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ON LINE OPTIMAL CONTROL FOR BIPED ROBOTS

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Abstract: This paper deals with the control of walking biped robots. An on line optimal control approach based on moving horizon strategy is developed. The problem is stated as an optimization one subject to physical coherent constraints issued from human behavior observation. The strategy is tested with a model of the BIP robot designed by INRIA.

Keywords: biped robot, optimal control, moving horizon, walking gait

1. INTRODUCTION

Receeding horizon control approaches proved their efficiency in a large variety of processes. Due to their ability to handle multivariable constraint problems through on line optimization, they seem a very interesting way of controlling biped robot dedicated to navigate in rough terrain and subject to unilateral constraints such as the ground contacts. In the past decade, model based predictive control for nonlinear system have been used very successfully in industry, especially on refining and chemical processes (Allgöwer F., 1999), (Nevistic V., 1996). More recently Richalet and al (Richalet J., 1997) have implemented their predictive controller on a KUKA robot with an approximated linear model of the robot. In (Poignet P., 2000), a predictive controller combined with a feedback linearization is compared to the computed torque control on a SCARA robot.

The major contribution of this work is to propose a model based technique with a moving horizon. Here, on line optimal control is performed considering the dynamic model of the BIP robot (fig.1) designed by INRIA (Azevedo C., 2000). Most of the approaches are based on two stages i) walking patterns synthesis and ii) control of the biped robot along these patterns (Huang Q., 2001). On the contrary, in the presented approach, the trajectory is not planned. Physical robot coherent constraints are added to the optimization problem and environment is also described as constraints. These constraints will ensure the biped motion through the minimization of a quadratic cost function involving the actuator torques.

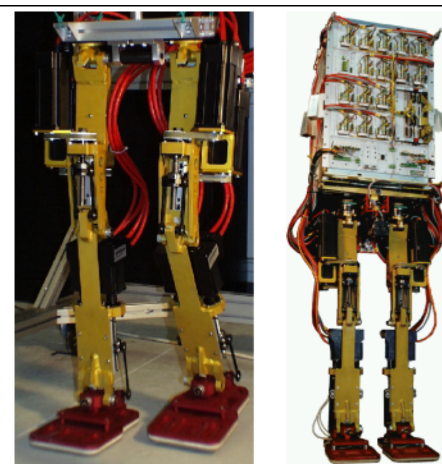

Fig. 1. *The INRIA biped robot: BIP*

The paper is organized as follows: section 2 presents the nonlinear dynamics biped robot modeling during the different gait phases and the physical constraints,

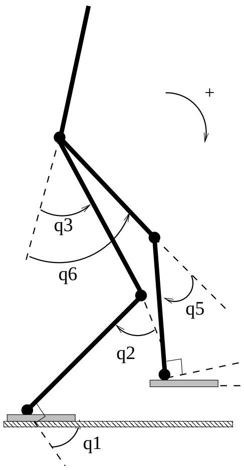

Fig. 2. *BIP 6 dof planar model*

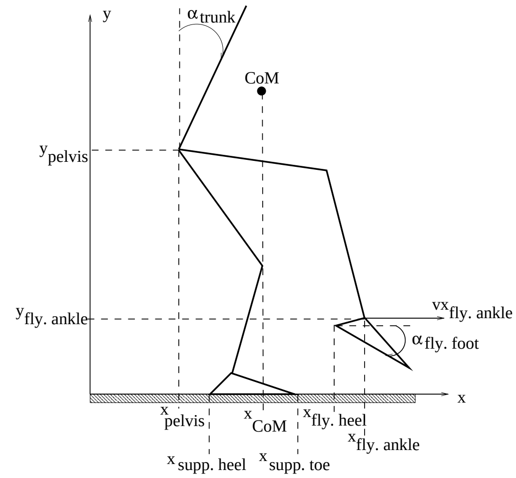

Fig. 3. *Notations used in the paper*

section 3 details the finite horizon nonlinear model predictive control. Finally section 4 exhibits the major numerical results obtained with different types of steps and thrust disturbances.

2. PROBLEM STATEMENT

In this paper, we consider a planar biped model with 7 rigid moving links connected to 6 purely rotational joints. Each joint is actuated (fig.2). We study the part of the walking cycle which lies in the sagittal plane and we assume that the system has always at least one feet in contact with the ground. The biped robot is a varying kinematic chain structure (Chessé S., 2001). This chain is successively opened or closed when tips of open kinematic chains come in contact with the supporting floor. It is also supposed that the gait consists in a succession of single support phases. As in (Grizzle J.W., 1999), the duration of the double support phase is assumed to be negligible. Finally, the biped model consists of two parts: the differential equations describing the dynamics during the swing phase, and an impulse model of the contact event for computing the velocity vector after impact.

2.1 Single support phase

During the swing phase, the stance foot is modeled as a pivot fixed on the ground. Using the method of Lagrange, the dynamic model of the robot can be written as (Spong M.W., 1989):

$$M(q)\ddot{q} + N(q,\dot{q})\dot{q} + G(q) = \Gamma \qquad (1)$$

This equation provides with the joint torques vector Γ as a function of the joint positions vector (fig.2) $q = (q_1, q_2, q_3, q_4, q_5, q_6)^T$ and joint velocity vector $\dot{q}$, with M, N and G respectively the inertia matrix, the Coriolis matrix and the gravity vector.

2.2 Impact Equation

During collision, the contact of the foot with the ground consists in unilateral constraint without sliding (Pfeiffer F., 1996). The foot is kept in the horizontal position. The impact is considered as occurring at the ankle. The condition of the geometric closure is given by:

$$\phi(q) = \begin{pmatrix} y_{fly.\ ankle} \\ x_{fly.\ ankle} - (x_{supp.\ ankle} + L_s) \end{pmatrix} = 0 \qquad (2)$$

where:
- $(x_{supp.\ ankle}, y_{supp.\ ankle})$ are the Cartesian coordinates of the ankle of the stance leg (fig.3),
- $(x_{fly.\ ankle}, y_{fly.\ ankle})$ are the Cartesian coordinates of the swing leg (fig.3),
- L_s is the step length.

By derivating ϕ:

$$\dot{\phi}(q) = \frac{\partial \phi(q)}{\partial q}\frac{\partial q}{\partial t} = 0 \qquad (3)$$

Let's $C(q)$ be the Jacobian of $\phi(q)$:

$$C(q) = \frac{\partial \phi(q)}{\partial q} = \begin{pmatrix} C_n(q) \\ C_t(q) \end{pmatrix} \qquad (4)$$

where $C_n(q)$ is the normal component and $C_t(q)$ the tangential component.
When the end point of the kinematic chain enters in contact with the ground, velocity vector suddenly changes (Wieber P.B., 2000), (Roussel L., 1998), (Zheng Y.F., 1984). Using the conservation of linear and angular impulse and momentum for the entire chain, one can write the impulsive dynamic model:

$$M(q)(\dot{q}^+ - \dot{q}^-) = C(q)^T \Lambda \qquad (5)$$

$\dot{q}^-$ and $\dot{q}^+$ are respectively the velocity of the joints before and after impact. From equation (5), $\dot{q}^+$ is given by:

$$\dot{q}^+ = \dot{q}^- + M^{-1}(q)C(q)^T \Lambda \qquad (6)$$

where $\dot{q}^-$, $M(q)$ and $C(q)$ are supposed to be known. Assuming that the motion is achieved without slipping or rebounding, equation (6) is supplemented by equation (3) written after impact:

$$\begin{cases} C_t(q)\dot{q}^+ = 0 \\ C_n(q)\dot{q}^+ = 0 \end{cases} \tag{7}$$

The Lagrangian Λ can then be computed by replacing equation (7) into equation (6):

$$\Lambda = -(C(q)M(q)^{-1}C(q)^T)^{-1}C(q)\dot{q}^- \tag{8}$$

Finally, equations (1), (4), (6) and (8) describe the dynamic model over the whole gait phases and will be used for the simulations.

2.3 *Physical dynamic constraints*

In addition to the classical robot constraints such as joint and torque limitations, coherent physical constraints are deduced by observing the human behavior. In (Hurmuzlu Y., 1993*a*), (Hurmuzlu Y., 1993*b*), the body erection posture and the overall progression are prescribed to characterize the motion. In (Patla A., 1991), a successful human locomotion is stated in 3 points: i) the production of a basic locomotor rhythm, ii) the control of the equilibrium and iii) the adaptation of the movements to meet the environmental demands and goals. In (Pratt J., 1998), the conditions for a planar robot to walk are defined in terms of height, pitch and speed stabilization. The role of the swing leg is presented as important to ensure the stabilization.

According to these intuitive but physical considerations, a set of equality and inequality constraints listed below has been established (fig.3):

• **Robot internal constraints:**

- Actuator torque limitations:

$$\Gamma_{min} \leq \Gamma \leq \Gamma_{max}$$

- Joint position limitations:

$$q_{min} \leq q \leq q_{max}$$

• **Erected Body Posture**

The biped should maintain an erected posture during locomotion. This condition is guaranteed by a constraint on the pelvis height:

$$y_{pelvis} \geq h_{pelvis_{min}}$$

• **Overall Progression Velocity**

The overall progression velocity is defined as the ankle speed of the flying foot in the positive x-direction.

$$v_{x_{min}} \leq v_{x_{fly.\ ankle}} \leq v_{x_{max}}$$

• **Static stability**

In this study, with the manipulator based model described in section 2, the locomotion we can consider is limited to static walking. The static stability is guaranteed by maintaining the projection of the center of mass within the soil of the support foot:

$$x_{supp.\ heel} \leq x_{CoM} \leq x_{supp.\ toe}$$

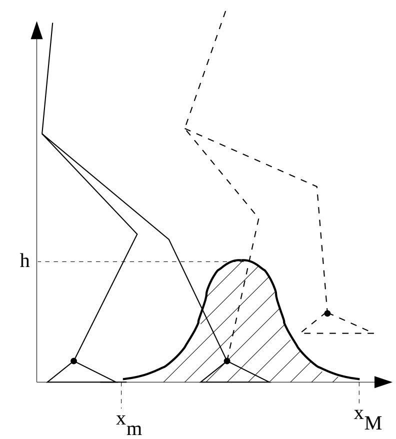

Fig. 4. *Constraints on the flying ankle position*

As the weight is mainly disposed in the trunk, a constraint on its inclination guarantees that it does not bend back and limits the forward bending:

$$0 \leq \alpha_{trunk} \leq \alpha_{max}$$

Additional constraint maintains the pelvis *x*-position within the heel of the flying foot and the toes of the support foot:

$$x_{fly.\ heel} \leq x_{pelvis} \leq x_{supp.\ toe}$$

It facilitates the support foot transition at the impact. As the pelvis *x*-position and the projection of the center of mass are located closely to in the BIP case, this constraint will also contribute to the static stability.

• **Trajectory of the swing limbs**

The swing limbs have to be lifted off the ground at the beginning of a step and landed back at the end of the step. During the step, the flying foot has to stay clear off the surface to avoid contact. Limit for the *y*-coordinates of the ankle (fig.3) is given by:

$$f_m(x_{fly.\ ankle}) \leq y_{fly.\ ankle} \leq f_M(x_{fly.\ ankle})$$

where f_m and f_M are chosen as 4^{th} order polynomial functions depending of the desired minimum tip clearance h, and the minimum step length $x_M - x_m$ (fig.4). The flying foot is maintained in the horizontal position:

$$\alpha_{fly.\ foot} = 0$$

3. FINITE HORIZON NONLINEAR MODEL PREDICTIVE CONTROL

Let's consider the class of nonlinear systems described by the following set of equations:

$$x_{k+1} = f(x_k, u_k) \tag{9}$$

with u_k and x_k respectively the input and the state of the system. The nonlinear model predictive control

(NMPC) (Allgöwer F., 1999) open loop optimization problem that is solved at each time is given by:

$$\min_{u_k^{N_c}} J(x_k, u_k^{N_c}) \tag{10}$$

subject to:

$$\begin{array}{l} x_{l+1|k} = f(x_{l|k}, u_{l|k}) \\ x_{0|k} = x_k \\ u_{l|k} \in U, l \in [0, N_c - 1] \\ x_{l|k} \in X, l \in [0, N_c] \end{array} \tag{11}$$

with:

$$J(x_k, u_k^{N_c}) = \Phi(x_{N_c|k}) + \sum_{i=0}^{N_c-1} L(x_{i|k}, u_{i|k}) \tag{12}$$

and:

$$\begin{array}{l} U := \{u_k \in R^m | u_{min} \leq u_k \leq u_{max}\} \\ X := \{x_k \in R^n | x_{min} \leq x_k \leq x_{max}\} \end{array} \tag{13}$$

Usually the stage cost L is a quadratic function in x and u:

$$L(x_{i|k}, u_{i|k}) = x_{i|k}^T Q x_{i|k} + u_{i|k}^T R u_{i|k} \tag{14}$$

In case of the biped robot, the criterion J is chosen as:

$$J(u_k^{N_c}) = \sum_{i=0}^{N_c-1} \Gamma_{i|k}^T \Gamma_{i|k} \tag{15}$$

where $\{\Gamma_{i|k}, i \in [0, ..., N_c - 1]\}$ are the actuator torque vectors over the receeding horizon and

$$u_k^{N_c} = \left[\Gamma_{0|k}, ..., \Gamma_{N_c-1|k}\right]$$

By neglecting the Coriolis effect, the nonlinear system (eq.1) is written as:

$$\ddot{q}_{l+1|k} = M^{-1}(q_{l|k}) G(q_{l|k}) + M^{-1}(q_{l|k}) \Gamma_{l|k} \tag{16}$$

At each time instance k, the initial conditions are:

$$x_{0|k} = \begin{bmatrix} q_{0|k} \\ \dot{q}_{0|k} \end{bmatrix} = x_k = \begin{bmatrix} q_k \\ \dot{q}_k \end{bmatrix} \tag{17}$$

Finally our optimization problem is resumed as:

$$\min_{u_k^{N_c}} J(u_k^{N_c}) = \min_{u_k^{N_c}} \sum_{i=0}^{N_c-1} \Gamma_{i|k}^T \Gamma_{i|k} \tag{18}$$

subject to:

$$\begin{array}{l} \ddot{q}_{l+1|k} = M^{-1}(q_{l|k}) G(q_{l|k}) + M^{-1}(q_{l|k}) \Gamma_{l|k} \\ \Gamma_{min} \leq \Gamma_{l|k} \leq \Gamma_{max} \\ q_{min} \leq q_{l|k} \leq q_{max} \\ h_{pelvis_{min}} \leq y_{pelvis_{l|k}} \\ v_{x_{min}} \leq v_{x_{fly.\ ankle_{l|k}}} \leq v_{x_{max}} \\ x_{supp.\ heel} \leq x_{CoM_{l|k}} \leq x_{supp.\ toe} \\ 0 \leq \alpha_{trunk_{l|k}} \leq \alpha_{max} \\ x_{fly.\ heel_{l|k}} \leq x_{pelvis_{l|k}} \leq x_{supp.\ toe_{l|k}} \\ f_m(x_{fly.\ ankle_{l|k}}) \leq y_{fly.\ ankle_{l|k}} \leq f_M(x_{fly.\ ankle_{l|k}}) \\ \alpha_{fly.\ foot_{l|k}} = 0 \end{array} \tag{19}$$

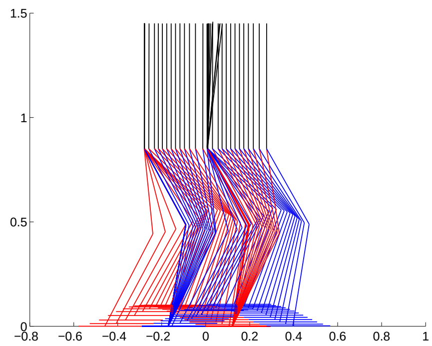

Fig. 6. *Two steps with same height (plot every 100ms)*

At each time instance k, we apply the first element of the solution sequence $u_k^{N_c*}$ of the optimal open loop control problem.

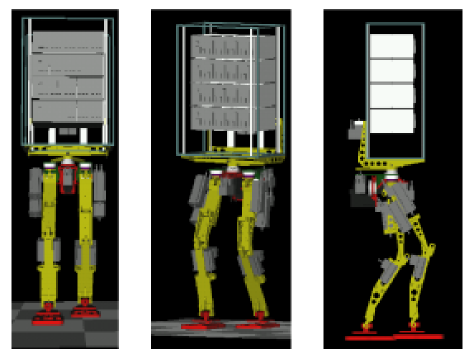

Fig. 5. *Views of the initial position used in simulations*

4. NUMERICAL RESULTS

The optimization problem is solved using the *fmincon* [1] function of the Matlab Optimization Toolbox dedicated to the minimization of a constrained nonlinear multivariable function. *fmincon* is based on the sequential quadratic programming (SQP) algorithm. SQP is an iterative technique in which the objective is replaced by a quadratic approximation and the constraints by linear approximations (Allgöwer F., 1999). The BIP dynamic (fig.1) model is used for the simulations. Detailed parameters can be founded in (Espiau B., 2000).

Motion parameters are given by: $h_{pelvis_{min}} = 0.85$m, $v_{x_{min}} = 0.2 m.s^{-1}$, $v_{x_{max}} = 0.6 m.s^{-1}$, $\alpha_{max} = 0.2$rad. The simulation results are computed with a sampling period of 0.01s. The moving control horizon corresponds to 5 sampling periods. Fig.5 shows different views of the initial position.

The simulation of two consecutive steps (fig.6) is performed by constraining the minimum height of the flying foot to 10cm. The trajectory of the flying ankle is shown on figure 7. The average velocity of the flying foot is of 0.93$km.h^{-1}$ and the computation time for

[1] http://www.mathworks.com/products/matlab

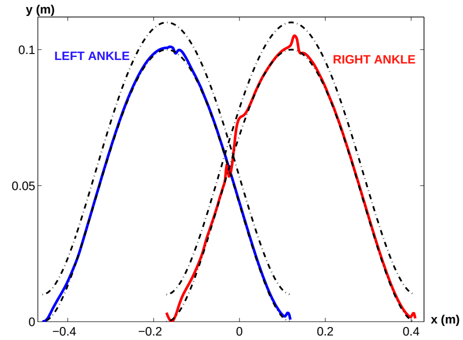

Fig. 7. *Trajectory of the flying ankle*

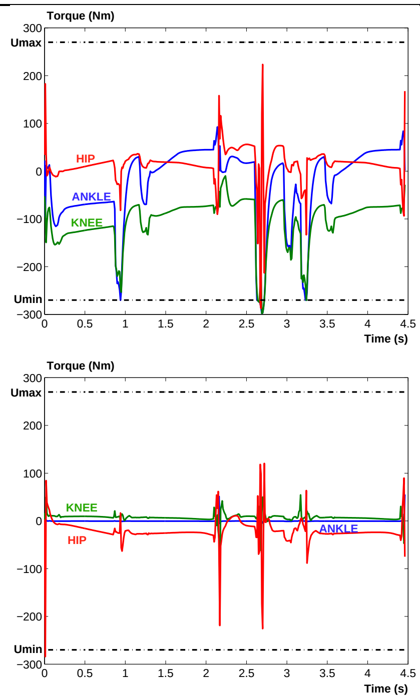

Fig. 8. *Joint Torques* Γ *(support and swing legs)*

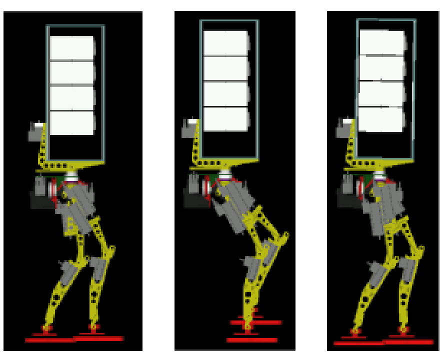

Fig. 9. *3D Simulation of one step*

one sampling period of simulation is $31s$. In figure 8, the upper figure shows the joint torques of the stance limbs and the lower figure presents the torques of the flying leg. Figure 9 presents a 3D simulation of one step.

The simulation of two consecutive steps with different heights (fig.10) exhibits the ability of the control strategy to negotiate on line obstacles or to match the roughness of terrain. The minimum step height is $2cm$ for the first one and $10cm$ for the second one (fig.11). The average velocity of the flying foot is $1.04km.h^{-1}$ and the computation time for one sampling period of simulation is of $37.08s$.

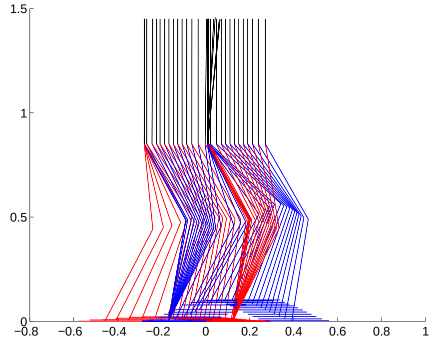

Fig. 10. *Two steps of different heights (plot every 100ms)*

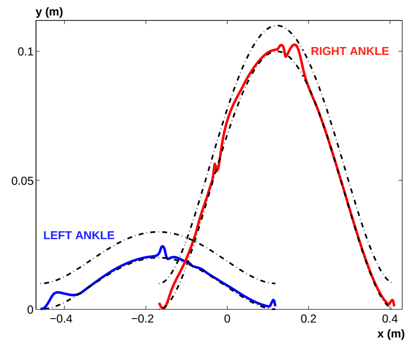

Fig. 11. *Trajectory of the flying ankle*

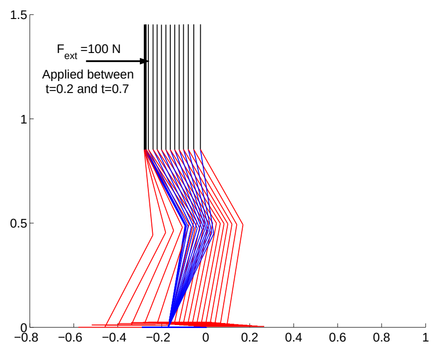

Fig. 12. *Thrust disturbance* $F_{ext} = 100N$

The simulation of an unknown and unmeasured thrust disturbance applied in the back of the trunk at the middle height during a step (fig.12) exhibits the robustness of the control law. The simulation is performed with a thrust equal to $F_{ext} = 100N$ applied during 0.5s. In figure 13, the upper figure shows the joint torques of the stance limbs and the lower figure presents the torques of the flying leg.

5. CONCLUSION

In this paper, we present a new control approach for walking biped robots. The strategy is developed with

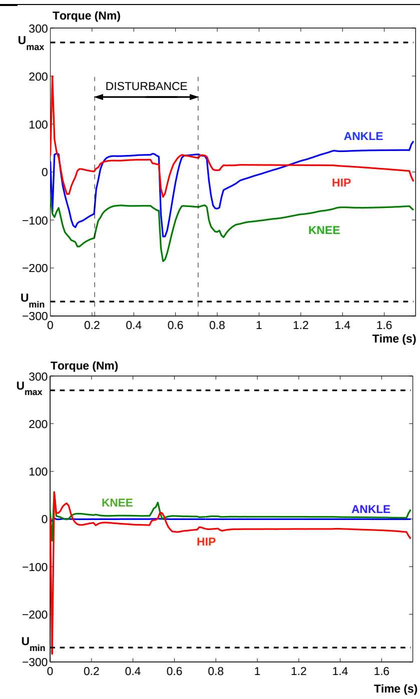

Fig. 13. *Joint torques* Γ *(support and swing legs)*

an on line optimal computation over a receeding horizon using physical constraints ensuring the biped motion.

Simulations exhibit efficient results in case of a biped walking on a rough terrain or subject to a thrust disturbance. The key advantage of the proposed technique is that we do not need any reference trajectory.

The manipulator based model used in this paper limits the motion to static walking. Indeed, we are currently developing a moving reference base model, which will allow us to consider dynamic walking taking into account the support forces. We are also investigating solutions to reduce the computation times in order to implement real-time strategy on BIP robot.

REFERENCES

Allgöwer F., Badgwell T.A., Qin J.S. Rawlings J.B.-Wright S.J. (1999). Nonlinear predictive control and moving horizon estimation - an introductory overview. *Advances in control. Highlights of ECC'99, Springer* pp. 392–449.

Azevedo C., the Bip team (2000). Control architecture and alghorithms of the anthropomorphic biped bip2000. *Proc. of International Symposium on Climbing and Walking Robots* pp. 285–293.

Chessé S., Bessonnet G. (2001). Optimal dynamics of constrained multibody systems - application to bipedal walking synthesis. *International Conference on Robotics and Automation IEEE* pp. 2499–2505.

Espiau B., Sardain P. (2000). The anthropomorphic biped robot bip2000. *International Conference on Robotics and Automation*.

Grizzle J.W., Abba. G., Plestan. F. (1999). Proving asymptotic stability of a walking cycle for a five dof biped robot model. *Conference on Walking and Climbing Robots*.

Huang Q., Yokoi K., Kajita S. Kaneko K.-Arai H. Koyachi N. Tanie K. (2001). Planning walking patterns for a biped robot. *IEEE Transactions on Robotics and Automation* **17**, 280–289.

Hurmuzlu Y. (1993*a*). Dynamics of bipedal gait: Part1- objective functions and the contact event of a planar five–link biped. *Journal Of applied Mechanics* **60**, 331–336.

Hurmuzlu Y. (1993*b*). Dynamics of bipedal gait: Part2- stability analysis of a planar five-link biped. *Journal Of applied Mechanics* **60**, 337–343.

Nevistic V., Morari M. (1996). Robustness of mpc based schemes for constrained control of nonlinear systems. *13th IFAC Triennal World Congress* pp. 25–30.

Patla A. (1991). Understanding the control of human locomotion: A prologue. *Adaptability Of Human Gait, Elsevier Science Publishers*.

Pfeiffer F., Glocker C. (1996). *Multibody Dynamics With Unilateral Contacts*. Wiley-Interscience Publication, John Wiley and Sons Inc., New York, NY, USA.

Poignet P., Gautier M. (2000). Nonlinear model predictive control of a robot manipulator. *6th International Workshop on Advanced Motion Control* pp. 401–406.

Pratt J., Pratt G. (1998). Intuitive control of a planar bipedal walking robot. *IEEE International Conference on Robotics and Automation*.

Richalet J., Arber C., Kuntze H.B. AbuElAta-Doss S. Jacubasch A. Schill W. (1997). Predictive functional control.application to fast and accurate robots. *10th IFAC World Congress*.

Roussel L. (1998). Génération de trajectoires de marche optimales pour un robot bipède. Thèse de doctorat. INPG.

Spong M.W., Vidyasagar M. (1989). *Robot Dynamics And Control*. John Wiley and Sons Inc., New York, NY, USA.

Wieber P.B. (2000). Modélisation et Commande d'un Robot marcheur Anthropomorphe. Thèse de doctorat.

Zheng Y.F., Hemami H. (1984). Impact effects of biped contact with the environment. *IEEE Trans. Syst. Man Cyber.* **14-3**, 437–443.