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ADAPTIVE STABILIZATION FOR NON-CONTROLLABLE TIME-VARYING PLANTS BY USING MULTIESTIMATION

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Abstract: This paper presents an indirect adaptive control scheme for nominally stabilizable, but possible non-controllable, continuous and time-varying systems with unmodelled dynamics. The control objective is the adaptive stabilization of the system. The scheme includes several estimation algorithms and a supervisor which selects the appropriate estimator keeping it in operation during at least a minimum residence time. *Copyright©2002 IFAC*

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1. INTRODUCTION

Robust adaptive stabilization, under unmodelled dynamics and bounded noise, of linear time-invariant stabilizable, but possibly non controllable, plants has been successfully developed by De la Sen and Alonso-Quesada (1999) and Alonso-Quesada (2001). In such works the assumptions of the ideal adaptive control problem (see Narendra *et al*., 1980) and the controllability of the nominal plant were relaxed. The adaptive control algorithm had to include a relative adaptation dead-zone, for robustness purposes, and 'a posteriori' modification of the estimated parameters to ensured the controllability of the estimated and modified estimated plant model since such property is crucial for stabilization via adaptive pole-placement (see Chen and Cao, 1997).

This paper extends the study of Alonso-Quesada (2001) to the case of nominally stabilizable plants with piecewise constant unknown parameters. The control objective is the stabilization, under the presence of unmodelled dynamics, of the system via an adaptive pole placement scheme. Several estimation algorithms running in parallel are designed with a supervisor which selects one of them depending on a criterion relative to the identification errors (Narendra and Balakrishnan, 1997). That estimator is used for the adaptation controller synthesis and it is kept in operation during a certain time interval so that closed-loop stability is guaranteed.

All of the adaptive control algorithms to be designed have to include a relative dead-zone and a 'a posteriori' modification of the plant estimated model parameters as in the time-invariant plant case. The stabilizability of the nominal model of the plant at all time instant and the knowledge of an upper-bound function on the contribution of the unmodelled dynamics and, eventually, bounded noise to the output are the only assumptions required to establish the stability of the closed-loop system.

2.PROBLEM STATEMENT

Consider the following time-varying plant

$$A(D,t)y(t)=B(D,t)u(t)+\eta(t) \qquad (1)$$

where η is the contribution of unmodelled dynamics and, possibly, bounded noise to the output, $A(D,t)=D^n+\sum_{i_1=1}^{n} a_{i_1}(t)D^{n-i_1}$, $B(D,t)=\sum_{i_2=n-m}^{n} b_{i_2}(t)D^{n-i_2}$ with $m\le n$, and $a_{i_1}(t)$, $b_{i_2}(t)$, for $i_1\in\{1,\cdots,n\}$ and $i_2\in\{n-m,\cdots,n\}$, being unknown piecewise constant functions and D being the time-derivative operator. In the following, the time argument is omitted for notational abbreviation.

The following filtered signals subscripted by $"f"$ are introduced

$$F(D)u_f=u \ ; \ F(D)y_f=y \ ; \ F(D)\eta_f=\eta \qquad (2)$$

for some stable filter $F(D)=D^n+\sum_{i=1}^{n} f_i D^{n-i}$. Then, the filtered plant output is given by

$$\begin{aligned}&A(D,t)y_f=B(D,t)u_f+\eta_f+\xi(t,t_{zp+j})\Rightarrow\\&y_f^{(n)}=\boldsymbol{\theta}^T\boldsymbol{\varphi}+\eta_f+\xi(t,t_{zp+j}) \quad for\ t\in\left[t_{zp+j},t_{zp+j+1}\right)\end{aligned} \qquad (3)$$

where t_{zp+j} and t_{zp+j+1}, for any bounded or unbounded integer $z\ge 0$ and $j\in\{0,\ldots,p-1\}$ with $p>0$ being some prefixed arbitrary bounded integer, denote two consecutive instants at which at least one of the functions $a_{i_1}(t)$ or $b_{i_2}(t)$ switches. It is supposed that the time interval between two consecutive plant switches is higher than a minimum threshold Δt_{min}, residence time used for stability purposes. The signal $\xi(t,t_{zp+j})$ is a exponentially decaying term which depends on the parameterized conditions of the plant and the filters at the initial instant of each interval $\left[t_{zp+j},t_{zp+j+1}\right)$. The plant parameter vector $\boldsymbol{\theta}=[b_{n-m}\ \cdots\ b_n\ a_1\ \cdots\ a_n]^T$ and $\boldsymbol{\varphi}=\left[u_f^{(n)}\ \cdots\ u_f\ -y_f^{(n-1)}\ \cdots\ -y_f\right]^T$ have been introduced for notational compactness.

<u>Assumption 1</u>: The signal η_f is the sum of a bounded term, plus a term related to u by a strictly proper exponentially stable transfer function. ***

From *Assumption 1* and in view of *Lemma 3.1* by Middleton *et al.* (1988), there exist real constants $\sigma_0\in(0,1)$, $\alpha_0\ge 0$ and $\alpha\ge 0$, and a constant vector $\boldsymbol{v}$, which are assumed known, such that

$$\begin{aligned}&\left|\eta_f(t)\right|\le\bar{\eta}_f(t)=\alpha\rho(t)+\alpha_0 \quad \forall t>0\\&for \quad \rho(t)=\sup_{0\le\tau\le t}\left\{\left|\boldsymbol{v}^T\boldsymbol{z}(\tau)\right|e^{-\sigma_0(t-\tau)}\right\}\end{aligned} \qquad (4)$$

with $\boldsymbol{z}=\left[\varepsilon_f^{(n-1)}\cdots\ \varepsilon_f\ \ u_f^{(n-1)}\cdots\ u_f\right]^T$, $\varepsilon_f=y_f-y_{mf}$ being the filtered tracking-error and y_{mf} the filtered reference signal obtained from $F(D)y_{mf}=y_m$. The reference signal y_m is the output of the exponentially stable filter $W_m(D)=B_m(D)/A_m(D)$ operating under any uniformly bounded and piecewise continuous input $r^*(t)$.

3. ADAPTIVE CONTROL

The control objective is *the adaptive stabilization with achievement of a bounded tracking-error between the system output and any uniformly bounded reference signal*. The control law is given by

$$u=\bar{K}(t)y_m-\bar{R}(D,t)u_f-\bar{S}(D,t)y_f \qquad (5)$$

where $\bar{R}=\sum_{i=0}^{n-1}\bar{r}_{n-i}(t)D^i$ and $\bar{S}=\sum_{i=0}^{n-1}\bar{s}_{n-i}(t)D^i$ with the control parameters $\bar{K}$, $\bar{r}_i$ and $\bar{s}_i$ being calculated from a plant estimated model. Several estimation algorithms are designed in order to obtain controllable estimated models of the plant at each instant. All of them are of least square-type or of gradient-type. They include a relative dead-zone and a parameter modification to ensure the robustness under disturbances and the controllability of the estimated models, respectively. These algorithms run in parallel while a supervisor selects the estimated model which optimizes certain cost function involving the identification error every certain time.

The motivation of the use of a multiestimation scheme is that the estimation model may be adjusted more closely to the true plant. This is very relevant when the plant varies or when the initial conditions of a unique estimator are very deviated from the unknown true plant parameters. In this sense, it is suitable to reset each estimator when the true plant presents a change in any of their parameters. This is crucial to improve the performance of the system signals before changes in the parameters of the nominal model of the plant. It has to be pointed out that only one of the parameterized controller acts as an effective controller on the plant during each time interval while all controllers are parameterized from their respective identifiers for all time.

Remark 1. In the ideal case of time-invariant known plant, the control parameters requested to meet the control objective can be obtained from the following diophantine equation,

$$(F(D)+R(D))A(D)+S(D)B(D)=C(D) \qquad (6)$$

where $C(D)=D^{2n}+c_1 D^{2n-1}+\ldots+c_{2n}$ is a *Hurwitz* polynomial. Besides, another parameter K can be considered to obtain a perfect tracking for some frequency range of interest. For example, if the external reference signal belongs to a low-frequency signals class, then K must be obtained from

$$Kf_n b_n=(f_n+r_n)a_n+s_n b_n \qquad (7)$$

Eqns. (6) and (7) can be compactly expressed by

$$\begin{aligned} &\boldsymbol{M(\theta)}[1 \;\; f_1+r_1 \;\cdots\; f_n+r_n \;\; s_1 \;\cdots\; s_n \;\; -Kf_n]^T = \\ &=[1 \;\; c_1 \;\cdots\; c_{2n} \;\; 0]^T \end{aligned} \qquad (8)$$

where

$$\boldsymbol{M(\theta)}=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ a_1 & 1 & 0 & \cdots & 0 & 0 & b_{n-m} & 0 & \cdots & 0 & 0 & 0 \\ a_2 & a_1 & 1 & \cdots & 0 & 0 & b_{n-m+1} & b_{n-m} & \cdots & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 & 0 & b_{n-m2+} & b_{n-m+1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_1 & 1 & b_{n-1} & b_{n-2} & \cdots & b_{n-m+1} & b_{n-m} & 0 \\ 0 & a_n & a_{n-1} & \cdots & a_2 & a_1 & b_n & b_{n-1} & \cdots & b_{n-m+2} & b_{n-m+1} & 0 \\ 0 & 0 & a_n & \cdots & a_3 & a_2 & 0 & b_n & \cdots & b_{n-m+3} & b_{n-m+2} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_n & a_{n-1} & 0 & 0 & \cdots & b_n & b_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & a_n & 0 & 0 & \cdots & 0 & b_n & 0 \\ 0 & 0 & 0 & \cdots & 0 & a_n & 0 & 0 & \cdots & 0 & b_n & b_n \end{bmatrix} \qquad (9)$$

The objective is achieved if $|Det(\boldsymbol{M(\theta)})| \geq \delta > 0$, for some real constant δ. Such an expression is referred to as the plant *controllability condition*. ***

3.1 Estimation algorithms

Assume there is a finite number l of estimators. Each estimation algorithm consist of two steps.

**Step1 *(Parameter estimation)*: The equations which define each algorithm are the following

$$\dot{\boldsymbol{P}}_i=\begin{cases} \dfrac{-g_i s_i \boldsymbol{P_i \varphi_n \varphi_n^T P_i}}{1+\gamma_i \boldsymbol{\varphi_n^T P_i \varphi_n}} & if \quad \lambda_{min}(\boldsymbol{P_i}(t)) \geq \lambda_0 > 0 \\ 0 & otherwise \end{cases} \qquad (10)$$

for some prefixed real constant λ_0 with $\lambda_{min}(\boldsymbol{P_i})$ being the minimum eigenvalue of the matrix $\boldsymbol{P_i}$, and

$$\dot{\hat{\boldsymbol{\theta}}}_{\boldsymbol{i}}=\frac{s_i \boldsymbol{P_i \varphi_n} e_{n_i}}{1+\gamma_i \boldsymbol{\varphi_n^T P_i \varphi_n}} \qquad (11)$$

for $i \in \{1,\ldots,l\}$ where $\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}=\left[\hat{b}_{0_i} \;\cdots\; \hat{b}_{n_i} \;\; \hat{a}_{1_i} \;\cdots\; \hat{a}_{n_i}\right]^T$, $\boldsymbol{\varphi_n}=\boldsymbol{\varphi}/(1+\|\boldsymbol{\varphi}\|)$, $e_{n_i}=e_i/(1+\|\boldsymbol{\varphi}\|)$ with $e_i=(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^T\boldsymbol{\varphi}+\eta_f=-\tilde{\boldsymbol{\theta}}_{\boldsymbol{i}}^T\boldsymbol{\varphi}+\eta_f$ being the identification error corresponding to the i-th estimator, $0<\boldsymbol{P_i}(t_0)=\boldsymbol{P_i}^T(t_0)$ and bounded, $\gamma_i:[t_0,\infty)\to[\gamma_{i_1},\gamma_{i_2}]$ for some $0 \leq \gamma_{i_1} \leq \gamma_{i_2} < \infty$, $g_i=0$ (for a least-square type algorithm which becomes to a gradient type one) or $g_i=1$ (a gradient type algorithm) and $s_i:[t_0,\infty)\to[0,1)$ a relative adaptation dead-zone defined by

$$s_i(t)=\begin{cases} 0 & if \quad w_i \leq \mu_i \overline{\eta}_{fn} \\ (w_i-\mu_i\overline{\eta}_{fn})/w_i & otherwise \end{cases} \qquad (12)$$

for any arbitrary constant $\mu_i>1$, with $\overline{\eta}_{fn}=\overline{\eta}_f/(1+\|\boldsymbol{\varphi}\|)$, and an augmented error $w_i(t)=(e_{n_i}^2+g_i\boldsymbol{\varphi_n^T P_i^2 \varphi_n})^{1/2}$.

**Step2 *(Estimation modification)*: The modified estimates are obtained from

$$\overline{\boldsymbol{\theta}}_{\boldsymbol{i}}=\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}+\pi_i\boldsymbol{P_i\beta_i} \qquad (13)$$

with $\overline{\boldsymbol{\theta}}_{\boldsymbol{i}}=\left[\overline{b}_{0_i} \;\cdots\; \overline{b}_{n_i} \;\; \overline{a}_{1_i} \;\cdots\; \overline{a}_{n_i}\right]^T$, $\pi_i:[t_0,\infty)\to\Re$ and $\boldsymbol{\beta_i}:[t_0,\infty)\to\Re^{(2n+1)\times 1}$, see (Alonso-Quesada, 2001; De la Sen and Alonso-Quesada, 1999) for details. The components of $\boldsymbol{\beta_i}(t)$ are given by

$$\beta_{i_j}=\frac{Det([\boldsymbol{p_{i_1}} \;\; \boldsymbol{p_{i_2}} \;\cdots\; \boldsymbol{v} \;\cdots\; \boldsymbol{p_{i_{2n}}} \;\; \boldsymbol{p_{i_{2n+1}}}])}{Det(\boldsymbol{P_i})} \qquad (14)$$

where $\boldsymbol{v}=\left[0 \;\cdots\; 0 \;\; \overset{n+1}{\widehat{1}} \;\; 0 \;\cdots\; 0\right]^T$ is replacing the j-th column of the matrix $\boldsymbol{P_i}$, i.e. $\boldsymbol{p_{i_j}}$, for $j\in\{1,\ldots,2n+1\}$.

The switching functions $\pi_i(t)$, for $i\in\{1,\ldots,l\}$, are zero at t_0 and for $t>t_0$ are defined by

$$\pi_i=\begin{cases} \pi_i(t^-) & if \quad \left|Det\left(\boldsymbol{M}\left(\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}+\pi_i(t^-)\boldsymbol{P_i}\boldsymbol{\beta}_i\right)\right)\right| \geq \delta > 0 \\ \pi_{0_i}(t) & otherwise \end{cases} \qquad (15)$$

for some prefixed real constant δ. i.e., (15) implies that $\pi_i(t)$ only changes its value when the i-th plant estimated model is near to a non-controllable model.

****Algorithm 1*** *to compute* π_{0_i} *

Step1: Set $\pi_{0_i}=0$, compute $\left|Det\left(\boldsymbol{M}\left(\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}+\pi_{0_i}\boldsymbol{P_i\beta_i}\right)\right)\right|$ and go to *Step2*, Step2: If $\left| Det\left(M\left(\hat{\theta}_i + \pi_{0_i} P_i \beta_i\right)\right)\right| \geq \delta$ then ***end***, else go to *Step3*,

Step3: Increase the value of π_{0_i} as $\pi_{0_i} = \pi_{0_i} + \delta_i$, with $0 < \delta_i << 1$, compute $\left| Det\left(M\left(\hat{\theta}_i + \pi_{0_i} P_i \beta_i\right)\right)\right|$ and go to *Step4*,

Step4: If $\left| Det\left(M\left(\hat{\theta}_i + \pi_{0_i} P_i \beta_i\right)\right)\right| \geq \delta$ then set $\pi_{1_i} = \pi_{0_i}$ and go to *Step5*, else go to *Step3*,

Step5: Set $\pi_{0_i} = 0$ and go to *Step6*,

Step6: Decrease the value of π_{0_i} according to $\pi_{0_i} = \pi_{0_i} - \delta_i$, compute $\left| Det\left(M\left(\hat{\theta}_i + \pi_{0_i} P_i \beta_i\right)\right)\right|$ and go to *Step7*,

Step7: If $\left| Det\left(M\left(\hat{\theta}_i + \pi_{0_i} P_i \beta_i\right)\right)\right| \geq \delta$ then go to *Step8*, else go to *Step6*,

Step8: If $\left|\pi_{1_i}\right| \leq \left|\pi_{0_i}\right|$ then set $\pi_{0_i} = \pi_{1_i}$, and ***end***. ***

Remark 2. The nominal plant is chosen of relative degree zero due to the knowledge of the relative degree of the true plant has not been assumed. Then, the adaptive problem can be overparameterized. ***

Remark 3. By substitution of (13) into the expression $e_i = (\theta - \hat{\theta}_i)^T \varphi + \eta_f$, the following estimated and modified plant models

$$y_f^{(n)} = \bar{\theta}_i^T \varphi + e_i - \pi_i \beta_i^T P_i \varphi = \bar{\theta}_i^T \varphi + e_{a_i} \tag{16}$$

are obtained, where $e_{a_i} = e_i - \pi_i \beta_i^T P_i \varphi$ is referred to as *a modified identification error*. The estimation and modification algorithm ensure the controllability of all plant estimated models, i.e, $\left| Det(M(\bar{\theta}_i))\right| \geq \delta > 0$ where $M(\bar{\theta}_i)$ is obtained from (9) by replacing θ by $\bar{\theta}_i$.***

3.2. Supervisor

This design device selects the plant estimated model which optimizes the following cost function,

$$J_i \equiv \lambda_3 \int_{t_0}^{t} e^{-\lambda_1(t-\tau)} e_i^2(\tau) d\tau + (1-\lambda_3) \int_{t_0}^{t} e^{-\lambda_2(t-\tau)} e_{a_i}^2(\tau) d\tau \tag{17}$$

for some real constants $\lambda_1, \lambda_2 \geq 0$ and $\lambda_3 \in [0,1]$. The switches between the estimators only take place after a minimum residence time T, in the current estimator, which is required to guarantee the system stability, see (Narendra and Balakrishnan, 1997).

Let $\{t_k, k \geq 1\}$ be the collection of instants where the adaptive controller switches the parameterization from one of the estimators to another one. Assume that $\bar{\theta}(t_k^+) = \bar{\theta}_i(t_k^+)$. Thus, for $t > t_k^+$:

$$\bar{\theta} = \begin{cases} \bar{\theta}_i & if \ t - t_k < T \ or \ J_i(t, e_i, e_{a_i}) = J_q(t, e_q, e_{a_q}) \\ \bar{\theta}_q & otherwise \end{cases} \tag{18}$$

where the q-th estimator is such that $J_q(t, e_q, e_{a_q}) \leq J_m(t, e_m, e_{a_m})$ for all $q, m \in \{1, \cdots, l\}$.

The plant estimated model issued by the supervisor is

$$y_f^{(n)} = \bar{\theta}^T \varphi + e_a \qquad \forall t \in [t_k, t_{k+1}) \tag{19}$$

where $e_a = e_{a_i}$ with i denoting the chosen estimator index during the time interval $[t_k, t_{k+1})$, $\bar{\theta} = \left[\bar{b}_0 \ \cdots \ \bar{b}_n \ \ \bar{a}_1 \ \cdots \ \bar{a}_n\right]^T$ and φ as in (3).

3.3. Adaptive control law parameters

The parameters of the adaptive control law, eqn. (5), are obtained by means of an equation similar to (8) replacing θ by $\bar{\theta}$ and K, r_i and s_i by $\bar{K}$, $\bar{r}_i$ and $\bar{s}_i$, respectively, for $i \in \{1, \ldots, n\}$. Such equation is uniquely solvable due to the fact that $M(\bar{\theta})$ is ensured as a non-singular matrix.

4. CONVERGENCE AND STABILITY RESULTS

All of algorithms of eqns. (10-15), for $i \in \{1, \cdots, l\}$, have the following properties:

Lemma 1

(i) $\|P_i\| \in L_\infty$, $P_i > 0 \quad \forall t \geq t_0$, it is symmetric and it converges asymptotically as time tends to infinity,

(ii) $\bar{\eta}_{fn} \in L_\infty$ and $\left|\eta_{fn}\right| \in L_\infty$,

Besides, if

$$\sum_{j=1}^{p} \Delta t_{I_i^{zp+j}} \geq \geq \frac{1}{\varepsilon'} \frac{\mu_i}{\mu_i - 1} \sum_{j=1}^{p} \Delta^T(t_{zp+j}) P_i^{-1}(t_{zp+j}) \left(\Delta(t_{zp+j}) - 2\tilde{\theta}_i(t_{zp+j}^-)\right) \tag{20}$$

where $\Delta(t_{zp+j}) \equiv \theta(t_{zp+j}^+) - \theta(t_{zp+j}^-)$, $I_i^{zp+j} \equiv \left\{t \in (t_{zp+j-1}^+, t_{zp+j}^-) \mid s_i(t) \geq \varepsilon' > 0\right\}$, with $\varepsilon' << 1$, and $\Delta t_{I_i^{zp+j}}$ is sum of the time intervals belonging to I_i^{zp+j}, for $i \in \{1, \cdots, l\}$ and $j \in \{1, \cdots, p\}$, then:

(iii) $\left\|\hat{\theta}_i\right\| \in L_\infty$, $w_i \in L_\infty$, $s_i w_i^2 \in L_\infty$ and $\left\|\dot{\hat{\theta}}_i\right\| \in L_\infty$

(iv) $\pi_i \in L_\infty$ and it is time-differentiable for all t except at the switching time instants,

(v) $\|\bar{\boldsymbol{\theta}}_i\| \in L_\infty$ and it is time-differentiable $\forall t$ except at the time instant at which π_i switches. ***

The proof is given in *Appendix A*.

Remark 4. Note that (20) implies that a minimum time between some finite number *p* of consecutive switches in the true parameter vector is imposed to guarantee the properties (iii) to (v). This is accomplished if

$$\begin{aligned} & V_i(t_{zp}^{+}) - V_i(t_{(z+1)p}^{+}) = \\ & = \sum_{j=1}^{p} \left(V_i(t_{zp+j-1}^{+}) - V_i(t_{zp+j}^{-}) \right) + \sum_{j=1}^{p} \left(V_i(t_{zp+j}^{-}) - V_i(t_{zp+j}^{+}) \right) \geq 0 \end{aligned} \quad (21)$$

where $\{t_{zp+j}\ ;\ j = 1,2,\dots,p\}$ are the time instants at which the plant parameters switches and $V_i(t) = \tilde{\boldsymbol{\theta}}_i^T(t)\boldsymbol{P}_i^{-1}(t)\tilde{\boldsymbol{\theta}}_i(t) + Tr\,(\boldsymbol{P}_i(t))$ is a Lyapunov's-like function for the *i*-th parametrical error. ***

The estimated model of the plant has the following properties:

Lemma 2

(i) $\|\bar{\boldsymbol{\theta}}\| \in L_\infty$ and it is a piecewise continuous function,

(ii) The controller parameters $\bar{K}$, $\bar{r}_i$ and $\bar{s}_i$, for $i \in \{1,\dots,n\}$, are piecewise continuous bounded functions. ***

The proof is given in *Appendix B*.

Introducing the control law (5) in (19), one obtains

$$\dot{\boldsymbol{z}}(t) = \bar{\boldsymbol{A}}(t)\boldsymbol{z}(t) + \boldsymbol{B_1}\vartheta_1(t) + \boldsymbol{B_2}\vartheta_2(t) \quad (22.a)$$

with

$$\vartheta_1(t) = e_a(t) + [\bar{b}_0(\bar{K}F(D) - \bar{S}(D,t)) - \bar{A}(D,t)]\, y_{mf}(t)$$

$$\vartheta_2(t) = (\bar{K}F(D) - \bar{S}(D,t))y_{mf}(t)$$

$$\bar{\boldsymbol{A}}(t) = \begin{bmatrix} -(\bar{a}_1 + \bar{b}_0\bar{s}_1) & -(\bar{a}_2 + \bar{b}_0\bar{s}_2) & \dots & -(\bar{a}_n + \bar{b}_0\bar{s}_n) & \bar{b}_1 - \bar{b}_0(f_1 + \bar{r}_1) & \dots & \bar{b}_n - \bar{b}_0(f_n + \bar{r}_n) \\ 1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ -\bar{s}_1 & -\bar{s}_2 & \dots & -\bar{s}_n & -(f_1 + \bar{r}_1) & \dots & -(f_n + \bar{r}_n) \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}$$

$$\boldsymbol{B}_1^T = [1 \;\; 0 \;\; \cdots \;\; 0]\ ;\ \boldsymbol{B}_2^T = \left[0 \;\; \cdots \;\; 0 \;\; \underset{n+1}{\underbrace{1}} \;\; \cdots \;\; 0\right] \quad (22.b)$$

Remark 5. (Stability of the homogeneous system). The time-varying system $\dot{\boldsymbol{z}}(t) = \bar{\boldsymbol{A}}(t)\boldsymbol{z}(t)$ is asymptotically stable in view of *Lemma 3.1* of Ioannou and Datta (1991) since: $\bar{\boldsymbol{A}}(t)$ is bounded, the eigenvalues of $\bar{\boldsymbol{A}}(t)$ are strictly inside the stability boundary $\forall t$ and $\int_t^{t+T} \|\dot{\bar{\boldsymbol{A}}}(\tau)\| d\tau \leq k_0 T + k_1 \quad \forall t$, some $k_0, k_1 > 0$, with T being the minimum residence time interval between two consecutive switches of estimator, and where k_0 is sufficiently small. This property is fulfilled for a k_0 sufficiently small, even though $\|\dot{\bar{\boldsymbol{A}}}(t)\|$ can present impulsive *Dirac*-type discontinuities in the time interval $(t, t+T)$ since the jumps in the entries of $\bar{\boldsymbol{A}}(t)$ are bounded. Note that the discontinuities in $\bar{\boldsymbol{A}}(t)$ are due to switches from one estimator to another one or switches due to parameter estimator modification. ***

***Theorem** (Stability result)*. The adaptive control law (5) stabilizes the stabilizable plant (1), in the sense that the signals u and y are bounded for all time, any finite initial state and any piecewise continuous bounded reference signal r^*, subject to *Assumption 1*, provided α in (4) and T in (18) are such that, $T > \frac{1}{\sigma} ln \dfrac{K}{1 - \dfrac{K'\alpha\, \beta_{(k+1)_z}(t) \underset{i\in\{1,\dots,l\}}{Max}\{\mu_i\}\|\boldsymbol{v}\|}{\sigma}}$ for all $t \in [t_{m_z} + (k+1)T, t_{m_z} + (k+2)T]$, with any integer $z \geq 0$ and any $k \in \{0,1,2,\dots\}$, where $\beta_{(k+1)_z}(t) \equiv \underset{t_{m_z}+kT \leq \tau \leq t}{Sup} \left\{ \underset{i\in\{1,\cdots,l\}}{Max} \{1, \|\pi_i(\tau)\boldsymbol{\beta}_i(\tau)\|\} \right\}$. The instant t_{m_z} is the last one in the interval $[t_{zp}, t_{(z+1)p})$ at which some of the functions π_i switches. The constants K and K' are relative to an upper bound of the transition matrix associated with $\bar{\boldsymbol{A}}$ and $\sigma > 0$ is a lower bound of the absolute values of the real parts of the eigenvalues of $\bar{\boldsymbol{A}}$ for all $t \in [t_{m_z} + (k+1)T, t_{m_z} + (k+2)T]$ and any $k \in \{0,1,\dots\}$. ***

The proof is given in *Appendix C*.

5. SIMULATION RESULTS

The plant to be controlled is given by (1) with $b_0 = 1$, $b_1(t) = \begin{cases} 1 & if \quad 0 \leq t \leq 10 \;\; or \;\; t > 30 \\ 1.1 & if \quad 10 < t \leq 30 \end{cases}$, $b_2(t) = \begin{cases} -6 & if \quad 0 \leq t \leq 10 \\ -5.7 & if \quad 10 < t \leq 30 \\ -5.51 & if \quad t > 30 \end{cases}$, $a_1(t) = \begin{cases} 2 & if \quad 0 \leq t \leq 20 \\ 1.9 & if \quad 20 < t \leq 30 \\ 1.8 & if \quad t > 30 \end{cases}$, $a_2(t) = \begin{cases} -3 & if \quad 0 \leq t \leq 20 \\ -3.3 & if \quad 20 < t \leq 30 \\ -3.19 & if \quad t > 30 \end{cases}$ and the initial condition $y(0) = 0$. The signal η arises from a multiplicative and an additive unmodelled dynamics acting on the nominal plant $W_0(s) = \dfrac{b_0 s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$. Such unmodelled dynamics are given by $\Delta_1(s)=\dfrac{0.1}{s+3.5}$ and $\Delta_2(s)=\dfrac{0.1}{s+4.5}$. The stable filter $W_m(s)=\dfrac{s^2+9s+4}{s^2+5s+4}$ and the input $r^*(t)=\begin{cases}0.5\ (\sin 20\pi t+\sin 30\pi t) & for\ \ 0\le t\le 40\\ 1 & for\quad t>40\end{cases}$ are considered. The system signals are filtered by means of eqns. (2) with $F(D)=D^2+9D+12$ and initial conditions zero. The parameters $\alpha=0.005$, $\alpha_0=0.01$, $\sigma_0=0.25$ and $\mathbf{v}=[-0.7\ \ 0.6\ \ -0.4\ \ 0.5]^T$ are chosen in (4). A supervisory system with two estimation algorithms running in parallel are used to obtain a controllable estimated model of the plant at each time instant. The *estimator 1* is an algorithm of a least-squares type which becomes gradient one with $\lambda_0=0.01$, $\gamma_1(t)=0.1$ for all t, $\mu_1=1.01$ and $\delta_1=0.001$ in *Algorithm 1* to compute π_{0_1}. The initial conditions of this estimator are

$$\mathbf{P_1}(0)=10000000\times\begin{bmatrix}1 & 0.9 & 0.9 & 0.9 & 0.9\\ 0.9 & 1 & 0.9 & 0.9 & 0.9\\ 0.9 & 0.9 & 1 & 0.9 & 0.9\\ 0.9 & 0.9 & 0.9 & 1 & 0.9\\ 0.9 & 0.9 & 0.9 & 0.9 & 1\end{bmatrix};\hat{\boldsymbol{\theta}}_1(\mathbf{0})=\begin{bmatrix}-1\\ -1.5\\ 2.5\\ -0.5\\ 1.5\end{bmatrix}.$$

The *estimator 2* is a gradient algorithm type with

$$\mathbf{P_2}=100000\times\begin{bmatrix}1 & 0.1 & 0.1 & 0.1 & 0.1\\ 0.1 & 1 & 0.1 & 0.1 & 0.1\\ 0.1 & 0.1 & 1 & 0.1 & 0.1\\ 0.1 & 0.1 & 0.1 & 1 & 0.1\\ 0.1 & 0.1 & 0.1 & 0.1 & 1\end{bmatrix},\quad \gamma_2(t)=0 \text{ for all } t,$$

$\mu_2=1.01$ and $\delta_2=0.001$ in *Algorithm 1* to compute π_{0_2}. The initial condition of this estimator is $\hat{\boldsymbol{\theta}}_2(0)=[-1.5\ \ -1\ \ 2\ \ -1.2\ \ 1]^T$. The parameter $\delta=0.005$ is chosen as the controllability lower-bound for the estimated models of the plant. The supervisory systems selects the estimator which minimizes the function cost (17) with $\lambda_i=1$, for $i\in\{1,2,3\}$. The minimum residence time interval T in (18) is taken as 30 times the minimum integration step of the simulator program used (*Simulink*, version 5.3). Finally, the desired dynamics for the closed-loop system is defined by the *Hurwitz* polynomial $C(s)=s^4+4s^3+6s^2+5s+2$.

Figure 1 displays the tracking error and *Figure 2* shows the index of the estimator chosen by the supervisor, with $i=1$ corresponding to the least-squares algorithm and $i=2$ to the gradient algorithm.

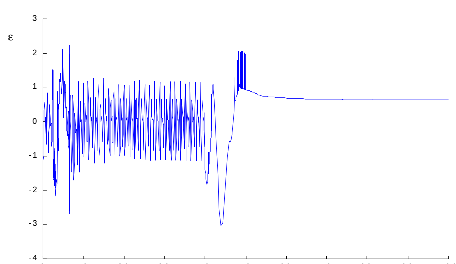

Fig.1. Tracking error signal.

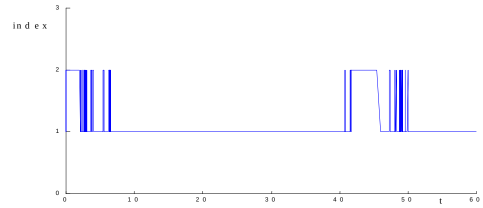

Fig.2. Index of the estimator chosen by the supervisor.

6. CONCLUSIONS

An adaptive control algorithm that stabilizes a stabilizable, but possibly non-controllable, continuous plant with piecewise constant parameters, in the presence of unmodelled dynamics has been presented. The algorithm includes the use of several estimation algorithms running in parallel and a supervisory system which selects the most appropriate estimator at certain time instants. Such estimator is then kept in operation during a minimum residence time interval so that closed-loop stability is guaranteed.

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REFERENCES

Alonso-Quesada, S. (2001). *Estabilización robusta adaptable de sistemas con modelos nominales de planta estabilizables pero, posiblemente, no controlable*, Chap. 4. Ph. Dissertation, UPV/EHU Univ., Leioa, Spain.

Chen, H-F and X-R, Cao (1997). Controllability is not necessary for adaptive pole placement control. *IEEE Trans. Automat. Contr.* **42**, 1222-1229.

De la Sen, M. and S. Alonso-Quesada (1999). Robust adaptive regulation of potentially inversely unstable first-order hybrid system. *Journal of the Franklin Institute*, **336**, 627-648.

Ioannou, P. A. and A. Datta (1991). Robust adaptive control: A unified approach. *Proceedings of the IEEE*, **79**, 1736-1768.

Middleton, R. H., G. C. Goodwin, D. J. Hill and D. Q. Mayne (1988). Design issues in adaptive control. *IEEE Trans. on Automat. Contr.*, **33**, 50-58.

Narendra, K. S., Y. H. Lin and L. S. Valavani (1980). Stable adaptive controller design, Part II: Proof of stability. *IEEE Trans. on Automat. Contr.*, **AC-25**, 440-448.

Narendra, K. S. And J. Balakrishnan (1997). Adaptive control using multiple models. *IEEE Trans. on Automat. Contr.*, **42**, 171-187.

<u>*Note*</u>: The appendixes A, B and C have been suppressed because of lack of space.