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MODEL BASED PREDICTIVE CONTROL OF RIVER RESERVOIRS

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Abstract: The operational water management of river reservoirs has to consider different and sometimes competitive objectives, e. g. navigation conditions, hydro-power generation, flood protection and environmental aspects. Despite of problem specific extensions to cover the non-linear process behavior the application of local standard (PI) controllers to each reservoir can fulfill only basic management requirements. MBPC algorithms have the capability to give a wide and time proper exploitation of the active storage of the whole cascade. A two-layer control structure, consisting of a predictive controller in the upper layer and local controllers at each barrage, allows to provide the management decisions under real time conditions. Simulation results for three reservoirs of the river Moselle show the aptitude of the proposed approach.

Keywords: river reservoir systems, model predictive control, distributed-parameter systems

1. INTRODUCTION

Many rivers are improved with barrages to make them open for navigation throughout the year and to generate hydropower. Besides these objectives the operational water management of river reservoirs has to consider further aspects, e. g. flood protection and ecology. Up to now standard SISO controllers are applied to keep the water level upstream of each barrage within desired bounds (Cuno, 1998). The reference level depends on the current flow situation to fulfill navigation demands. A predetermined time schedule for the change of these target levels in the reservoirs is utilized to attenuate waves traveling along the river. However, the compilation of an operating instruction, representing the sometimes contradicting objectives in a suitable manner, is very difficult.

A single central model predictive controller for the whole cascade could take into account all the operational requirements, but the necessary complexity of the underlying process model prevents its application under real time conditions.

Therefore, a two-layer control structure for the management of a cascade of river reservoirs is proposed, see fig. 5. The model predictive controller of the upper layer calculates optimal trajectories for both the water level of each reservoir and the discharges of weirs and turbines. The operational requirements are formulated as an objective function and as constraints for the process variables. The control decisions are derived using a nonlinear model of the whole reservoir cascade based on a suitable discretization of the Saint Venant equations. The local controllers of the lower layer keep the prescribed reference trajectories for water level and flow. They compensate for uncertainties of the process model of the upper layer and guarantee safe control system operation in failure case, e. g. communication breakdown.

This paper presents simulation results for the proposed control strategy for three reservoirs of the river Moselle, Germany, with a total length of 72 km. Besides the two-layer structure this approach differs from other known applications of predictive controllers to river reservoirs (Compas *et al.*, 1994; Ackermann *et al.*, 1997) in the complexity of the process model and the direct treatment of the level constraints in the optimization solver.

2. PROCESS MODEL

The process model has to reproduce the propagation of waves along the river and their effects on the local water level with sufficient accuracy. Therefore, a conservative formulation of the Saint Venant equations

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ Q \end{pmatrix} + \frac{\partial}{\partial \xi}\begin{pmatrix} Q \\ Q^2/A + P \end{pmatrix} = \begin{pmatrix} q \\ gAS_\xi - gAS_f \end{pmatrix} \quad (1)$$

is used to describe the process behavior (Weiyan, 1992), where A is the wet cross section of the river bed, Q is the flow and P is the total pressure on A depending on the time t and the position ξ. The lateral inflow is denoted by q. The frictional slope S_f, which depends on the bottom structure, is calculated by the Chezy/Manning formula and S_ξ is the bottom slope. Two nested trapezoids are used to approximate the relation between the cross section of the river bed and the water level h $(A = f(h))$ for the interesting range as well as to take real flow velocity profile into account, see fig. 1. Thereby Q is assumed to be zero outside of the 'inner' trapezoid. This profile representation requires only one set of model parameters over the whole flow range and has an explicit relation $h = f^{-1}(A)$, which is useful with respect to the applied optimization algorithm within the predictive controller. The model parameters were estimated by a least squares method in order to reproduce the water surface under static flow conditions at 200 $\mathrm{m^3/s}$ and 1340 $\mathrm{m^3/s}$, see fig. 2.

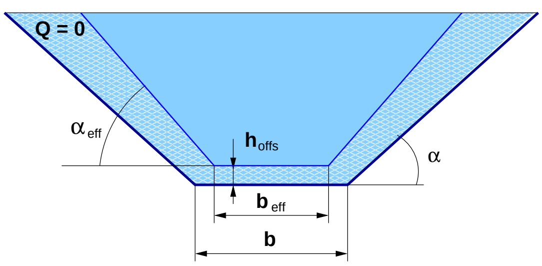

Fig. 1. Approximation of the river bed.

The first order Godunov method, which is based on a discrete formulation of the integral form of

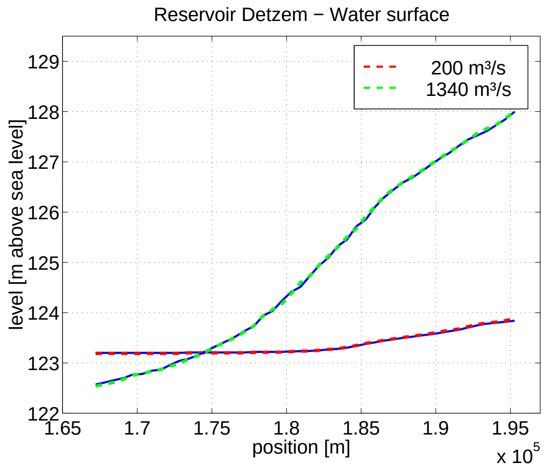

Fig. 2. Water surface of the reservoir Detzem. Measurements (solid lines) compared to simulation results (dashed lines) with optimized model parameters ($\Delta\xi$=600 m).

the underlying hyperbolic system, is used for the numerical solution of (1). Therefore, each reservoir is divided into equidistant cells with the state variables A and Q defined in a cell averaged manner. This defines a sequence of Riemann problems, which are solved using a local linearization approach due to Roe (Roe, 1981).

Fig. 3 shows simulation results for two model realizations of different grid sizes, which are used for the simulation of the impoundments ($\Delta\xi$=600 m) and the process model within the model predictive controller ($\Delta\xi$=3 km). The time step is chosen taking into consideration the Courant-Friedrichs-Lewy condition. Even the coarse grid model is suitable for control purposes, see fig. 3.

3. LOCAL CONTROLLER

Usually, the water level of cascades of river reservoirs is controlled by a local controller for each of the reservoirs. The main operational goal of the local controller is a constant and steady discharge within narrow water level tolerances (Cuno, 1998). Operational rules are set up to manage, e. g. set point changes for the whole cascade, which demand communication between the human operators.

Typically, a PI controller controls the upstream water level h_{up} of each reservoir. The manipulated variable of the SISO control system (2) is the discharge of the reservoir Q_{out} which serves as a command variable for the subsidiary weir sector and turbine flow control loops. Control saturation has to be taken into account to avoid windup in the integral path of the PI controller.

The reference value h_{ref} is changed with the flow situation. With increasing flow, this target value

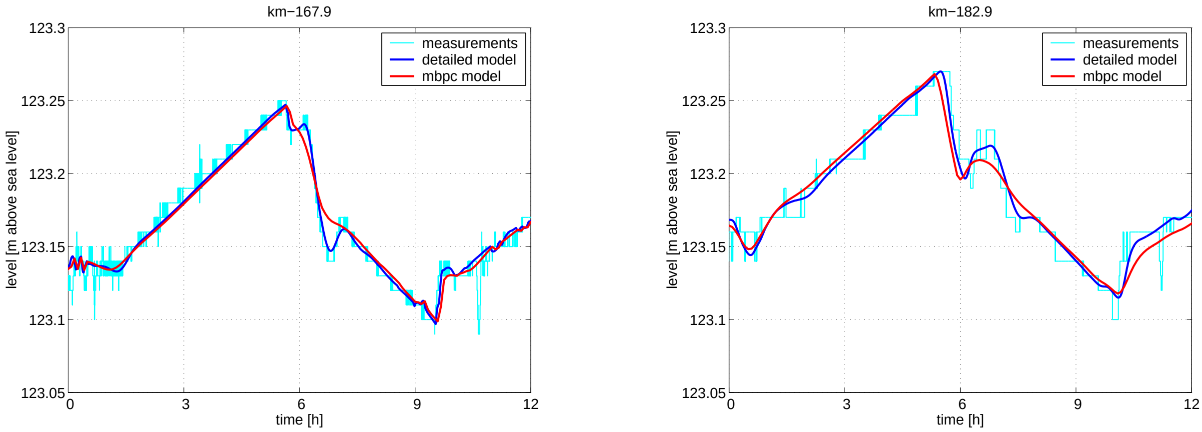

Fig. 3. Reservoir Detzem: measurements compared to simulation results for the detailed model ($\Delta\xi$=600 m) and the model used within the model based predictive controller ($\Delta\xi$=3 km).

of the upstream water level is reduced gradually to fulfill the navigation demands.

The behavior of the local control system is improved substantially by an additional feedforward control path Q_{ff}, see e. g. (Theobald, 1998*a*), where the delayed entering flow into the reservoir is added to the manipulated variable.

$$Q_{out}(s) = \frac{K_R(1+sT_N)}{sT_N}\left(h_{up}(s) - h_{ref}(s)\right) + \underbrace{\frac{\exp\left(-sT_t\right)}{(1+sT_f)^2} Q_{in}(s)}_{Q_{ff}(s)} \qquad (2)$$

Careful adaption of both flow-dependent controller and filter parameters to the flow situation avoids the amplification of inflow waves. However, there is no clear correlation between control goals and controller parameters, which makes the tuning process expensive.

Fig. 4 shows the upstream water level and the discharge of the reservoir Detzem during a medium flood event in March 2001. The actual water level follows the changing reference value and stays safely within the desired bounds of ± 0.05 m.

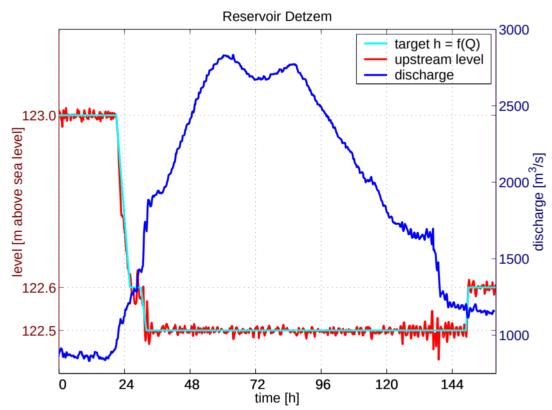

Fig. 4. Reservoir Detzem: measurement data from local controller operation starting from 03/20/2001 16:00.

Unfortunately, the inclusion of additional control goals, e. g. damping of water level variations resulting from setpoint changes or regularization of the downstream water level, into this SISO control scheme is not straightforward or even impossible. The main reason is the lack of communication between the local control units of the adjacent reservoirs. Therefore, the local controllers are embedded in a hierarchical control structure, see fig. 5.

The coordinator described in detail in section 4 sets the upstream water level reference trajectory as well as the discharge reference trajectory of the feedforward path. h_{ref} is replaced by the optimal h^*_{up} and Q_{ff} by Q^*_{out}, respectively.

$$Q_{out}(s) = \frac{K_R(1+sT_N)}{sT_N}\left(h_{up}(s) - h^*_{up}(s)\right) + Q^*_{out}(s) \qquad (3)$$

The feedforward path enables the local control system to follow changing discharge trajectories almost immediately. The feedback path allows attenuation of local disturbances and compensates uncertainties of the process model of the upper layer.

This process model includes a simplified model of the local control units only. The dynamics of the control loop as well as the details of the actuators are neglected. The actuator limits are translated into constraints of the model variables.

4. MODEL BASED PREDICTIVE CONTROL

Predictive controllers optimize the future process behavior using a (nonlinear) process model and a forecast of the non-controllable inputs. At each time step an optimal control problem is solved, where the operational requirements are formulated as objective function and constraints for the process variables:

$$\min_{\mathbf{x}^k,\mathbf{u}^k} \left\{ \sum_{k=0}^{K-1} J^k(\mathbf{x}^k,\mathbf{u}^k) + J^K(\mathbf{x}^K) \right\} \quad (4)$$

subject to:

$$\begin{aligned} \mathbf{x}^0 &= \mathbf{x}(t_0), \\ \mathbf{x}^{k+1} &= \mathbf{f}^k(\mathbf{x}^k,\mathbf{u}^k,\mathbf{z}^k),\ k=0,\ldots,K-1 \quad (5) \\ \mathbf{g}^k(\mathbf{x}^k,\mathbf{u}^k,\mathbf{z}^k) &\le \mathbf{0}, \qquad k=0,\ldots,K-1 \quad (6) \\ \mathbf{g}^K(\mathbf{x}^K) &\le \mathbf{0}, \qquad (7) \end{aligned}$$

with the discrete time index k, the control horizon K, the state variables (flow and water level of the cells of the discretized Saint Venant equations) $\mathbf{x}^k \in \mathbb{R}^n$, the control variables (outflow through the turbines or over the weir) $\mathbf{u}^k \in \mathbb{R}^m$ and the non-controllable inputs $\mathbf{z}^k$ (predicted inflow into the river reservoirs).

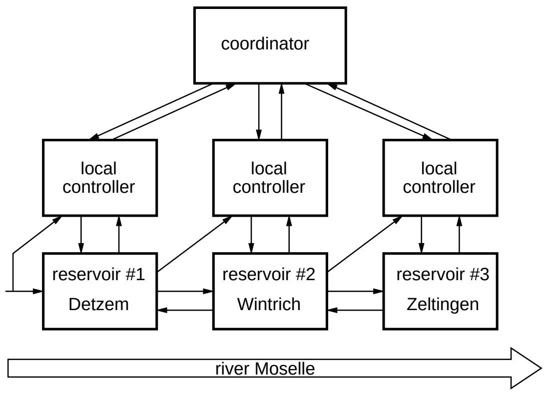

Fig. 5. Two-layer structure of the control system.

While usually the first element of the computed control sequence is applied directly to the process, in this case these values serve as reference values for the local controllers. In this way, a suitable reaction to the predicted inflow can be achieved.

The considered reservoirs belong to the lower reaches of the river Moselle and posses no important lateral inflows. Therefore, the inflow prediction can be obtained from analyzing the flow situation upstream of the first reach over the last hours.

One important aspect of the water management is to guarantee safe navigation conditions. Therefore, the water level h_{up} upstream of the control facilities must be within predetermined bounds

$$|h_{up}^k - h_{ref}^k| \le \Delta h\ ,\ k=1,\ldots,K, \quad (8)$$

where the target level h_{ref} depends on the current flow situation. In addition the objective function (4) includes a term to evaluate the deviation of the water level from the target

$$J^k(h_{ref}) = \rho_{ref}^k (h_{up}^k - h_{ref}^k)^2\ ,\ k=1,\ldots,K \quad (9)$$

as well as another term to damp waves traveling along the river

$$J^k(Q_{out}) = \sum_{i=l_0}^{l_1} \rho_{out}(Q_{out}^k - Q_{out}^{k-i})^2\ , \quad (10)$$
$$k=1,\ldots,K-1\,.$$

The generation of electrical energy is another relevant goal of the management of river reservoirs. The obtained profit, which is to be maximized, depends on the outflow, the head and a time varying charge c^k

$$J_{el} = \sum_{k=0}^{K-1} c^k f\left(Q_{out}^k, h_{up}^k, h_{down}^k\right) \Delta t. \quad (11)$$

The operational range and the efficiency of the turbines are described in dependence on the flow and head using a set of linear constraints and a quadratic approximation, respectively.

Predictive controllers are especially suitable for reservoirs with a large active storage, because the resulting volume to be managed can be used to adapt the control strategy to time varying requirements.

The discrete time optimal control problem is solved as a large scale, structured nonlinear programming problem in the state and control variables. Therefore, a specially tailored SQP algorithm with an interior-point solver for the quadratic subproblem is used, see (Franke and Arnold, 1999).

5. SIMULATION RESULTS

Fig. 6 and 7 show simulation results of the coordinated operation of the three reservoirs compared to the sole application of the local controllers. The artificially generated inflow into the reservoirs varies between $250\,\mathrm{m^3/s}$ and $1500\,\mathrm{m^3/s}$ (see fig. 6) and was used as forecast for the predictive controller. The inflow peak exceeds the admissible limit for navigation slightly.

One crucial point for the management of river reservoirs is the generation of a suitable target level for each reservoir. Up to now this is done at the central process control station in dependence on the inflow into the first reservoir (Detzem) based on an operating instruction, which comprise the experiences of the staff.

The standard SISO controller holds the reference values with a high accuracy, but this leads to a slight amplification of the flood wave at the end of the first reservoir. Within the following two reservoirs the flood wave is damped.

The target level for the local controller is used within the predictive controller, too. But in periods with variations of the target the constraints for this level are modified to allow changes of the

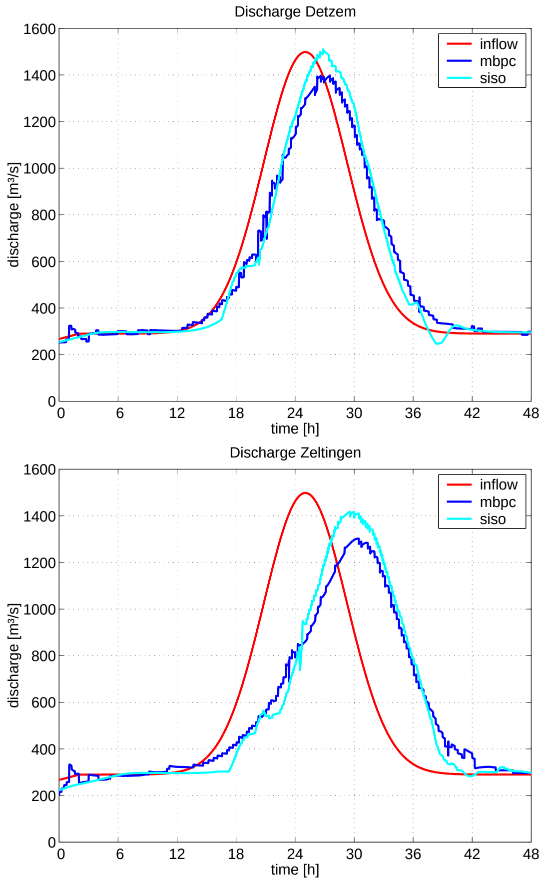

Fig. 6. Reservoir Detzem and reservoir Zeltingen (third reservoir): inflow into the chain of reservoirs and outflow generated from local (SISO) and MBPC controller, respectively.

set point within a larger time interval (± 4 h). The predictive controller is able to deliver the reference values, which lead to a noticeable attenuation of the flood wave. The weight of the according term (10) of the objective function was chosen to dominate the term for keeping the reference values. During the transient to normal flow conditions (time interval from 36 h to 42 h) the local controller in Detzem generates an undershoot of the flow, which impacts especially the level at the downstream face of the weir. In drought periods this controller is not able to attenuate flow variations, which leads to a similar behavior and may endanger the navigation on the river, as reported in (Linke and Arnold, 2000). The model based predictive controller avoids such effects.

A discrete time step of 0.5 h and an optimization horizon of 12 h are used. The reference trajectories are updated each hour. The current state $\mathbf{x}(t_0)$ of each reservoir is reconstructed from water level measurements at both ends of the impoundment and model inputs over the last hours using a moving horizon state estimator.

The solution of the optimal control problem (4)–(7) takes approximately 20 SQP steps, and about 15 minutes on a PC Pentium III (600 MHz).

6. CONCLUSIONS

A hierarchical two-layer control system for a cascade of river reservoirs is developed. At the upper layer, a model predictive controller sets water level and discharge references for the local control systems at the lower layer.

The process model of the model predictive controller is obtained by a suitable discretization of the Saint Venant equations, which describe water level and flow dynamics of the river sections. The open-loop optimal control problem which is solved at each sample instant includes a proper formulation of the navigation demands as well as maximization of hydro-power generation in the objective function and in the constraints.

The two-layer structure enables a substantial improvement of the control decisions of the lower layer compared with a pure local control. In particular, the wave attenuation of the cascade and the behavior during setpoint (target) changes is improved. On the other hand, the existence of the local controllers of the lower layer allows simplifications of the process model and subsequently essential savings of computational effort in the upper layer. Furthermore, the local controllers guarantee safe operation, even if communication breaks down in the distributed control system.

As a first step towards the implementation of the whole control system, the local controller of the reservoir Detzem was put in operation in November 2000.

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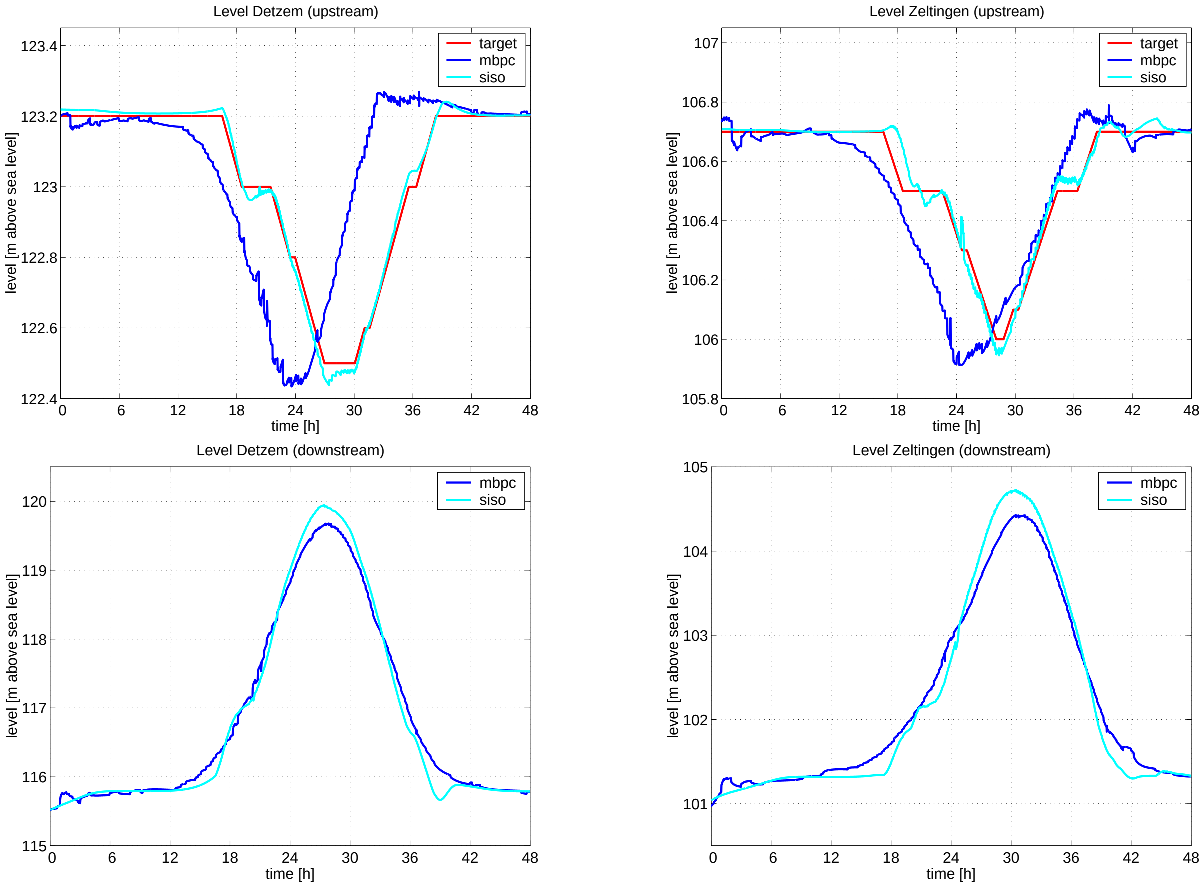

Fig. 7. Reservoir Detzem and reservoir Zeltingen: coordinated operation using the MBPC algorithm (mbpc) compared to the local controller (siso) in combination with flow-dependent target level resulting from operational instructions (target).

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