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DESIGN OF RECEDING-HORIZON FILTERS FOR DISCRETE-TIME LINEAR SYSTEMS USING QUADRATIC BOUNDEDNESS

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Abstract: State estimation for discrete-time linear systems is addressed by developing a filter that provides an estimate of the state depending only on a batch of recent measurement and input vectors. This problem has been solved by introducing a receding-horizon objective function that includes also a weighted penalty term related to the prediction of the state. Convergence results and unbiasedness properties have been proved for this estimator in a previous work. In this paper, the focus is on the problem of designing such a filter using results related to quadratic boundedness of the estimation error. Upper bounds on the norm of the estimation error have been found by constructing a suitable positively invariant set. Moreover, these bounds may be expressed in terms of Linear Matrix Inequalities (LMIs), and are well-suited to being minimized for the purpose of design. *Copyright ©2002 IFAC*

Keywords: state estimation; filtering; receding horizon; quadratic boundedness; Linear Matrix Inequalities.

1. INTRODUCTION

Receding-horizon estimation for discrete-time linear systems has been objective of investigations from the pioneering work of Jazwinski (1968). A receding-horizon estimator is well-suited in the presence of modeling uncertainties or numerical errors as, for example, in identification problems (Niedźwiecki and Guo, 1991; Houacine, 1992). Moreover, in the area of fault diagnosis, receding-horizon filters are used for the purpose of residual generation as reported, among others, in (Medvedev, 1998). Most recent results regard the application of such methodologies to constrained estimation [see (Rao *et al*, 2001)].

In this paper, we assume the system state and measurement equations to be linear, time-invariant, affected by additive norm-bounded disturbances that take their values from known compact sets. A generalized least-squares approach is considered that consists in minimizing a quadratic estimation cost function defined on a sliding window composed of a finite number of time stages (Alessandri *et al*, 1999). The cost function is made of two contributions: the first one is the usual prediction error computed on the basis of the more recent measurements; the second one is a weighted term penalizing the distance of the current estimated state from its prediction (both computed at the beginning of the sliding window).

The scalar weight is given by a positive scalar parameter. This cost function is more general with respect to the standard least-square method and the trade-off between the two components of the cost can be tuned by suitably selecting the scalar weight. The receding-horizon estimator has been derived by imposing the necessary condition on the minimization of the cost function, which can be solved analytically for linear systems (Alessandri *et al.*, 2001). If the system and measurement noises are absent, this filter reduces to an observer whose estimation error is exponentially convergent to zero. If disturbances affect the system and measurement equations and are regarded as random variables together with the initial state, the resulting filter is asymptotically unbiased.

The focus of this work is on the problem of finding upper bounds on the estimation error. To this end, in Section 2, the results on quadratic boundedness for continous-time linear systems [see, among others, (Brockman and Corless, 1998)] have been extended to discrete-time linear systems. Section 3 is devoted to the description of the receding-horizon estimator. The design of the estimator is considered in Section 4, where, using the results of Section 2, a method is proposed to minimize an upper bound on the estimation error. This problem can be easily expressed in terms of LMIs, which enable one to solve it by means of efficient convex programming algorithms [the reader is referred to (Boyd *et al.*, 1994) for an introduction on this subject]. Finally, a simulation example is given in Section 5.

We conclude this section with some notations we use throughout this paper. Given a generic, symmetric, positive definite matrix S, let us denote by $\underline{\lambda}(S)$ and $\bar{\lambda}(S)$ the minimum and maximum eigenvalues of S, respectively. The superscript $'$ indicates matrix transpose. For a generic vector v, let us define $v_{t-N}^t \triangleq \text{col}\,(v_{t-N}, v_{t-N+1}, \ldots, v_t)$. Furthermore, given a generic matrix M, $\|M\|_{\max} \triangleq \|M\| = [\bar{\lambda}(M'M)]^{1/2}$ and $\|M\|_{\min} \triangleq [\underline{\lambda}(M'M)]^{1/2}$.

2. QUADRATIC BOUNDEDNESS FOR DISCRETE-TIME LINEAR SYSTEMS

Consider an uncertain dynamical system described by the state equation

$$z_{t+1} = Az_t + Gw_t\,, \quad t = 0, 1, \ldots \qquad (1)$$

where $z_t \in \mathbb{R}^n$ and $w_t \in \mathbb{R}^l$ are the state and the noise vector, respectively. We assume that the noise vector belongs to a compact set $\Omega \subset \mathbb{R}^l$.

Definition 1. System (1) is quadratically bounded with Lyapunov matrix P if

(i) P is a symmetric positive definite matrix;
(ii) $z \in \mathbb{R}^n$, $z'Pz \geq 1$ implies $(Az + Gw)'\,P\,(Az + Gw) \leq z'Pz\,, \quad \forall w \in \Omega$.

If, instead, $z \in \mathbb{R}^n$, $z'Pz > 1$ implies $(Az + Gw)'\,P\,(Az + Gw) < z'Pz\,, \quad \forall w \in \Omega$, then system (1) is said to be strictly quadratically bounded with Lyapunov matrix P.

Remark 1. Consider the function $V(z) \triangleq z'Pz$. Strict quadratic boundedness of system (1) ensures that the function $V(z_t)$ decreases, i.e., $V(z_{t+1}) < V(z_t)$, for any possible value of the system noise when $V(z_t)$ is greater than 1.

Proposition 1. If there is at least one vector $w \in \Omega$ such that $Gw \neq 0$ (i.e., we are not in the trivial case), then the following facts are equivalent:

(i) system (1) is quadratically bounded with Lyapunov matrix P;
(ii) system (1) is strictly quadratically bounded with Lyapunov matrix P.

□

Definition 2. The set S is a robustly positively invariant set for system (1) if $z \in S$ implies $Az + Gw \in S\,, \quad \forall w \in \Omega$.

Theorem 1. The following facts are equivalent:

(i) system (1) is quadratically bounded with Lyapunov matrix P;
(ii) ellipsoid $\varepsilon_P \triangleq \{z \in \mathbb{R}^n : z'Pz \leq 1\}$ is a robustly positively invariant set for system (1).

□

Theorem 2. Suppose that the noise vector belongs to the compact set

$$\varepsilon_Q = \left\{ w \in \mathbb{R}^l : \; w'Qw \leq 1\,, \; Q > 0 \right\}\,,$$

then system (1) is quadratically bounded with Lyapunov matrix P if and only if there exists $\alpha \geq 0$ such that

$$\begin{bmatrix} A'PA - P + \alpha P & A'PG \\ G'PA & G'PG - \alpha Q \end{bmatrix} \leq 0\,. \qquad (2)$$

□

Remark 2. Clearly, Theorem 1 together with Theorem 2 and Proposition 1 ensures that, if there exist $\alpha \geq 0$ and $P > 0$ such that inequality (2) is verified, then any possible trajectory for system (1) converges to ε_P as $t \to \infty$, i. e., the set ε_P results to be an attractive invariant set for system (1). Moreover, an upper bound on the ultimate value of $\|z_t\|$ is given by $\underline{\lambda}(P)^{-1/2}$ as it turns out that

$$\lim_{t\to\infty} \|z_t\| \le \underline{\lambda}(P)^{-1/2} .$$

Remark 3. It is worth noting that inequality (2) is an LMI in P for a fixed α. Thus, the selection of the parameters may be easily accomplished by means of efficient standard routines [see (Boyd *et al.*, 1994)].

In order to prove Theorem 2 we need the following technical lemma.

Lemma 1. Suppose that the noise vector belongs to the compact set ε_Q, then the following facts are equivalent:

(i) system (1) is quadratically bounded with Lyapunov matrix P;
(ii) $w'Qw \le x'Px$ implies $V(Ax+Gw) - V(x) \le 0$.

□

Proof of Lemma 1. Clearly $(ii \Rightarrow i)$, as $V(x) \ge 1$ and $w'Qw \le 1$ imply $w'Qw \le x'Px$.

$(i \Rightarrow ii)$ Suppose that system (1) is quadratically bounded with Lyapunov matrix P and Fact (ii) does not hold, then there exist x_0 and w_0 such that

$$w_0'Qw_0 \le x_0'Px_0 \tag{3}$$
$$V(Ax_0+Gw_0) - V(x_0) > 0 \tag{4}$$

Clearly $x_0 \ne 0$, as if $x_0 = 0$, then it can be deduced that $w_0 = 0$ from (3) and consequently $V(Ax_0+Gw_0) - V(x_0) = 0$, which contradicts hypothesis (4). Let us now define the following quantities

$$\lambda \triangleq \frac{1}{(x_0'Px_0)^{1/2}}, \quad x_1 \triangleq \lambda x_0, \quad w_1 \triangleq \lambda w_0 .$$

Clearly

$$x_1'Px_1 = 1, \quad w_1'Qw_1 \le 1$$

and

$$V(Ax_1+Gw_1) - V(x_1) = \\ = \lambda^2 V(Ax_0+Gw_0) - \lambda^2 V(x_0) > 0 ,$$

thus proving that system (3) is not quadratically bounded with Lyapunov matrix P.

■

Proof of Theorem 2. If we rewrite the condition stated in Fact (ii) of Lemma 1 as

$$\begin{bmatrix} x \\ w \end{bmatrix}' \begin{bmatrix} -P & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \le 0 \quad \Longrightarrow \tag{5}$$
$$\begin{bmatrix} x \\ w \end{bmatrix}' \begin{bmatrix} A'PA-P & A'PG \\ G'PA & G'PG \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \le 0 ,$$

then we can invoke the S-procedure, see (Boyd *et al.*, 1994) and conclude that (5) is satisfied if and only if

$$\exists\, \alpha \ge 0 : \begin{bmatrix} A'PA-P & A'PG \\ G'PA & G'PG \end{bmatrix} - \alpha \begin{bmatrix} -P & 0 \\ 0 & Q \end{bmatrix} \le 0 .$$

■

For an introduction to set invariance problems, the reader is referred to (Blanchini, 1999). In the next section, these results on quadratic boundedness will be applied to a receding-horizon estimation problem for discrete-time linear systems.

3. RECEDING-HORIZON ESTIMATION FOR DISCRETE-TIME LINEAR SYSTEMS

Let us consider the discrete-time state equation

$$x_{t+1} = A\,x_t + B\,u_t + G_1 w_t , \quad t = 0,1,\ldots \tag{6}$$

observed through the noisy measurement equation

$$y_t = C\,x_t + G_2 w_t \quad , \quad t = 0,1,\ldots \tag{7}$$

where $x_t \in \mathbb{R}^n$ is the state vector (the initial state x_0 is unknown), $u_t \in \mathbb{R}^m$ is the control vector, $w_t \in \mathbb{R}^l$ is the noise vector, and $y_t \in \mathbb{R}^p$ is the measurement vector. We assume the statistics of the random variables $x_0, w_0, w_1, \ldots$ to be unknown, and consider them as disturbances of unknown character that take their values from known compact sets. Moreover, we assume the estimator to be a finite-memory one. Then, for any $t \ge N$, we define the information vector as

$$I_t^N \triangleq \text{col}\,(y_{t-N},\ldots,y_t,u_{t-N},\ldots,u_{t-1}) .$$

$N+1$ is the number of measurements made within a "sliding window" $\{t-N,t\}$.

We will follow the approach to receding-horizon estimation described in (Alessandri *et al.*, 1999), i.e., the goal is to estimate the state vectors $x_{t-N},\ldots,x_t$, at any stage t, on the basis of the information vector I_t^N and of a prediction $\bar{x}_{t-N}$ on the state x_{t-N}. Let us define as $\hat{x}_{t-N,t},\ldots,\hat{x}_{tt}$ the estimates of $x_{t-N},\ldots,x_t$, respectively, made at stage t. We assume that the prediction $\bar{x}_{t-N}$ is determined via the state equation (6) by the estimate $\hat{x}_{t-N-1,t-1}$, that is, $\bar{x}_{t-N} = A\,\hat{x}_{t-N-1,t-1} + B\,u_{t-N-1}$, $t = N+1, N+2,\ldots$. $\bar{x}_0$ denotes an a priori prediction on x_0.

Let us now denote by W the set from which the vector w_t takes its values. As we have assumed the statistics of the disturbances to be unknown, a natural criterion to derive the estimator consists in resorting to a least-squares approach. Towards this end, we introduce the following loss function

$$J_t = \mu \| \hat{x}_{t-N,t} - \bar{x}_{t-N} \|^2 + \sum_{i=t-N}^{t} \| y_i - C\,\hat{x}_{it} \|^2$$

where $\|\cdot\|$ is the Euclidean norm and μ is a positive scalar by which we express our belief in the prediction $\bar{x}_{t-N}$ with respect to the observation model. Then, at any stage $t = N, N+1, \ldots$, the following problem has to be solved:

Problem 1. For a given pair $(\bar{x}^\circ_{t-N}, I_t^N)$, find the optimal estimate $\hat{x}^\circ_{t-N,t}$, ..., $\hat{x}^\circ_{t,t}$ that minimize the cost J_t and satisfies the constraints

$$\hat{x}^\circ_{i+1,t} = A\,\hat{x}^\circ_{i,t} + B\,u_i\,, \quad i = t-N, \ldots, t-1\,.$$

The optimal prediction is determined as

$$\bar{x}^\circ_{t-N} = A\,\hat{x}^\circ_{t-N-1,t-1} + B\,u_{t-N-1} \qquad (8)$$

for $t = N+1, N+2, \ldots$. □

Clearly, as to the propagation of the estimation procedure from Problem 1 at time t to Problem 1 at time $t+1$, only the estimate $\hat{x}^\circ_{t-N,t}$ has to be retained. This estimate becomes the optimal prediction $\bar{x}^\circ_{t-N+1}$ for Problem 1 at time $t+1$ through the use of (8). When the measures given by y_{t+1} and the input u_t become available, we can refer to the new information vector $I_{t+1}^N = \mathrm{col}\left(y_{t-N+1}, \ldots, y_{t+1}, u_{t-N+1}, \ldots, u_t\right)$ and generate the new estimates $\hat{x}^\circ_{i,t+1}$, $i = t-N+1, \ldots, t+1$. The same mechanism is applied stage after stage.

Let us introduce the following assumptions:

A1. W is a compact set.
A2. The pair (A, C) is completely observable.
A3. $N \geq n$.

Let us define

$$F_N \triangleq \begin{bmatrix} C \\ CA \\ \vdots \\ CA^N \end{bmatrix}, \quad \bar{F}_N \triangleq \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix}$$

$$\hat{B}_N \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB & 0 & \cdots & 0 & 0 \\ CAB & CB & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{N-1}B & CA^{N-1}B & \cdots & CAB & CB \end{bmatrix},$$

$$\tilde{T}_N \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C & 0 & \cdots & 0 & 0 \\ CA & C & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{N-2} & CA^{N-3} & \cdots & CA & C \end{bmatrix},$$

$$T_N \triangleq \left[\, F_{N-1} \middle| \tilde{T}_N \,\right], \; O \triangleq F_N' F_N\,, \; \bar{O} \triangleq \bar{F}_N' F_{N-1}\,,$$

$$\hat{G}_{i,N} \triangleq \underbrace{\begin{bmatrix} G_i & 0 & \cdots & 0 \\ 0 & G_i & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & G_i \end{bmatrix}}_{N}, \; i = 1, 2\,,$$

$$r_\xi \triangleq \max_{w_t \in \Omega} \|G_1 w_t\|\,, \; r_\eta \triangleq \max_{w_t \in \Omega} \|G_2 w_t\|\,,$$

$$f_{min} \triangleq \|F_N\|_{\min}\,, \; f_{max} \triangleq \|F_N\|\,, \; f_m \triangleq \|\bar{F}_N\|\,,$$

$$\bar{f}_{max} \triangleq \|F_{N-1}\|\,, \; k_f \triangleq \|A\|\,, \; \tilde{t} \triangleq \|\tilde{T}_N\|\,.$$

Using the above-written definitions, we can state the following results [see (Alessandri *et al.*, 2001)].

Theorem 3. Suppose that $\mu > 0$. Then Problem 1 at time t has a unique solution given by

$$\hat{x}^\circ_{t-N,t} = (\mu\,I + O)^{-1}\left[\mu\,\bar{x}^\circ_{t-N} + F_N'\,(y^t_{t-N} - \hat{B}_N\,u^{t-1}_{t-N})\right]$$

for $t = N, N+1, \ldots$. Moreover, if we denote the estimation error by $e_{t-N} = x_{t-N} - \hat{x}^\circ_{t-N,t}$, then

$$\begin{cases} e_0 = (\mu I + O)^{-1}\Big[\mu(x_0 - \bar{x}_0) - (CA^N)'\,G_2 w_N \\ \quad - \left(\bar{F}_N' T_N \hat{G}_{1,N} + F_{N-1}' \hat{G}_{2,N}\right) w_0^{N-1}\Big] \\ e_{t-N} = (\mu I + O)^{-1}\Big[\mu A e_{t-N-1} + \mu G_1 w_{t-N-1} \\ \quad - \left(\bar{F}_N' T_N \hat{G}_{1,N} + F_{N-1}' \hat{G}_{2,N}\right) w_{t-N}^{t-1} \\ \quad - (CA^N)'\,G_2 w_t\Big], \quad t \geq N+1\,. \end{cases}$$

The error dynamics is stable if

$$\frac{\mu\,k_f}{\mu + f_{min}^2} < 1 \qquad (9)$$

i.e., $\forall \mu > 0$ if $k_f \leq 1$ and $\forall \mu : 0 < \mu < \dfrac{f_{min}^2}{k_f - 1}$ if $k_f > 1$. □

Theorem 3 has been used to show some nice properties of the receding-horizon state estimator, e.g., the exponential convergence when no noise acts on the system and measurement equations and unbiasedness in the presence of disturbances.

4. DESIGN OF THE RECEDING-HORIZON FILTER USING LMI

A crucial issue about the design of the proposed estimator is the choice of the scalar weight μ. Such a parameter can be chosen to tune the relative trust in the prediction of the state estimates or in the measures of the cost function. If the measurements are affected by a high amount of uncertainty, a bigger value of μ may be preferable. Otherwise, if the measurement noises are negligible and/or the process noises are large, it is well-suited to choose a smaller μ. In qualitative terms, μ expresses our belief in the prediction $\bar{x}_{t-N}$ with respect to the observation model. However, in order to devise a quantitative procedure to select an appropriate value of μ, it is customary to introduce some kind of performance index to be optimized with respect to μ. Among several possible choices, one possibility is to rely on the analysis of the asymptotic behavior of the receding-horizon estimator. We can state the following proposition [see, for details, (Alessandri *et al.*, 2001)].

Proposition 2. Suppose that Assumptions A1, A2, and A3 are verified. Then the norm of the estimation error is bounded as

$$\|e_{t-N}\| \leq \tilde{e}_{t-N} + \frac{f_m \bar{f}_{max} r_\xi}{\mu + f^2_{min}} \ , \ t \geq N \ ,$$

where the sequence $\tilde{e}_t$ is defined as

$$\begin{cases} \tilde{e}_0 = b_0 \\ \tilde{e}_t = a\,\tilde{e}_{t-1} + b \, , \quad t \geq 1 \end{cases}$$

and

$$\begin{aligned} a &\triangleq \frac{\mu k_f}{\mu + f^2_{min}} \ , \\ b &\triangleq \frac{1}{\mu + f^2_{min}} \left[\left\| I - A\,(\mu I + O)^{-1}\,\bar{O} \right\| \mu r_\xi + \right. \\ &\quad \left. + \tilde{t} f_m \sqrt{N-1} r_\xi + f_{max}\sqrt{N+1} r_\eta \right] , \\ b_0 &\triangleq \frac{1}{\mu + f^2_{min}} \Big(\mu \|x_0 - \bar{x}_0\| + \tilde{t} f_m \sqrt{N-1} r_\xi + \\ &\quad + f_{max}\sqrt{N+1} r_\eta \Big) \ . \end{aligned}$$

Then, if condition (9) is satisfied, the sequence $\tilde{e}_t$ has the following properties:

$$(a) \lim_{t\to\infty} \tilde{e}_t \triangleq \tilde{e} = \frac{1}{\mu(1-k_f) + f^2_{min}} \left[\tilde{t} f_m \sqrt{N-1} r_\xi \right. \\ \left. + f_{max}\sqrt{N+1} r_\eta + \left\| I - A\,(\mu I + O)^{-1}\,\bar{O} \right\| \mu r_\xi \right] ;$$

$$(b) \qquad \tilde{e}_{t-1} \geq \tilde{e} \Longrightarrow \tilde{e}_t \leq \tilde{e}_{t-1} \ .$$

An asymptotic upper bound on the error e_{t-N} is

$$e_{\infty,1}(\mu) \triangleq \tilde{e} + \frac{f_m \bar{f}_{max} r_\xi}{\mu + f^2_{min}} \ . \qquad (10)$$

□

An alternative approach follows from an application of the results on quadratic boundedness presented in Section 2 to the error dynamics. To this end, let us suppose that the compact set W is an elliptic one, i.e., $W \triangleq \{w : w^T Q w \leq 1, \, Q > 0\}$ and define

$$z_t \triangleq e_{t-N} \quad , \quad \tilde{w}_t \triangleq \begin{bmatrix} w_{t-N} \\ \vdots \\ w_{t+1} \end{bmatrix} \ .$$

Then we can rewrite the error dynamics as

$$z_t = \tilde{A} z_{t-1} + \tilde{G} \tilde{w}_{t-1} \qquad (11)$$

where

$$\begin{aligned} \tilde{A} &\triangleq (\mu I + O)^{-1} \mu A \ , \\ \tilde{G} &\triangleq (\mu I + O)^{-1} \times \\ &\times \left[G_1 \middle| -\left(\begin{array}{c} \bar{F}'_N T_N \hat{G}_{1,N} \\ + F'_{N-1} \hat{G}_{2,N} \end{array} \right) \middle| - \left(CA^N\right)' G_2 \right] \ . \end{aligned}$$

We now exploit the results of Section 2 by stating the following proposition.

Proposition 3. If system (11) is quadratically bounded with Lyapunov matrix P, then $\underline{\lambda}(P)^{-1/2}$ is an asymptotic upper bound on the error e_{t-N}.

□

If we define

$$\hat{Q} = \frac{1}{N+2} \underbrace{\begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & Q & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & Q \end{bmatrix}}_{N}$$

then we have that $w'_{t-N} Q w_{t-N} \leq 1, \ldots,$ $w'_{t+1} Q w_{t+1} \leq 1$ implies $\tilde{w}'_t \hat{Q} \tilde{w}_t \leq 1$. Therefore, we can solve the following problem.

Problem 2. Find $(P^o, \alpha^o) = \text{argmax}\,\{\underline{\lambda}(P)\}$ such that $P > 0$, $\alpha \geq 0$, and

$$\begin{bmatrix} \tilde{A}^T P \tilde{A} - P + \alpha P & \tilde{A}^T P \tilde{G} \\ \tilde{G}^T P \tilde{A} & \tilde{G}^T P \tilde{G} - \alpha \hat{Q} \end{bmatrix} \leq 0 \ .$$

□

The solution of Problem 2 is of particular interest for the design of the receding-horizon estimator as the upper bound on the asymptotic estimation error

$$e_{\infty,2}(\mu) = \underline{\lambda}(P^o)^{-1/2} \tag{12}$$

turns out to be minimized.

5. A NUMERICAL EXAMPLE

In this section, a simulation example is presented to illustrate the proposed approach to receding-horizon estimation. We considered the second-order system described in (Kwon *et al.*, 1999), by means of the equations (6) and (7) with

$$A = \begin{bmatrix} 0.9950 & 0.0998 \\ -0.0998 & 0.9950 \end{bmatrix}, \; B = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; G_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

We suppose the noise vector w belongs to the ellipsoid $\varepsilon_Q = \{w : w'Qw \leq 1\}$, where

$$Q = \begin{bmatrix} 156.25 & 0 & 0 \\ 0 & 156.25 & 0 \\ 0 & 0 & 312.5 \end{bmatrix}.$$

In order to evaluate how conservative the asymptotic bounds presented in this paper are, a nonlinear programming problem may be solved that consists in determining the worst-case performance, i.e.,

$$e_{worst}(\mu) \triangleq \max_{w_t' Q w_t \leq 1,\, t\in[0,T-1]} \left\| x_{T-N} - \hat{x}^{\circ}_{T-N} \right\| ,$$

where the simulation horizon T has to be chosen sufficiently large with respect to the dynamics of the error. $e_{worst}(\mu)$ provides a simulation-based practical evaluation of the maximum asymptotic estimation error. As shown in Fig. 1, the bound given by (12) and obtained via LMI is much less conservative than that provided by (10).

6. CONCLUSIONS

A method to perform receding-horizon estimation for discrete-time linear systems has been presented. A complete analysis of stability and of the unbiasedness properties for such a filter has been provided in (Alessandri *et al.*, 2001)). Here we focused on (i) a method to find less conservative upper bounds on the estimation error based on quadratic boundedness and (ii) a design methodology that aims at minimizing such upper bounds using LMI procedures.

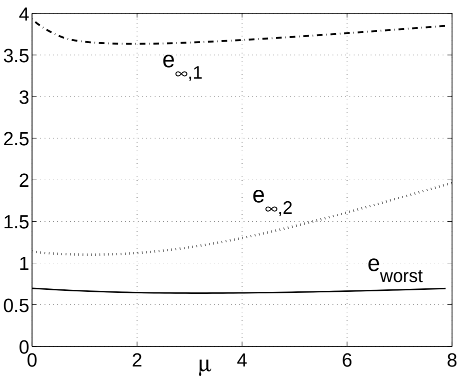

Fig. 1. Diagrams of $e_{\infty,1}$, $e_{\infty,2}$ and e_{worst} as functions of μ, for T equal to 70.

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