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AUTOPILOT STUDY FOR AN ASYMMETRIC AIRFRAME

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Abstract: We discuss the design of an autopilot for an asymmetric missile. The autopilot is obtained from controllers designed using H_∞ synthesis. Due to important cross-coupling in the pitch, yaw and roll channels, it was not possible to work on a set of three decoupled models. A multivariable H_∞ loop-shaping control law design is proposed at given operating points. The performance of the control law is evaluated through fully non-linear simulations and is found to be superior to that obtained from an LPV design in (Prempain *et al.*, 2001).

Keywords: Missile Control; H_∞ Optimisation

1. INTRODUCTION

The main requirement of a missile autopilot is to track, as fast as possible, lateral acceleration demands produced by the guidance loop. Tracking must be accurate over a large flight envelope whilst maintaining good stability margins. An autopilot design for an asymmetric airframe is a challenging problem since the missile exhibits highly nonlinear dynamics with significant cross-couplings between the pitch, yaw and roll channels. A major difficulty arises from the complexity of the aerodynamic forces acting on the missile body. These forces and moments are usually expressed in terms of aerodynamic coefficients, which are complex functions of the shape of the missile and of physical parameters such as Mach number, dynamic pressure, angle of attack, sideslip angle and fin deflections.

Missile autopilots are very often designed using some form of gain scheduling. Gain scheduling design can be dived in two main groups: the classical gain scheduling e.g. (Khalil, 1996) or the so-called LPV robust gain scheduling eg. (Apkarian *et al.*, 1995), (Papageorgiou and Glover, 2000). Gain scheduling methods extend the validity of the linearization approach to a range of operating points. Typically, the system is linearized at some point of the flight envelope (e.g. altitude, Mach number). Linear controllers are designed at each equilibrium point. The resulting family of controllers is implemented as a single control with parameters changing according to the scheduling variables. This paper summarises a recent study into the design of a multivariable H_∞ performance gain scheduled autopilot for a class of missiles developed by Matra BAe Dynamics. The design method investigated here is based on the design procedure of McFarlane and Glover (McFarlane and Glover, 1992). Due to the strong couplings present in the roll, pitch and yaw channels the design of a multivariable controller is considered. The results demonstrate that the autopilot can maintain acceptable tracking performance across the flight envelope.

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The paper is structured as follows. Section 2 presents the non-linear missile model and the design specifications. Section 3 presents the H_∞ designs and simulations are given in section 4. Conclusions are given in section 5.

2. NON-LINEAR MISSILE MODEL AND PROBLEM FORMULATION

2.1 *Missile Model*

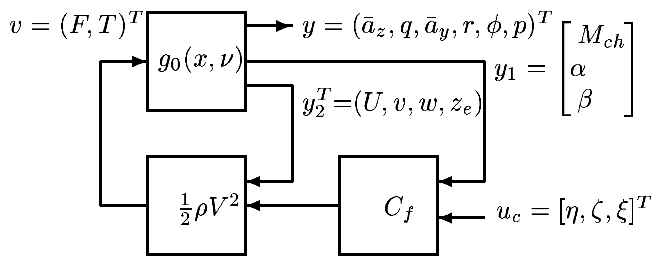

Fig. 1. Non-linear Missile Interconnection

An asymmetric, tail-controlled missile is considered here and its non-linear interconnection structure is shown in Fig. 1. $g_0(x, \nu)$ is the non-linear system that describes the rigid-body motions. The differential equations are derived with respect to orthogonal axes fixed in the missile with origin at the centre of gravity (C). The input to $g_0(x, \nu)$ is $\nu = [F, T]^T$ where $F = (X, Y, Z)^T$ and $T = (L, M, N)^T$ denote respectively the forces and the moments expressed in body axes.

$g_0(x, \nu)$ is governed by

$$g_0(x, \nu) : \begin{cases} \dot{x} = f(x, \nu) \\ y_g = h(x, \nu) \end{cases} \qquad (1)$$

where $x = [U, v, w, p, q, r, \phi]^T$ is the state vector; $V = (U, v, w)^T$ and p, q, r, are respectively the speed and the body angular rates; ϕ is the bank angle.

$$f(x, \nu) = \begin{bmatrix} X/m - qw + rv \\ Y/m - ru + pw \\ Z/m + qU - pv \\ L/I_x \\ rp + M/I_y \\ -pq + N/I_z \end{bmatrix} \qquad (2)$$

m is the missile mass, g the acceleration due to gravity and I_x, I_y and I_z are the moments of inertia about the x, y and z axes. Note that time dependencies of mass and moments of inertia are not considered in (2).

In the non-linear model defined above, the rigid body equations are relatively simple because C_{xz} is a plane of symmetry and thus the two products of inertia I_{xy}, I_{yz} are zero. Moreover, the missile has one axis of symmetry (C_x) and it is nearly symmetric about (C_y); thus $I_{yz} \approx I_{zx} \approx I_{xy} \approx 0$.

Moment terms are dominant in affecting the responses of the missile. For this reason, the gravity terms which affect the force vector F are negligible and are not included in (2).

The output of g_0 is $y_g = [y, y_1, y_2]$ (see Fig. 1) where y is the output actually available for control purposes. $\bar{a}_y$ and $\bar{a}_z$ are the measured side and normal accelerations. In fact, the missile accelerations at the centre of gravity are defined as $a = F/m$, but the inertial measurement unit (IMU) is set well forward of the centre of gravity and thus the lateral accelerations actually measured are:

$$\begin{aligned} \bar{a}_y &= a_y + l_{arm}(\dot{r} + pq) \\ \bar{a}_z &= a_z + l_{arm}(-\dot{q} + pr) \end{aligned} \qquad (3)$$

where l_{arm} is the accelerometer moment along the x-axis (i.e. the distance from the accelerometer to the centre of gravity).

The non-linear aerodynamic coefficient vector $C_f = (C_x, C_y, C_z, C_l, C_m, C_n)^T$ is approximately a function of six parameters: Mach number ($M_{ch} := \frac{V}{V_s}$, where V_s denotes the speed of sound), α the angle of attack, β the side-slip angle, and the three components of the control input vector, namely η, ζ and ξ which represent the elevator, rudder and aileron positions respectively. C_f is determined from experimental data which are stored in look-up tables. As a result, the derivatives of C_f may be significantly influenced by the method of interpolation, the number of data points, etc.. To avoid problems, uncertainties in the derivatives of C_f have to be considered when designing the control law.

The forces and moments are non-linearly related to the aerodynamic coefficients as

$$\nu = \bar{q}.S.diag(1, 1, 1, d, d, d)C_f(M_{ch}, \alpha, \beta, \eta, \zeta, \xi) \qquad (4)$$

where $\bar{q} := \frac{1}{2}\rho(z)V^2$ is the dynamic pressure, ρ the density of the air which depends on altitude (z), $V = \sqrt{U^2 + v^2 + w^2}$ total speed, d the missile diameter and $S := \frac{\pi d^2}{4}$ the reference area.

2.2 *Autopilot objectives*

The objective is to design an autopilot which responds as fast as possible to input demands at all points over the flight envelope. Two modes are of particular interest: Skid-to-Turn (STT) and Bank-to-Turn (BTT). In STT mode no constraint is placed on the side slip angle $\beta = arctan(v/U)$

and the lateral acceleration demands are $a_y^{STT} = a_y^{dem}$ and $a_z^{STT} = a_z^{dem}$. In BTT mode a sideslip constraint is placed on the maximum magnitude of β (less than 1.5 deg.). The missile is thus required to fly co-ordinated turns like an aircraft. In BTT mode the side acceleration is required to be small, $a_y^{BTT} = 0$ and the normal acceleration should follow the demand $a_z = -\sqrt{a_y^{dem} a_y^{dem} + a_z^{dem} a_z^{dem}})$. Moreover, in BTT, the roll angle error is defined as $tan(\phi_{error}) = -a_y/a_z$.

2.3 *Linearization*

To apply the control technique, the non-linear missile dynamics are linearized. The missile equations have been linearized at certain flight conditions (Mach number and altitude). To simplify the design procedure, the dependence of the aerodynamic derivatives on side-slip angle, angle of attack and control deflection are not considered. The aerodynamic coefficient variations will be taken into account by designing a sufficiently robust autopilot. Mass and inertia variations are neglected in the linear models.

The state space vector governing the rigid body is

$$x = [w, q, v, r, \phi, p]^T \tag{5}$$

The input vector is $u = [\xi, \eta, \zeta]^T$ (aileron, elevator and rudder angle deflections) and the output vector is $y = [\bar{a}_z, q, \bar{a}_y, r, \phi, p]^T$.

The dynamics of the actuators and sensors are taken into account in linear models as well as flexure dynamics which corrupt measured accelerations and body rates. Actuators are modelled as second order systems with a bandwidth of 188rad/s and a damping of 0.7. Similarly each sensor has a bandwidth of 628 rad/s and a damping of 0.7. Corruption of the measured lateral accelerations, pitch and yaw rates due to the fin forces can be described as second order functions. For example, the normal acceleration measurement is corrupted by the normal fin force according to

$$\frac{a_{zfin}}{F_{zfin}} = \frac{mks^2}{s^2 + 2\xi_{flex}\omega_{flex} + \omega_{flex}^2} \tag{6}$$

with lateral flexure frequency of 240 rad/s and damping 0.005 and where m denotes the missile mass. Therefore, the linearized model of the missile is of 26th order.

The singular value plot of the open-loop linearized model about Mach=2 and $x_0 = [150, 0, 10, 0, 0, 5]^T$ is given in figure 2. This operating point corresponds to an angle of attack of $\alpha = 12.5deg$ and a side-slip angle $\beta = 0.8deg$. The linearized plant is stable but exhibits badly damped modes:

- flexure modes at 240 rad/s
- badly damped mode at 15rad/s with damping of 0.09.
- badly damped mode at 62 rad/s with a damping of 0.017.

The plant is badly conditioned at all frequencies.

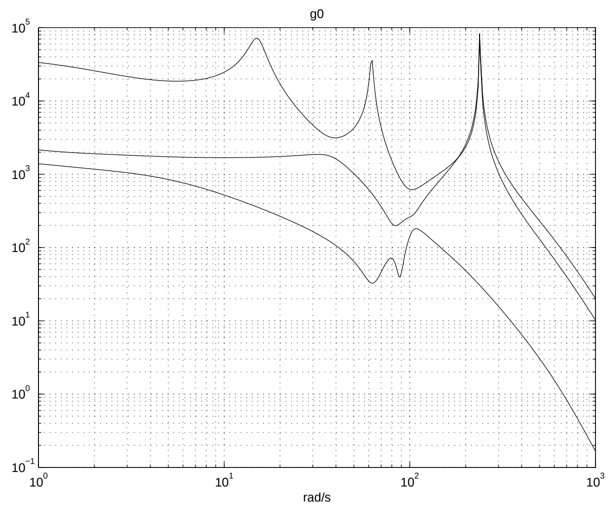

Fig. 2. Singular value plot of the missile linearized about Mach=2, $\alpha = 12.5$ deg, $\beta = 0.8$, $\rho = 1.1\ kg/m^3$

3. CONTROL LAW DESIGN USING THE GLOVER MACFARLANE APPROACH

The design of the controller follows a classic approach: controllers are designed for linearised plants at frozen points and then scheduled together via linear interpolation. In this application, only two operating points are considered which depend on the pressure situations. The scheduling variable is thus the dynamic pressure. The control law is designed using the Glover MacFarlane approach. The first linearized model corresponds to a high pressure situation.

For this operating point the weights were chosen as

$$W_1 = diag(.43, 1.3, 0.87) \tag{7}$$

$$W_2 = diag(w_{21}, .07.w_{23}, 0.07, w_{25}, 0.87) \tag{8}$$

with

$$w_{21} = w_{25} = .0007\frac{s+40}{s} \tag{9}$$

$$w_{25} = .1\frac{s+40}{s} \tag{10}$$

The reader is refereed to e.g. (Skogestad and Postlethwaite, 1996) for comprehensive introduction about the method and the weighting function selection.

The corresponding closed-loop sensitivity functions are given in figure 3. S_i, T_i are the sensitivity and the complementary sensitivity functions at the plant input and S_o is the sensitivity function at the plant output.

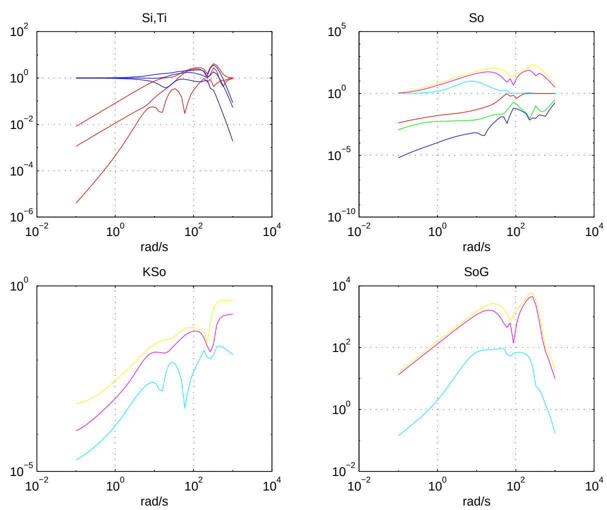

Fig. 3. Singular value plot of the sensitivity functions

3.1 *Low pressure design*

Another design is proposed for an altitude of 10km. The weighting functions for this design are

$$W_1 = diag(1, 3, 2) \qquad (11)$$

$$W_2 = diag(w_{21}, .07.w_{23}, 0.07, w_{25}, 0.87) \qquad (12)$$

with

$$w_{21} = w_{25} = .0007\frac{s+40}{s} \qquad (13)$$

$$w_{25} = .1\frac{s+50}{s} \qquad (14)$$

Essentially the same weights are used excepted for W_1.

4. NON-LINEAR SIMULATIONS

The performance and robustness of the designed autopilot are assessed on the non-linear model for two modes of operation: BTT180 (positive pitch incidence) and STT. Robustness with respect to the aerodynamic derivative variations could have been investigated with μ analysis see e.g. (Ferreres *et al.*, 1996). However, while very useful these tools may be misleading in presence of time varying uncertainties. Thus, in any cases, it is necessary to assess the autopilot on a full non-linear model.

In the non-linear model, the aerodynamic coefficients are interpolated from look-up-tables and take into account the current value of angle of attack, side slip angle and control surface deflection. Actuator models including rate limits and deflection limits, as well as measurement noise are included.

Figure 4 and 5 show the missile responses in BTT and STT modes of operation respectively for large accelerations demands. We can see that the autopilot allows fast and accurate lateral acceleration demands. One can note that the roll angle error response is quite small in STT mode. However, in BTT mode, the roll angle error response is a little bit slow. Ideally the dynamic of the roll channel should be close to the lateral accelerations dynamics. It was found difficult to speed up the roll channel without affecting the robustness of the closed loop. Responses in this channel can be, however, improved by using a two degree of freedom controller. Note that, in BTT mode, the side-slip constraint is respected and the actuators never enter into saturation for large demands.

Figures 6 and 7 show the missile responses in the BTT and STT modes respectively when the missile is operating at high altitude. The responses are satisfactory excepted for the response of the roll angle error in BTT mode (same as before).

We can note that the autopilot has a tendency to produce less damped responses for small incidences rather than for higher incidences. This is due to anomalies in the aerodynamic data at very low incidence.

The responses of the interpolated controller for an altitude of 5km are shown in figure 8. Again, acceleration responses are satisfactory.

Remarks

- The non-linear simulations demonstrate good robustness properties of the autopilot with respect to aerodynamic uncertainties.
- An LMI/LPV design conducted on the two augmented plants corresponding to the previous designs returns an excessively big H_∞ cost with a significant deterioration in the linear performance (high control (Prempain *et al.*, 2001)). In a non-linear environment, the plant enters immediately into saturation and instability follows.

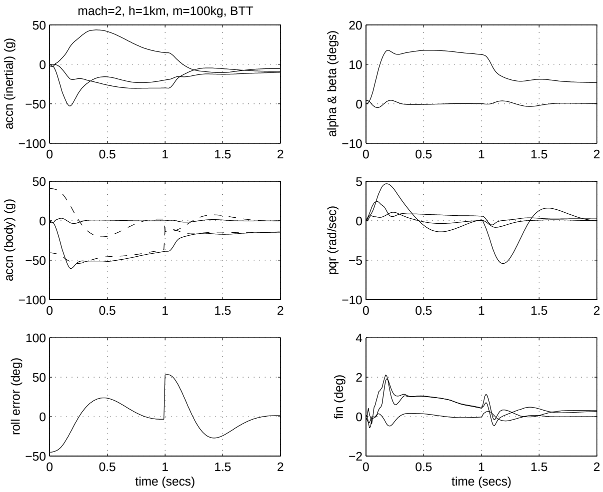

Fig. 4. Non-linear responses of the missile autopilot. Mach=2, 1km altitude. BTT mode. Large demands.

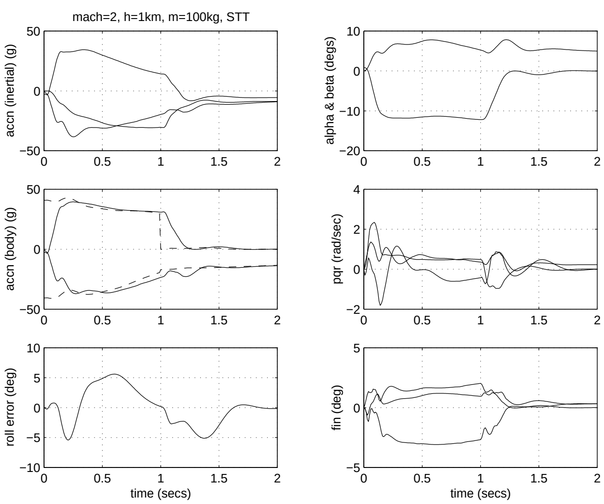

Fig. 5. Non-linear responses of the missile autopilot. Mach=2, 1km altitude. STT mode. Large demands.

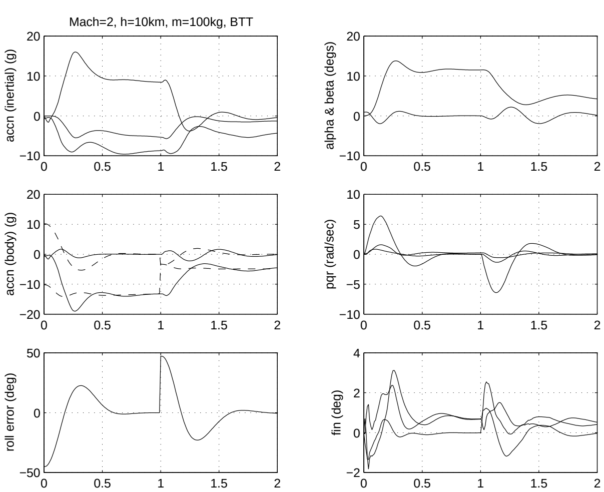

Fig. 6. Non-linear responses of the missile autopilot. Mach=2, 10km altitude. BTT mode. Small demands.

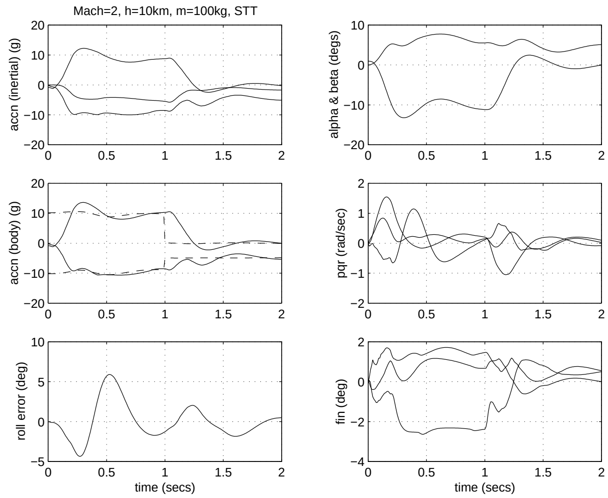

Fig. 7. Non-linear responses of the missile autopilot. Mach=2, 10km altitude. STT mode. Small demands.

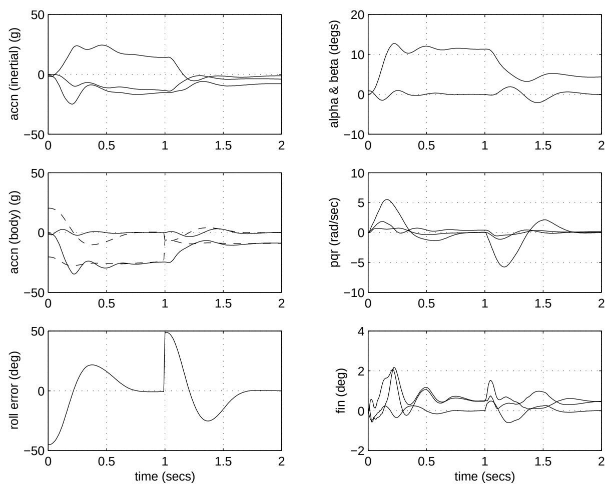

Fig. 8. Non-linear responses of the missile autopilot. Mach=2, 5km altitude. BTT mode.

Non-linear simulations demonstrated that only few LTI designs are sufficient to cover a large part of the flight envelope. The scheduling variables we have to consider are essentially the dynamic pressure and the longitudinal speed. The design demonstrated that it may not be useful, in many situations, to schedule the controller with respect to the angle of attack providing that the angle of attack is relatively small (less than 25 deg.). This contrast with (Papageorgiou and Glover, 2000).

5. CONCLUSIONS

This paper has presented an interpolated autopilot design for an asymmetric missile. A Glover-MacFarlane H_∞ type design was conducted on linearized models of the missile at two dynamic pressure situations. The nonlinear performance of the interpolated regulator for frozen values was found to be acceptable and the autopilot seemed to be robust to uncertainties in the aerodynamic derivatives. This study demonstrates that a classical gain-scheduling approach can be still seen as an effective way to handle autopilot designs. The results presented here are much better than a previous LPV design we had carried out on the same plant. In future work, it might be beneficial to make use of a more sophisticated two degree of freedom control structure to improve performance especially in the roll channel.

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