perfectly mixed buffer tank, with incompressible content, has the standard form

$$g_B(s) = \frac{1}{\tau_B s + 1} \tag{9}$$

where the residence time $\tau_B = V/q$ is given by the ratio between the nominal buffer volume V and the volumetric flow q. Note that $g_B(0) = 1$, and hence the buffer has no effect on the system at steady state. At higher frequencies, however, $|g_B(j\omega)| < 1$, and it is this nature which can be exploited to attenuate disturbances by cascading a buffer next to the process.

In order to minimize the buffer size, the main task should be to reduce the disturbance sensitivity to be less than 1 where the feedback control system can not be made effective, i.e. such that $|Sg_d g_B(j\omega)| \leq 1, \forall\omega$. If we assume $|S(j\omega)| = 1, \omega > \omega_B$, i.e. the control system has no effect above the bandwidth ω_B, and that $g_d(s)$ has low-pass characteristics, then the buffer design problem simply corresponds to solving the equation

$$|g_d g_B(j\omega_B)| = 1 \tag{10}$$

Solution of this equation gives the required residence time (Skogestad and Postlethwaite, 1996)

$$\tau_B = \frac{\sqrt{k_d^2 - 1}}{\omega_B} \tag{11}$$

where

$$k_d = |g_d(j\omega_B)| \tag{12}$$

Sometimes the disturbance sensitivity is so large that the required buffer volume becomes impractical. To achieve more efficient attenuation, several buffers in series may be employed (Skogestad and Postlethwaite, 1996), i.e.

$$g_B(s) = \frac{1}{(\frac{\tau_h}{n}s + 1)^n} \tag{13}$$

where τ_h, the required total residence time of the buffer tanks, is then given by Skogestad and Postlethwaite (1996)

$$\tau_h = \frac{n\sqrt{k_d^{2/n} - 1}}{\omega_B} \tag{14}$$

The optimal number of tanks is derived, in terms of minimizing the total residence time, by Cui (2000)

$$n = \frac{log_{10}(k_d)}{0.356} \tag{15}$$

In practice, fewer tanks may be favorable in order to lower the investment and maintenance cost.

3. BUFFER DESIGN FOR PROCESSES WITH RECYCLE

The procedure for buffer design as outlined for cascaded process in the previous section could of course be applied also to a plant with recycle, i.e. by cascading a buffer with the plant. However, for plants with recycle, there exists an extra degree of freedom represented by the placement of the buffer within the process flow sheet structure. Below we discuss how this extra degree of freedom can be used to further reduce the required size of buffers in plants with recycle.

3.1 Location of Buffer

As an example of a process with recycle, we consider a reactor-separator system as shown in Figure 1. The

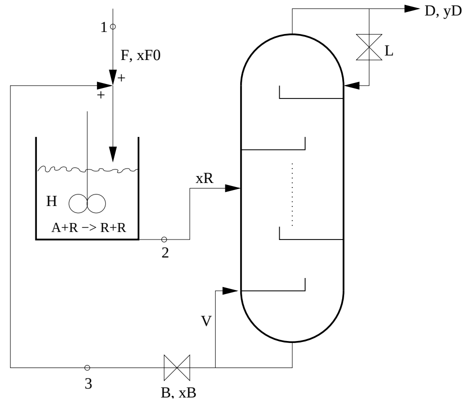

Fig. 1. Reactor-separator system.

recycling tends to increase the disturbance sensitivity, and a buffer may be needed to reduce this sensitivity. By considering the structure of the process in Figure 1, we see that the buffer could be placed in either location 1, 2 or 3. However, it is not obvious what represents the best location in terms of required buffer size for a given level of disturbance attenuation. Intuitively, one might expect that location 2 is optimal, since it attenuates not only external disturbances coming in with the feed flow, but also those returning with the recycle flow. However, as we show below, this is not necessarily correct.

To ease the analysis, especially for cases with the buffer placed inside the recycle loop, we decompose the plant by tearing the process at the recycle loop and analyze the system using linear systems theory. This allows us to relate the behavior of the overall plant to the properties of the individual units.

A block diagram for a typical recycle process is shown in Figure 2, where $\hat{g}_0$ is the transfer function for

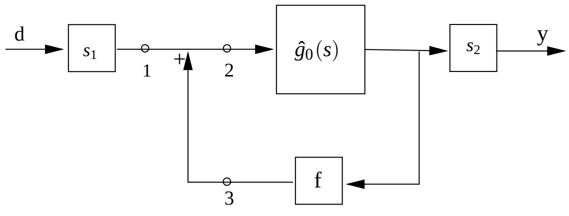

Fig. 2. Block diagram for recycle system structure.

the process with the recycle loop teared, $0 < f <$

1 represents the degree of recycling, and s_1, s_2 are scaling factors so that $|d| \leq 1$ and $|y| \leq 1$ corresponds to acceptable performance. The transfer function for the overall scaled system becomes

$$g_d(s) = \frac{y(s)}{d(s)} = k\frac{g_0(s)}{1-g_0(s)} \quad (16)$$

with $k = s_1 s_2/f$ and $g_0(s) = f\hat{g_0}(s)$ is the loop transfer function of the recycle loop.

Assume that the bandwidth limitation for the control system is ω_B, e.g. due to measurement delays, and that the disturbance sensitivity exceeds one at this frequency, i.e. $|g_d(j\omega_B)| = k_d > 1$. Hence a buffer system is required to further attenuate the effect of disturbances. To determine the best location of the buffer in terms of required buffer size, we examine all three possibilities and compare them with the original recycle system g_d. In all cases we assume stability of the overall system. The transfer-function from disturbance d to output y, for different cases, becomes

- structure 0: $g_d = \frac{kg_0}{1-g_0}$
- structure 1: $g_1 = \frac{kg_0 g_B}{1-g_0} = g_d g_B$
- structure 2: $g_2 = \frac{kg_0 g_B}{1-g_0 g_B} = g_3 g_B$
- structure 3: $g_3 = \frac{kg_0}{1-g_0 g_B}$

The numbering of the structures corresponds to the locations in Figure 1 and 2. ğIndex 0 refers to the case without a buffer. From the transfer functions for the different structures above, we see that $g_2 = g_3 g_B$. Since $|g_B(j\omega)| \leq 1$, we get $|g_2| \leq |g_3|$, i.e. better disturbance attenuation with structure 2 than with structure 3 if buffers with the same residence time are applied for both structures. However, recall that the required *buffer volume* V_i is equal to $\tau_{Bi} q_i$ where q_i is the volumetric flow rate. For the case of energy recycling (heat integration) the flow rates will generally be the same in all locations, and hence location 3 will always be less effective than location 2, for a given buffer volume, and can therefore be excluded for further considerations. However, for material recycling, the flow rates are typically different in each location. Since the flow rate q_2 in this case is always larger than q_1 and q_3, the minimum holdup may thus in principle be achieved with any structure. In the following, we derive the required buffer residence time for each location. The optimal location can then easily be determined, for a given process, by computing the corresponding buffer volume.

3.2 Required Residence Time

We consider here the required buffer residence time for a given level of disturbance attenuation $k_d = |g_d(j\omega_B)|$ at the frequency ω_B, using n equal-sized buffer tanks in series.

Structure 1: As shown for a cascaded system, the total residence time of n buffer tanks is (Skogestad and Postlethwaite, 1996)

$$\tau_{h1} = n\sqrt{k_d^{2/n} - 1}/\omega_B \quad (17)$$

Structure 2: The goal of the buffer design for structure 2 is to find buffers, which satisfy

$$|g_2(j\omega_B)| = |\frac{kg_0 g_{B2}}{1-g_0 g_{B2}}(j\omega_B)| = 1 \quad (18)$$

i.e.
$$|\frac{1}{g_{B2}(j\omega_B)} - g_0(j\omega_B)| = k|g_0(j\omega_B)| \quad (19)$$

Assume the frequency response of the teared system g_0 and the buffer system g_{B2} with n tanks may be written

$$g_0(j\omega_B) = re^{j\theta}, \quad g_{B2}(j\omega_B) = (r_2 e^{j\theta_2})^n \quad (20)$$

where $\theta = \angle g_0(j\omega_B), r = |g_0(j\omega_B)|, \theta_2 = \angle g_{B2}(j\omega_B)/n$ and $r_2 = \sqrt[n]{|g_{B2}(j\omega_B)|}$. Note that for the buffer system $g_{B2}(j\omega_B) = 1/(\frac{\tau_{h2}}{n} j\omega_B + 1)^n$ it holds $r_2 = cos\theta_2$. By inserting (20) into (19), we get

$$cos^{-2n}\theta_2 - 2rcos^{-n}\theta_2 cos(\theta + n\theta_2) + (1-k^2)r^2 = 0 \quad (21)$$

By solving the above nth order equation wrt. θ_2, the required total residence time of the buffer system g_{B2} can be calculated as

$$\tau_{h2} = -n\, tan\theta_2/\omega_B \quad (22)$$

Structure 3: Similar to structure 2, buffers in location 3 should satisfy

$$|g_3(j\omega_B)| = |\frac{kg_0}{1-g_0 g_{B3}}(j\omega_B)| = 1 \quad (23)$$

i.e.
$$|\frac{1}{g_0(j\omega_B)} - g_{B3}(j\omega_B)| = k \quad (24)$$

Defining $g_{B3}(j\omega_B) = (r_3 e^{j\theta_3})^n = cos^n\theta_3 e^{j(n\theta_3)}$ and insert it into (24), the total residence time of n buffer tanks in structure 3 is

$$\tau_{h3} = -n\, tan\theta_3/\omega_B \quad (25)$$

where θ_3 satisfies the following nth order equation derived from (24)

$$cos^{2n}\theta_3 - 2r^{-1}cos^n\theta_3 cos(\theta + n\theta_3) + r^{-2} - k^2 = 0 \quad (26)$$

In some cases there may not exist a positive real solution to (25), which then implies that the required dampening effect can not be achieved with n buffers placed in location 3 alone.

Observe that the buffer phase lag (θ_2 and θ_3) plays an important role in structure 2 and 3. Note also that, s_1, s_2 and f in Figure 2 can be extended from constant to transfer function with k in (19), (21), (24) and (26) set to $|k(j\omega_B)|$, if there are some process units in the path.

3.3 Optimal Location of Buffer

The optimal buffer location for a given process can now be determined simply by computing the required residence time for each location and choosing the structure corresponding to the smallest volume. However, in order to obtain some insight into which conditions that favor the respective locations, we here analyze the influence of a buffer placed in different locations.

A buffer cascaded with the process utilizes its low pass property, i.e. gain reduction, to attenuate the disturbance sensitivity. When placed within the recycle loop, however, the phase lag as well as the gain of the buffer will contribute to the "closed-loop" disturbance sensitivity. This is easily deduced from linear systems theory, and is due to the fact that the phase property determines the feedback effect in the loop, i.e. disturbance amplification or attenuation. This implies that, in principle, even though the buffer gain is less than unity, the additional phase contribution of the buffers may change the property of the feedback effect in the recycle loop, e.g. from amplifying the sensitivity to dampening it.

Since a buffer affects the feedback properties of a recycle loop, we consider when one can expect the buffer to improve, or deteriorate, the feedback properties of the loop. We first consider the effect of recycle on the disturbance sensitivity of a process *prior to adding a buffer*. Let the frequency response of the loop, prior to adding a buffer, at a given frequency ω be

$$g_0(j\omega) = re^{j\theta} \tag{27}$$

Then the feedback effect, imposed by the recycle, will serve to decrease the disturbance sensitivity at ω if

$$|1 - re^{j\theta}| > 1 \tag{28}$$

and otherwise increase the disturbance sensitivity. From (28) we derive the condition

$$cos(\theta) < \frac{r}{2} \tag{29}$$

for when the feedback provides disturbance damping at ω. Since by definition $r > 0$, we get from (29) that the recycle provides disturbance damping at frequencies for which the phase lag $\theta \in [-3\pi/2, -\pi/2]$, while it provides disturbance amplification for $\theta \in [-\pi/3, \pi/3]$ if we also assume $r < 1$. For other values of θ the conclusion depends also on the value of r.

Next consider adding buffers, with a frequency response given by $g_B(j\omega) = r_B e^{j\theta_B}$, in the feedback (recycle) loop. Since θ_B contributes further phase lag and $r_B \in (0, 1]$, we find from (29) that the buffers typically will deteriorate the feedback effect when $\theta \in [-3\pi/2, -\pi]$. The closer θ to $-\pi$, the less attenuation effect is achieved by integrating the buffers in the recycle loop. Similarly, for $\theta \in [-\pi/3, 0]$, the buffers usually contribute to disturbance dampening. The closer θ is to 0, the more advantageous it becomes to integrate the buffers in the recycle loop.

In conclusion we find that, depending on the process properties at the critical frequency ω_B, placing a buffer inside the recycle loop may either improve or deteriorate the disturbance damping provided by the feedback effect imposed by the recycle flow. In the former case, it may be advantageous to place a buffer inside the recycle loop while in the latter case the buffer should be placed outside the loop.

Example 1: Application of buffer design to a reactor-separator system, see Figure 1. Linear reduced order models of the individual units, with the recycle flow teared, are

reactor:

$$dx_R = \underbrace{\frac{0.4}{5s+1}}_{g_r}(dx_{F0} + dx_B) \tag{30}$$

distillation column:

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \underbrace{\frac{1}{30s+1}\begin{pmatrix} 0.04 \\ 2.2 \end{pmatrix}}_{G_c} dx_R \tag{31}$$

The recycle flow rate B is equal to the fresh flow rate F_0. The nominal distillate composition is $y_D = 0.995$ and the aim is to maintain the deviation in $y_D < 0.001$ for disturbances up to 0.1 in the feed composition x_{F0}.

The overall transfer-function from disturbance to output without scaling is

$$dy_D = \frac{G_{c1} g_r}{1 - G_{c2} g_r} dx_{F0} \tag{32}$$

With the proper scaling, (32) can be written on the form (16) with

$$k = 100\frac{G_{c1}}{G_{c2}}, \quad g_0 = G_{c2} g_r$$

The scaled steady-state disturbance sensitivity is 13.3 and disturbance attenuation is needed up to a frequency $\omega_d \approx 0.045$. Assume the bandwidth limitation is $\omega_B < 0.029$, at which $k_d = 1.6$, and a buffer system is hence needed. Note that the feedback imposed by the recycle increases the disturbance sensitivity at this frequency since $|1 - re^{j\theta}| = 0.75$. From (17), (22) and (25), we find that the required total residence time is minimum with only one tank for location 1 and 2, and the residence times $\tau_{h1} = 42min$ and $\tau_{h2} = 19min$, respectively. For location 3, at least two tanks with $\tau_{h3} = 30min$ are required, and the size is gradually reduced when increasing the number of tanks (i.e. from $\tau_{h3} = 27min$ for 3 tanks to $\tau_{h3} = 24min$ for 40 tanks). However, the smallest residence time is achieved with the buffer placed in the forward path of the recycle loop (location 2).

The fact that the best disturbance damping is achieved with an integrated buffer is as expected from the previous analysis, since the open-loop phase lag of the system without buffer at ω_B is approximately $-\pi/4$.

Figure 3 shows the effective disturbance attenuation achieved by a buffer tank with residence time $\tau_B = 20$ in location 1, 2 and 3, respectively. As seen from

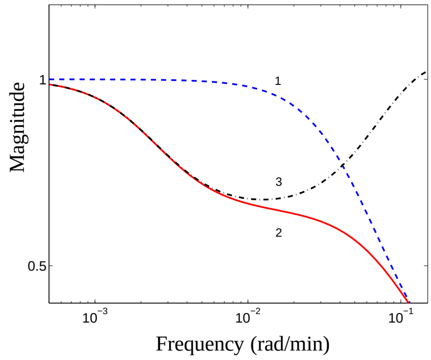

Fig. 3. Effective disturbance attenuation by a buffer with residence time $\tau_B = 20$ placed in location 1 (dashed), 2 (solid) and 3 (dash-dotted), respectively, in *Example 1*.

the figure, the buffer integrated in the forward path of the recycle loop (location 2) gives a significantly improved disturbance damping up to a frequency approximate 0.1, as compared to the cascaded buffer (location 1).

To convert the computed residence times into actual buffer volumes, it is necessary to know the flow rates in the different positions. In the example, with the flow rate $q_1 = q_3 = 0.05m^3/min$, the integrated buffer in location 3 has the minimum size, i.e. $V_1 = 2.1m^3$, $V_2 = 1.9m^3$ and $V_3 = 1.35m^3$ (assuming that 3 tanks are employed).

4. CONCLUSIONS

In this paper we have considered design modifications for disturbance attenuation, in particular the problem of determining the minimum size buffer tanks, in integrated plants. Design modifications should in principle serve as a complement to feedback control systems, e.g. adding buffer tanks with the aim of attenuating the effect of disturbances which can not be handled by the control system. Model based tools for deciding optimal size and location of buffers in processes with recycle of material and/or energy have been derived.

There exists a wealth of knowledge on how to design easily controllable process units, when operated individually or in cascaded plant (see e.g. Buckley (1964)). However, in order to enable this knowledge to be used in strongly integrated plants, it is essential to understand how the behavior of single units affects the overall plant dynamic properties. In this paper, it has been shown that modifying process units which are located in the recycle loop, affect the loop phase lag and hence the feedback effect imposed by the recycle loop. Thus, in order to provide, or further improve, disturbance attenuation properties imposed by the feedback effect (recycle), the phase as well as the gain property of the recycle loop should be optimized.

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