15th Triennial World Congress, Barcelona, Spain

PREDICTIVE CONTROL OF DRIVETRAINS

**Michael Back * Matthias Simons * Frank Kirschaum *
Volker Krebs ****

* DaimlerChrysler, *Research and Technology, REM/EP*
** *Universität Karlsruhe, Institut für Regelungs- und Steuerungssysteme*

Abstract: Telematics allow a prediction of the future driving conditions of a car along some time horizon. This prediction offers a knowledge of the torque request caused by the route ahead and can be used for implementing sophisticated operating strategies for Hybrid Electric Vehicles (HEVs). The intention of the present paper is to describe a method which minimizes the fuel consumption of the system beyond the prediction horizon. Therefore the strategy determines the best operating conditions of the combustion engine and the electric motor with respect to the predicted torque request and the SOC of the battery.

Keywords: Dynamic programming, Hybrid vehicles, Optimal control, Predictive control, Telematics

1. INTRODUCTION

Future car models will be equipped with several telematic devices which can provide the driver and the control units with various environmental information. Based on this data a prediction of the expected driving conditions ahead of the car can be acquired. The information for this prediction can be gained through:

- GPS and road maps containing information about road bends, speed limits and topographies
- traffic information obtained by external sources
- on-board sensors like Distronic, which is a radar system for detecting the distance to the car ahead.

The prediction offers the possibility of implementing advanced operating strategies for the drivetrain. This will help to reduce the fuel consumption of power-driven vehicles, which is the most important aim of all automotive development topics!

The knowledge of the torque request caused by the route ahead can be used to give the driver hints for a fuel efficient way of driving. An example for this is the advice to release the gas pedal very early when it is predicted that the speed has to be reduced soon. It can be shown that a prediction horizon of only 500 m helps saving up to 15 % of fuel. (Fried *et al.*, 2000) But it is obvious that this strategy is not satisfying for the driver, as he looses the power to set the velocity of the car. A solution to this problem is a hybrid electrical drivetrain. As the additional electrical power source together with the battery can be used for transforming and storing energy, the hybrid configuration offers an extra degree of freedom. Together with the prediction of the future torque request the electrical machine can be used for recuperating surplus kinetic energy by transforming it into electrical energy which can be stored in the battery of the vehicle. A shift of the engine's load condition to regions of higher efficiency is also possible. The energy acquired this way can be used for either pure electrical driving or for adding an electrically generated torque to the torque of the combustion engine. Every pro-

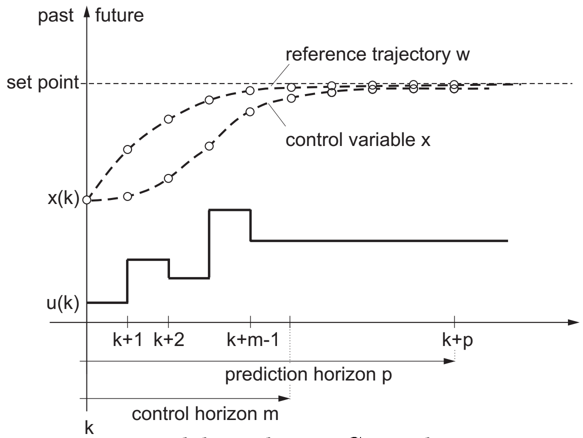

Fig. 1. Model Predictive Control

cedure can be done under the boundary condition that the driver's torque request will always be satisfied, which gives the driver the full control over his vehicle. As both the concept of predictive control and that of the hybrid drivetrain provides substantial potential savings in fuel on their owns, it can be expected that the combination of the two will lead to an even better result.

This paper's aim is to examine the potential of the combination of predictive control and a hybrid powertrain. As the modeling of a driver's behavior, which is necessary for predicting the torque request, is a very complicated task in itself, this paper first of all regards a kind of predictive cruise control. The speed is set to a constant value and the torque request is calculated only out of the knowledge of the incline profile ahead. This allows to show the potential of this approach. In a next step, of course, the prediction of the driver's actions to get a velocity prediction ahead has to be done.

2. PREDICTIVE CONTROL OF A HEV

If, for example, the driver of a car enters his desired destination into the navigation system, and the velocity over this distance could be predicted properly, then in theory the entire profile of the torque request at the wheel $T_{wheel}(t_0; t_\mathrm{p})$ in the time interval $[t_0\ t_\mathrm{p}]$ would be known. Using this knowledge, an optimal solution, with respect to some cost criterion, for the distribution of this torque request onto the electrical machine and the combustion engine can be found. The cost criterion is a measure of the cost of a control action and has to be minimized. When minimizing the fuel consumption of a HEV the criterion has to punish the fuel consumption and also to consider the charge of the battery.

In practice there are several problems which make it impossible to proceed this way. As there will be many disturbances caused by the traffic etc., there is no possibility of getting a suitable prediction of the velocity shape over the whole distance ahead. But even if the velocity could be predicted, there would be the need for solving a nonlinear dynamic optimization problem. However, this leads to an extraordinary high computational effort, which cannot be coped with in real-time applications.

Thus the method of *model predictive control* (MPC) is applied. The basic concept is described in figure 1 (Camacho and Bordons, 1999). Depending on the prediction of the desired velocity, a reference trajectory over p time-steps for the vehicle can be obtained. p is the so-called *prediction horizon*. The basic idea is to obtain the control variables by minimizing an objective function which can depend on the deviation to the reference and on the values of the control variables. If the vehicle is given in time-discrete state-space form

$$\begin{aligned} \underline{x}(k+1) &= \underline{f}\big(\underline{x}(k), \underline{u}(k), k\big) \\ \dim \underline{x} &= n \\ \dim \underline{u} &= p \end{aligned} \tag{1}$$

with the state-variable $\underline{x}(k)$ and the control input $\underline{u}(k)$ and the desired reference-trajectory is termed $\underline{x}_r(k)$ the cost criterion will take the shape of

$$\begin{aligned} J = &\sum_{\nu=k+1}^{\nu=k+p} \xi\big(\|\underline{x}_r(\nu) - \hat{\underline{x}}(\nu|k)\|\big) \\ &+ \sum_{\mu=k}^{\mu=k+m} \zeta\big(\underline{u}(\mu), \hat{\underline{x}}(\mu|k)\big) . \end{aligned} \tag{2}$$

$\hat{\underline{x}}(\nu|k)$ means "predicted $\underline{x}$ at time-step ν, when prediction is made at time-step k". The so-called control horizon m is normally chosen equal to the prediction horizon. The functions $\xi(\cdot)$ and $\zeta(\cdot)$ allow a weighting of the influence of the state-variables and the control variables. If the reduction of the fuel-consumption is the aim of the optimization, then $\zeta(\cdot)$ will be chosen as a function expressing the consumption and $\xi(\cdot)$ is for making sure that the reference trajectory of the state-variables is followed.

When minimizing J a sequence of optimal control values $[\underline{u}(k), \underline{u}(k+1), \cdots, \underline{u}(k+m)]$ will be obtained. Applying them to (1) leads to an open-loop control. In presence of disturbances an open-loop control would fail, therefore the loop has to be closed. This is done by applying the following algorithm:

1. Predict the system-trajectory over the prediction horizon. This trajectory is dependent on the unknown control variables $[\underline{u}(k), \underline{u}(k+1), \cdots, \underline{u}(k+m)]$ and the present state $\underline{x}(k)$.
2. Solve the optimization-problem phrased by (1) and (2). This leads to a sequence of optimal control inputs $[\underline{u}^*(k), \underline{u}^*(k+1), \cdots, \underline{u}^*(k+m)]$.
3. Apply the first value $\underline{u}^*(k)$ of the sequence of control-variables.

4. Go to **1.** and restart the algorithm with the measured current state.

By applying only the first control input (which of course can also be a vector) and then updating the current state a *closed-loop* controller is got. Due to the fact that the prediction horizon is always the same, this horizon will move forward. Consequently MPC is also called *receding horizon control*.

There are generally no limitations for the method which is used for solving the optimization problem (Allgöwer and Zheng, 1991). But as the actual problem is non-linear and as the number of states is small, BELLMAN*'s dynamic programming* (Bryson and Ho, 1975) has been found to be suitable.

3. THE MODEL OF THE HYBRID

3.1 *Configuration of the Hybrid Drivetrain*

The regarded hybrid drivetrain is a parallel one. It consists of a front-wheel driven common-rail-diesel powertrain with an automated manual gearing mechanism. Additional, there is an electrical engine included, which is connected via a second gearbox to the rear-wheel. Both gear-boxes are connected in a way that they always shift simultaneously. The electrical machine is a permanent magnetic synchronous machine with about the same power as the diesel engine has.

A NiMH-battery from *Panasonic* is held as an energy storage, the complete configuration is represented in figure 2

3.2 *General Assumptions*

Generally there are two different approaches to modeling vehicles:

- By using Newton's second law, several balances of power and electrical and chemical correlations, a set of differential equations can be obtained, which describe the behaviour of the car. The control actions like acceleration and breaking pedal position, choice of the gear or the desired torque of the electrical machine are the inputs to these equations; the rotation speed of the drivetrain or of the wheels, the output torque of the combustion engine and the state of charge of the traction battery are the outputs (Kiencke and Nielsen, 2000).
- By inverting the chain of causality and creating a model with a driving profile which consists of a velocity and acceleration shape and the route gradients as an input, most of the dynamic states can be neglected. This is based on the fact, that calculating *backwards* from accelerations, velocities and climbing resistances leads to the torque request to the two machines. Then the only control input is the distribution of the torque to the two torque sources, which then causes a fuel consumption and a change of the state of charge (SOC). Therefore the SOC is the only remaining state variable (Guzzella, 2000).

As the assigned task of the model is mere to describe the behavior of the vehicle when driving with a *known* velocity along a *known* acclivity trajectory, the second approach mentioned above, a so-called quasi-static model, can be applied for modeling the engines and the mechanical part of the vehicle. As the trajectory of the SOC is the *unknown* result of the optimization, the battery has to be modeled as a dynamic system, described by a differential equation of first order. It should be mentioned, that this first-order differential equation is the only dynamic part of the vehicle model.

3.3 *State Space Model of the Battery*

Using the quasi-static approach for modeling the mechanical system, it can be managed with only one single dynamic state describing the SOC of the traction-battery:

$$\dot{Q} = -I(Q, P_{EM}) \tag{3}$$

Q is the SOC, which is limited between 0 and 1, $I(\cdot)$ is describing the current which is charging or discharging the battery. The battery is modelled as a charge reservoir and an equivalent circuit whose parameters are a function of the remaining charge in the reservoir. The equivalent circuit accounts for the circuit parameters of the battery as if it were a perfect open circuit voltage source in series with an internal resistance (NREL, 2001). Therefore $I(\cdot)$ is a function of the present SOC and the power request of the electrical machine P_{EM}.

For simplicity thermal effects influencing the efficiency of the battery are neglected.

3.4 *Model of the Electric Machine*

The balance of power

$$T_{EM} \cdot \omega_{EM} = \eta(\omega_{EM}, T_{EM}) \cdot P_{EM} \tag{4}$$

leads to a model for the electric machine where T_{EM} is the torque of the electrical machine, ω_{EM} is it's angular velocity and $\eta(\cdot)$ is a testbed-map which describes the efficiency of the machine. Using $\omega_{EM} = j_{DTE}(i) \cdot \omega_{wheel}$ where ω_{wheel} is the

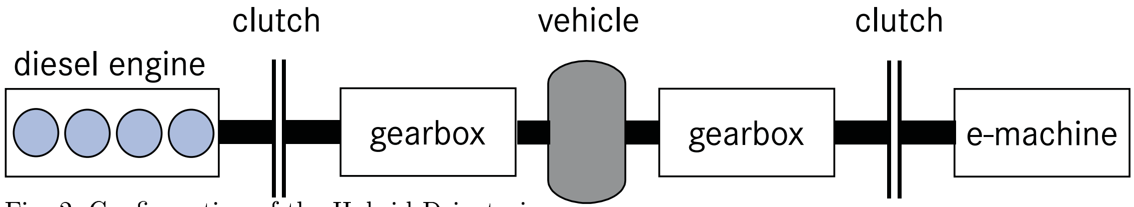

Fig. 2. Configuration of the Hybrid Drivetrain

angular velocity of the driven wheels and $j_{DTE}(i)$ is the gear-dependent gear-ratio of the drivetrain from the electrical machine to the wheel, leads to the description of the electric motor

$$P_{EM} = \frac{T_{EM} \cdot j_{DTE}(i) \cdot \omega_{wheel}}{\eta(\omega_{EM}, T_{EM})} \tag{5}$$

with the gear i and the mechanical torque T_{EM} as inputs and the electrical power request P_{EM} as output.

3.5 *Internal Combustion Engine and Drivetrain*

The internal combustion engine (ICE) and the losses of mechanical energy in the drivetrain are described with measured maps. As a backward-dynamic model is used for modelling the drivetrain, see section 3.2. The torque request at the wheel T_{wheel} is given by the predicted velocity and incline profile. Calculating this torque backwards through the drivetrain gives the gear-dependent torque request to the two engines, T_{EM} and T_{ICE}, respectively. As T_{EM} holds as an input variable to the system, T_{ICE} is only dependent from the driven cycle, the selected gear and the electric torque:

$$\begin{aligned} T_{ICE} = \eta_{DTC} \cdot j_{DTC}(i) \cdot \big(T_{wheel} - \\ - \eta_{DTE} \cdot j_{DTE}(i) \cdot T_{EM}\big) \end{aligned} \tag{6}$$

The variables η_{DTC}, j_{DTC} and η_{DTE}, j_{DTE} are the efficiencies and transmission ratios of the combustion engine and electrical machine, respectively. The drivertrain efficiencies are expressing the mechanical losses as gear, angular velocity and mechanical torque dependent coefficients.

As the torque of the combustion engine is given by (6) it is sufficient to describe the combustion engine by it's steady-state fuel consumption map. For each torque T_{ICE} and rotational speed ω_{ICE} of the combustion engine this map delivers the need for fuel $\Phi(\omega_{ICE}, T_{ICE})$.

3.6 *Route Dependent Model*

A time-based model has the disadvantage that it doesn't really match the problem: A map of the incline and a velocity prediction is used while predicting over some horizon. Both are not time-dependent, but dependent on the *position* of the vehicle! To handle this fact the time-dependent model is transformed into a position-dependent model by using the coherence between position s and velocity v:

$$\frac{ds}{dt} = v \Rightarrow dt = \frac{1}{v} ds \tag{7}$$

The result of using (7) for replacing the differential operator dt in (3) is

$$\frac{dQ}{ds} = -\frac{1}{v} \cdot I(Q, P_{EM}) \,. \tag{8}$$

This new route-dependent model is valid for all $v \neq 0$.

3.7 *Discretization of the Problem*

As MPC shall work together with *dynamic programming*, a discrete model has to be used. This will not be a time-discrete model but a *position discrete* model with the *position-step-size* Δs. As the problem is non-linear, an Euler-based approximative discretization is used [Oga87]. Derivations are replaced by difference quotients and integrals are replaced by sums. This leads to:

$$Q(s + \Delta s) = Q(s) + \Delta s \Big(-\frac{1}{v} \cdot I(Q, P_{EM}) \Big) \tag{9}$$

4. DESCRIPTION OF THE ALGORITHM

In this section a brief description of the underlying optimization algorithm, the used cost criterion and some ideas to decrease the computational effort will be done.

4.1 *Predictive Dynamic Programming*

Bellman's Dynamic Programming is a standard approach for the numerical solution of optimization problems for time-discrete dynamic systems expressed by the difference equation (1) with respect to the criterion (2), see for example (Bryson and Ho, 1975). When using Dynamic Programming, the state and control variables of the system will be quantised and a backward-calculating algorithm delivers a matrix which's elements are the optimal control, dependent on the present time-step and the present state of the system.

As Dynamic Programming is suited for solving non-linear problems with limited state and control variables and a *finite* control horizon, it can be easily used for solving the optimization problem in the model predictive control algorithm as it is described in section 2.

4.2 *The Cost Criterion*

The cost criterion has to prevent the battery's charge from getting too low and it has to weight the fuel consumption. To penalize a too low SOC at the end of the prediction $Q_{\text{end}} = Q(t_0 + t_p)$, a penalty function

$$h(Q_{\text{p}}) = \begin{cases} 0 & \text{for } Q_{\text{end}} \geq Q_{min} \\ \infty & \text{for } Q_{\text{end}} < Q_{min} \end{cases} \qquad (10)$$

is introduced. Using digital computers "∞" means "a very big number". For weighting the fuel consumption, the sum over the output of the steady-state engine map

$$\sum_{\tau=t_0}^{t_0+t_p} \Phi(\omega_{ICE}, T_{ICE})\Delta t \qquad (11)$$

is used. Here t_0 is the present time, $t_0 + t_p$ is the end of the prediction horizon in time domain and Δt is the sample time. Using (7) allows a transformation of (11) into the route domain. By adding (10) the cost criterion then gets it's final form

$$h(Q_{\text{p}}) + \sum_{\nu=s_0}^{s_0+s_p} \frac{1}{v(\nu)}\Phi(\omega_{ICE}, T_{ICE})\Delta s(\nu) \qquad (12)$$

where s_0 is the current position and s_p is the length of the prediction horizon.

4.3 *Reducing the Computational Effort*

The drawback of Dynamic Programming is the quite high computational effort, which increases exponentially with the number of states. As the problem is formulated as a difference equation with only one dynamic state, the need for computational power is not extensively high, but still too high for calculating over horizons of 1000 m and more. Therefore some approaches which specially match the predictive control are used to increase the computational efficiency:

- In the classical Dynamic Programming, a search through the complete state-space has to be done to find the optimal controller. In the problem of optimizing the hybrid vehicle this would mean calculating the optimal control for all possible SOC-conditions of the battery over the whole prediction horizon. But as the current SOC is known and the maximum possible power of the electrical motor generally doesn't allow to completely load or unload the battery, only the reachable SOC-area has to be regarded. This in itself leads to a significant reduction of the computational effort!
- The storage of the complete optimal control matrix over the whole prediction needs a lot of memory. But as only the first control input is applied to the system and then the optimization is started again (see 2), only the control values in the current position s_0 have to be kept in memory.
- Due to the concept of the receding horizon, only the area close to the present position has to be calculated with a very fine grid of quantization. The area far ahead can be calculated with a rougher grid.

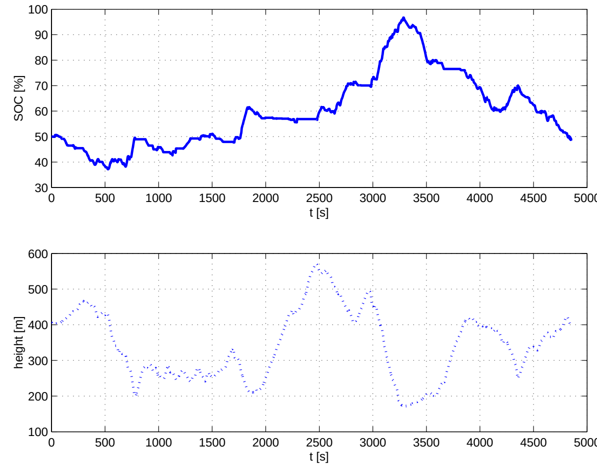

Fig. 3. trajectory of the optimal SOC and profile of the test track

5. SIMULATION RESULTS

In this section some results, obtained by simulating the HEV while driving on a test track, will be presented. The benchmark for the shown test-simulations is a round-course on a typical low-mountain area. For getting the x-, y- and z-coordinates of the course the measurements were made by using differential-GPS. This leads to a very high accuracy of all coordinates. Figure 3 shows the profile of the absolute height of the benchmark track and the optimal trajectory of the SOC. To obtain the global optimum for this calculations the prediction horizon was extended to the complete track. For the SOC the condition was set that it must reach at least it's initial value at the end. It can be seen, that the SOC stays in reasonable limits between 30 % and 100 % and that the final SOC is identically to the initial SOC. Also can be seen, that the battery is charged during all downhill driving conditions and that the stored energy is only used for driving the vehicle in not very steep uphill sections, but not in the steep ones. To explain the reason for this strategy, a section out of the test track will be regarded.

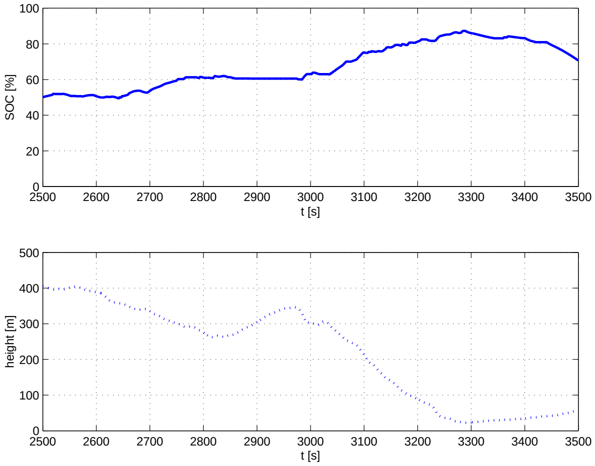

Fig. 4. a section out of the trajectories of the SOC and the profile

Figure 4 shows a section of the SOC and the height. Figure 5 shows the torque distribution in the upper part. The solid line represents the torque of the electrical machine *at the wheel* T_{EMw}, which means that the influence of the gearbox is already considered. The dotted line is the torque request at the wheel for enabling the car driving the test track. It can be seen that the strategy is using negative torque requests for regenerative breaking and that small positive torque requests are supplied by the electrical machine while bigger torque requests are the domain of the combustion engine. The second plot in figure 5 explains this behaviour: Here the trajectory of the efficiency of the diesel engine over the driven route is plotted, dotted when driving the cycle purely with the combustion engine, solid when using the hybrid strategy. If the thermal efficiency is high, like in the steep uphill parts mentioned above, then the electrical machine is off and the diesel engine is working alone. Only if the efficiency is small then the electrical motor is using the energy stored in the battery for generating the traction torque.

Extending the prediction horizon to the entire route shows a high potential of fuel saving, in a dimension of 20 %, compared to a non-hybrid configuration with a much smaller mass.

Of course the complete track, it's length is around 80 km, is an unrealistic long horizon. Therefore simulations with shorter horizons between 250 m and 3000 m were carried out. They showed that the saving of fuel is decreasing with a decreasing horizon, but that this effect is strongly dependent of the profile of the torque request ahead. There are also dependencies on the size of the battery, which is clear when considering that the potential of using the information of a profile ahead is dependent on the amount of energy which can be stored.

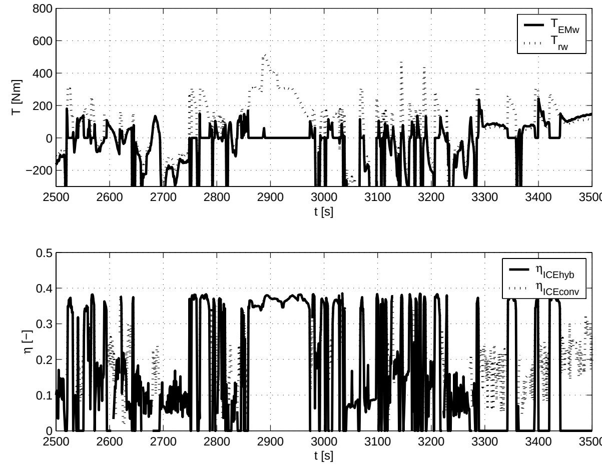

Fig. 5. a section out of the trajectories of the torque distribution and the combustion engine efficiency

6. CONCLUSIONS

In the present paper it could be shown that a predictive powertrain control for a HEV offers a possibility of saving fuel. A simplified model with only one dynamic state for a HEV is derived and transformed into the route-domain. Some ideas on how to use Dynamic Programming for solving the optimization problem in model predictive control and how to reduce the computational effort were presented. It is also mentioned that the success of minimizing the fuel consumption is dependent on the prediction horizon. How to choose this horizon when considering the route ahead and how to accelerate the algorithms are the topics of our future work.

7. REFERENCES

Allgöwer, F. and A. Zheng (1991). *Nonlinear Model Predictive Control*. Birkhäuser Verlag, Germany.

Bryson, Arthur E. Jr. and Y. Ho (1975). *Applied Optimal Control*. Taylor & Francis, USA.

Camacho, E. F. and C. Bordons (1999). *Model Predictive Control*. Springer Verlag, Germany.

Fried, O., M. Bargende and D. Hötzer (2000). *Fuel Consumption Reduction for Electromechanical Powertrains under Driving Conditions*. Stuttgarter Symposium, Germany.

Guzzella, L. (2000). *Fahrzeugantriebssysteme*. Lecture Notes, ETH-Zürich, Switzerland.

Kiencke, U. and L. Nielsen (2000). *Automotive Control Systems*. Springer-Verlag, Germany.

NREL, National Renewable Energy Laboratory (2001). *ADVISOR: Advanced Vehicle Simulator*. Online-Documentation, USA.