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APPLICATION OF ROBUST OBSERVER-BASED FDI SYSTEMS TO FAULT TOLERANT CONTROL

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Abstract: In this paper, an active fault tolerant control (FTC) strategy is presented for linear dynamic systems. The robust observer-based fault detection and isolation (FDI) systems are applied to guide the reconfiguration of controller parameters to achieve the optimal control performance during different operating conditions of the system: fault-free, fault detected and fault isolated. The selection of design parameters is achieved using the linear matrix inequality (LMI) optimization technique.

Keywords: Fault tolerant systems, fault detection, fault isolation, observers, uncertainty, robustness.

1. INTRODUCTION

During the last years, fault tolerant control (FTC) has received more and more attention (Patton, 1997; Blanke *et al.*, 2000; Isermann *et al.*, 2000). Due to its intimate relationship to the robust control theory, robust controllers based passive FTC received the first attention at the end of the 80's and is still an actual research topic on account of the development of the robust control theory (Wu and Chen, 1996). The major advantage of this scheme is its simplicity in implementation while its application is strongly limited. In comparison with it, active FTC may improve the fault tolerant performance by reconfigurating controller parameters or even structure. In the active FTC scheme, fault detection and isolation (FDI) plays an important role. Among the existing FDI approaches (Frank and Ding, 1997; Gertler, 1998; Chen and Patton, 1999), the parameter identification technique is widely integrated in the active FTC systems to achieve fault identification (Zhang and Jiang, 1999). On the other hand, the known limitations of the parameter identification technique (Chen and Patton, 1999; Frank and Ding, 1997) may restrict the application of such kind of FTC systems.

In this contribution, an FTC strategy is presented, whose core is an observer-based FDI system and a reconfiguration algorithm of controller parameters based on the information delivered by the FDI system. The basic idea is to ensure an optimal control performance in different operating conditions by switching the controller between different control laws. The output of the observer-based FDI system controls the switching action.

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It is evident that the FDI system should react very fast to fault and be able to deliver information about the operating conditions of the system as early as possible so that the controller parameters can be correspondingly adjusted and, as a result, the best control performance can be achieved. To ensure it, an optimally sensitive FDI system is needed. The design of such an FDI system from the FTC viewpoint is the main objective of this paper.

In practice it is very difficult to get an exact mathematical model of real systems. Thus the FTC problem of linear systems with model uncertainty is also treated using the linear matrix inequality (LMI) technique.

2. PRELIMINARY

In this section, the observer-based FDI schemes is briefly reviewed.

Consider linear time-invariant (LTI) processes without model uncertainty described by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + E_f f(t) + E_d d(t)\\ y(t) &= Cx(t) + Du(t) + F_f f(t) + F_d d(t)\end{aligned} \quad (1)$$

where x, u, d, f and y are vectors of states, control inputs, disturbances, faults and measured outputs, respectively. d is unknown but bounded by $\|d\|_2 \leqslant \Delta_d$. The transfer function matrices from u, d, f to y are denoted as $G_u(s), G_d(s)$ and $G_f(s)$ respectively. Without loss of generality, assume

A1. (C, A) is detectable;
A2. $\begin{bmatrix} A - j\omega I & E_d \\ C & F_d \end{bmatrix}$ has full row rank for all ω.

The first step to FD is residual generation. An observer-based residual generator can be constructed as

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))\\ r_b(t) &= y(t) - \hat{y}(t) = y(t) - C\hat{x}(t) - Du(t)\\ r(s) &= R(s)r_b(s)\end{aligned} \quad (2)$$

where L is the observer gain matrix, $R(s) \in RH_\infty$ is the so-called post filter which is an arbitrarily selectable parametrization matrix (Frank and Ding, 1997). Note that if $R(s) = V$ with V being a constant matrix, then residual generator (2) reduces to the standard fault detection filter (FDF).

It can be derived that the dynamics of the residual generator (2) is governed by

$$\begin{aligned}r(s) &= R(s)M_u(s)(G_d(s)d(s) + G_f(s)f(s))\\ M_u(s) &= I - C(sI - A + LC)^{-1}L\end{aligned} \quad (3)$$

To evaluate the residual, the 2-norm of the residual signal r is used as the evaluation function and the decision logic is the mostly used one

$$\begin{aligned}\|r\|_2 > J_{th} &\Longrightarrow fault\\ \|r\|_2 \leqslant J_{th} &\Longrightarrow no\ fault\end{aligned} \quad (4)$$

where J_{th} is the threshold selected as

$$J_{th} = \sup_{d, f=0} \|r\|_2 = \|R(s)M_u(s)G_d(s)\|_\infty \Delta_d \quad (5)$$

The main objective of designing residual generators is to improve the sensitivity of the FD system to faults without loss of the robustness to disturbances. Thus the selection of the design parameters L and $R(s)$ can be formulated as an optimization problem

$$\min_{R(s),L} \frac{\|R(s)M_u(s)G_d(s)\|_\infty}{\sigma_i(R(s)M_u(s)G_f(s))} \quad (6)$$

where $\sigma_i(R(s)M_u(s)G_f(s))$ denotes some nonzero singular value of $R(s)M_u(s)G_f(s)$.

Lemma 1. (Ding *et al.*, 2000) Given system (1) satisfying Assumption A1-A2. Suppose $M_u(s)G_d(s)$ has a co-inner-outer factorization (CIOF) as

$$M_u(s)G_d(s) = G_{do}(s)G_{di}(s) \quad (7)$$

where $G_{do}(s)$ is co-outer and has an RH_∞ left inverse $G_{do}^{-1}(s)$, $G_{di}(s)$ is co-inner satisfying $G_{di}(j\omega)G_{di}^T(-j\omega) = I$, then

$$R(s) = G_{do}^{-1}(s) \quad (8)$$

solves the optimization problem (6).

For the purpose of fault isolation, a bank of FD systems are designed. Each of them is sensitive to some faults while robust to the rest faults and disturbances. With a suitable decision logic the faults can be isolated (Chen and Patton, 1999).

In residual generator (2), post filter $R(s)$ plays an important role in that it releases the observer gain L from the task of optimizing FDI performance, as shown below. Motivated by this, in the FTC strategy described in the next section, FDI system and controller use the same observer without impairing either control or FDI performance.

Suppose that $\bar{R}(s)$ solves (6) and generates a residual $\bar{r}$ with optimal dynamics

$$\begin{aligned}\bar{r}(s) &= \bar{R}(s)\bar{M}_u(s)(G_d(s)d(s) + G_f(s)f(s))\\ \bar{M}_u(s) &= I - C(sI - A + \bar{L}C)^{-1}\bar{L}\end{aligned}$$

If now the observer gain is selected as $L \neq \bar{L}$, then the residual dynamics is governed by (3). Because there exists always a matrix (Ding and Guo, 1997)

$$Q(s) = I + C(sI - A + \bar{L}C)^{-1}(L - \bar{L})$$

such that $Q(s)M_u(s) = \bar{M}_u(s)$, the optimal residual dynamics $\bar{r}$ can always be achieved by letting $R(s) = \bar{R}(s)Q(s)$. Moreover, $R(s) \in RH_\infty$ since $Q(s) \in RH_\infty$. Thus, as long as L stabilizes $A - LC$, the optimal residual $\bar{r}(s)$ can always be obtained by a suitable selection of $R(s)$.

In the same way, it can be shown that the optimality of the residual generated by the FDF depends not only on V but also on L. This is also the reason why the post filter $R(s)$ is used in our robust observer-based FDI systems.

3. DESCRIPTION OF THE FTC STRATEGY

The FTC problem is considered for linear systems described by

$$\begin{aligned} \dot{x}(t) &= \tilde{A}x(t) + Bu(t) + E_f f(t) + E_d d(t) \qquad (9)\\ z(t) &= C_1 x(t) + D_1 u(t) + F_{f1} f(t) + F_{d1} d(t)\\ y(t) &= C_2 x(t) + D_2 u(t) + F_{f2} f(t) + F_{d2} d(t) \end{aligned}$$

where z denotes the controlled signal, x, u, d, f, y are the same as before. The system matrix $\tilde{A} = A$ in the nominal case and $\tilde{A} = A + \Delta A$ in the case of model uncertainty with ΔA structured as

$$\Delta A = M \Sigma N \qquad (10)$$

where M, N are known matrices, Σ is unknown but bounded by $\Sigma^T \Sigma \leqslant I$.

The controller is based on the observer

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))\\ \hat{y}(t) &= C_2 \hat{x}(t) + D_2 u(t)\\ r_b(t) &= y(t) - \hat{y}(t) \qquad (11) \end{aligned}$$

The control law is

$$u(t) = -K\hat{x}(t) + Hr_b(t) + v(t) \qquad (12)$$

where v is the reference input signal, H is a constant matrix

$$H = \begin{cases} 0, & \|r\|_2 \leqslant J_{th} \\ H, & \|r\|_2 > J_{th} \end{cases}$$

i.e. if a fault is detected, the residual signal will be taken as an input to the controller to compensate the influence of faults. The same observer is used for the FDI purpose with post filter $R(s)$ whose state space realization is (A_R, B_R, C_R, D_R).

In the normal operating conditions of the system, i.e. $\|r\|_2 \leqslant J_{th}$, the controller parameters are set to $H_1 = 0$ and K_1, L_1 which solve the optimization problem

$$\min_{K,L} \|[T_{zd}(s) \;\; T_{zv}(s)]\|_\infty \qquad (13)$$

If $\|r\|_2 > J_{th}$, a fault is detected. The controller is then reconfigured to K_2, L_2, H_2 which solve

$$\min_{K,L,H} \|[T_{zd}(s) \;\; T_{zf}(s) \; T_{zv}(s)]\|_\infty \qquad (14)$$

in order to tolerate all possible faults. And a bank of post filters are activated to isolate the faults.

After the occurring fault $\tilde{f}$ is isolated, the controller is reconfigured to K_3, L_3, H_3 which solve

$$\min_{K,L,H} \|[T_{zd}(s) \;\; T_{z\tilde{f}}(s) \; T_{zv}(s)]\|_\infty \qquad (15)$$

in order to tolerate the occurring fault. Correspondingly, the FDI system is also reconfigured to be able to detect new faults. Note that in each operating condition of the system, the stability of the system is the basic requirement.

During the transition phase that the fault has happened but not yet been detected by the FDI system, the closed-loop stability will not be influenced because the fault enters the system as an additive external signal. Moreover, the supremum of the 2-norm of the controlled signal z during this period can be determined as follows. Since $\|r\|_2 \leqslant J_{th}$, the fault is deduced to satisfy

$$\|f\|_2 \leqslant \frac{2J_{th}}{\sigma_{\min}(R(s)M_u(s)G_{f2}(s))}$$

Thus

$$\begin{aligned} \|z\|_{2,\tau} &\leqslant \|z\|_2\\ &= \|T_{zd}(s)d(s) + T_{zf}(s)f(s) + T_{zv}(s)v(s)\|_2\\ &\leqslant \|T_{zd}(s)\|_\infty \Delta_d + \frac{2\|T_{zf}(s)\|_\infty J_{th}}{\sigma_{\min}(R(s)M_u(s)G_{f2}(s))}\\ &\quad + \|T_{zv}(s)\|_\infty \|v\|_2 \end{aligned}$$

where τ denotes the detection delay.

It is worth noticing that in the above presented FTC strategy, the controller aims not only to ensure the stability of the overall system, but also to achieve the best possible performance making use of the information delivered by the FDI system.

4. PROBLEM FORMULATION

The design parameters are K, L, H and $R(s)$. Remembering that L can be devoted fully to improving the control performance, the design for each operating condition of the system is completed in two steps. At first, $K, L, (H)$ are selected to optimize the control performance. Then, based on the resulting $L, (H)$, the post filter $R(s)$ is selected to improve the FDI performance.

In the following, our attention is focused on two sub-problems:

- Design of the optimal post filter;
- Solution to the optimization problem (14)

since the solution to (13) is well-known (Wang and Shieh, 1992) and the solution to (15) is similar to the solution to (14).

5. NOMINAL DESIGN

In this section, we consider the design of optimal post filter for LTI systems (9) without model uncertainty, which is also called nominal design. The solution to the optimization problem (14) can be derived by simplifying the algorithm given in Section 6.1, thus it is omitted here.

From Lemma 1, the key to get the optimal post filter is to do the CIOF (7). Note that

$$\begin{aligned} M_u(s)G_{d2}(s) &= N_d(s) \\ &= F_{d2} + C_2(sI - A + LC_2)^{-1}(E_d - LF_{d2}) \end{aligned}$$

The following theorem is obtained by applying the CIOF approach given in Francis (1987).

Theorem 2. Suppose that L is determined by the controller design, the post filter $R(s)$ that solves (6) is given by

$$R(s) = \bar{V} + \bar{V}C_2(sI - A + \bar{L}C_2)^{-1}(L - \bar{L}) \quad (16)$$

where

$$\begin{aligned} \bar{L} &= [E_d F_{d2}^T + XC_2^T](F_{d2}F_{d2}^T)^{-1} \\ \bar{V} &= (F_{d2}F_{d2}^T)^{-\frac{1}{2}} \end{aligned} \quad (17)$$

and $X \geq 0$ solves the Riccati equation

$$\begin{aligned} &\bar{A}X + X\bar{A}^T - XC_2^T(F_{d2}F_{d2}^T)^{-1}C_2X \\ &\quad + E_d(I - F_{d2}^T(F_{d2}F_{d2}^T)^{-1}F_{d2})E_d^T = 0 \end{aligned} \quad (18)$$

with $\bar{A} = A - E_d F_{d2}^T(F_{d2}F_{d2}^T)^{-1}C_2$.

Correspondingly, the threshold is set to be

$$J_{th} = \|R(s)M_u(s)G_{d2}(s)\|_\infty \Delta_d = \Delta_d$$

Remark 1. The dynamics of the optimal residual is independent of L and governed by

$$r(s) = \bar{V}\bar{N}_d(s)d(s) + \bar{V}\bar{N}_f(s)f(s) \quad (19)$$

where

$$\begin{aligned} \bar{N}_d(s) &= F_{d2} + C_2(sI - A + \bar{L}C_2)^{-1}(E_d - \bar{L}F_{d2}) \\ \bar{N}_f(s) &= F_{f2} + C_2(sI - A + \bar{L}C_2)^{-1}(E_f - \bar{L}F_{f2}) \end{aligned}$$

The optimal residual (19) will be called nominal optimal residual in the following sections.

Remark 2. We would like to point out that $\bar{L}$ and $\bar{V}$ in (17) is also the unique solution to the optimization problem formulated for the standard FDF design

$$\min_{V,L} \frac{\|VM_u(s)G_{d2}(s)\|_\infty}{\sigma_i(VM_u(s)G_{f2}(s))}$$

6. ROBUST DESIGN

In this section, the two sub-problems formulated in Section 4 are solved for LTI systems with model uncertainty described by (9)-(10).

6.1 Solution to the optimization problem (14)

The control loop dynamics is governed by

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + \Delta A - BK & BK + BHC_2 \\ \Delta A & A - LC_2 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$
$$+ \begin{bmatrix} E_d + BHF_{d2} \\ E_d - LF_{d2} \end{bmatrix} d + \begin{bmatrix} E_f + BHF_{f2} \\ E_f - LF_{f2} \end{bmatrix} f + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

$$z = \begin{bmatrix} C_1 - D_1K & D_1K + D_1HC_2 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$
$$+ (F_{d1} + D_1HF_{d2})d + (F_{f1} + D_1HF_{f2})f + D_1v$$

The optimization problem (14) is re-formulated as

$$\min_{K,L,H} \alpha \quad (20)$$

where $\|[T_{zd}(s)\ T_{zf}(s)\ T_{zv}(s)]\|_\infty < \alpha$ and the control loop is stable. Because of the limitation of space, only the algorithm is given below.

Step 1: For a given value of $\alpha > 0$, find a positive definite matrix Q, a matrix $\bar{K}$ and a positive real number ϵ, which solve the LMI

$$\begin{bmatrix} U_{11} & E_d & E_f & B & U_{15} & QN^T \\ E_d^T & -\alpha I & 0 & 0 & F_{d1}^T & 0 \\ E_f^T & 0 & -\alpha I & 0 & F_{f1}^T & 0 \\ B^T & 0 & 0 & -\alpha I & D_1^T & 0 \\ U_{15}^T & F_{d1} & F_{f1} & D_1 & -\alpha I & 0 \\ NQ & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix} < 0 \quad (21)$$

$$U_{11} = AQ + QA^T - B\bar{K} - \bar{K}^T B^T + \epsilon MM^T$$
$$U_{15} = (C_1Q - D_1\bar{K})^T$$

and let $K = \bar{K}Q^{-1}$. If (21) has no solution, go directly to Step3.

Step 2: Based on the resulting K and using the alternating projection algorithm (Skelton *et al.*, 1998), find positive definite matrices R_1, S_1, S_2 and positive real number ε which satisfy $R_1S_1 = I$ and the LMI's

$$W_1^T\Phi W_1 < 0,\ W_2^T\Psi W_2 < 0 \quad (22)$$

with

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & E_d & E_f & B & M & R_1N^T \\ \Phi_{12}^T & -\alpha I & F_{d1} & F_{f1} & D_1 & 0 & 0 \\ E_d^T & F_{d1}^T & -\alpha I & 0 & 0 & 0 & 0 \\ E_f^T & F_{f1}^T & 0 & -\alpha I & 0 & 0 & 0 \\ B^T & D_1^T & 0 & 0 & -\alpha I & 0 & 0 \\ M^T & 0 & 0 & 0 & 0 & -\delta I & 0 \\ NR_1 & 0 & 0 & 0 & 0 & 0 & -\delta^{-1}I \end{bmatrix}$$

$$\Phi_{11} = (A - BK)R_1 + R_1(A - BK)^T$$
$$\Phi_{12} = R_1(C_1 - D_1K)^T$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & S_1E_d & S_1E_f & S_1B & \Psi_{16} & S_1M \\ \Psi_{12}^T & \Psi_{22} & S_2E_d & S_2E_f & 0 & \Psi_{26} & S_2M \\ E_d^TS_1 & E_d^TS_2 & -\alpha I & 0 & 0 & F_{d1}^T & 0 \\ E_f^TS_1 & E_f^TS_2 & 0 & -\alpha I & 0 & F_{f1}^T & 0 \\ B^TS_1 & 0 & 0 & 0 & -\alpha I & D_1^T & 0 \\ \Psi_{16}^T & \Psi_{26}^T & F_{d1} & F_{f1} & D_1 & -\alpha I & 0 \\ M^TS_1 & M^TS_2 & 0 & 0 & 0 & 0 & -\delta I \end{bmatrix}$$

$$\Psi_{11} = S_1(A - BK) + (A - BK)^TS_1 + \delta N^TN$$
$$\Psi_{12} = S_1BK,\ \Psi_{16} = (C_1 - D_1K)^T$$
$$\Psi_{22} = A^TS_2 + S_2A,\ \Psi_{26} = (D_1K)^T$$

$$W_1 = diag\{W_{11}, I, I\},\ W_2 = \begin{bmatrix} 0 & I & 0 \\ W_{22} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

where W_{11} and W_{22} are the bases of null spaces of $\begin{bmatrix} B^T & D_1^T \end{bmatrix}$ and $\begin{bmatrix} C_2 & F_{d2} & F_{f2} \end{bmatrix}$ respectively.

Step 3: Reduce or increase the value of α, iterate the above process till the minimal α is achieved.

Step 4: Substitute S_1, S_2 into the LMI to get Θ

$$\Psi + \digamma^T \Theta \Xi + \Xi^T \Theta^T \digamma < 0 \tag{23}$$

$$\digamma = \begin{bmatrix} B^T S_1 & 0 & 0 & D_1^T & 0 \\ 0 & -S_2 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi = \begin{bmatrix} 0 & C_2 & F_{d2} & F_{f2} & 0 & 0 & 0 \end{bmatrix}$$

Step 5: Partition Θ into $\begin{bmatrix} H \\ L \end{bmatrix}$.

Thus the controller design is completed.

6.2 Design of the optimal post filter

To deal with model uncertainty ΔA, the nominal optimal residual is taken as a rule, since it represents the best compromise between the robustness to disturbances and sensitivity to faults in the ideal nominal case. The design problem is formulated as: Find the optimal post filter $R(s)$ so that in the face of model uncertainty ΔA, the residual approximates the nominal optimal residual to preserve the best trade-off property.

To this aim, define $\xi(t) = r(t) - \bar{r}(t)$. $\xi(t)$ reflects the difference between $r(t)$ and nominal optimal residual $\bar{r}(t)$. From Remark 1, the dynamics of $\bar{r}(t)$ with respect to disturbance and fault is

$$\begin{aligned} \dot{x}_n(t) &= (A - \bar{L}C_2)x_n(t) + (E_f - \bar{L}F_{f2})f(t) \\ &\quad + (E_d - \bar{L}F_{d2})d(t) \\ \bar{r}(t) &= \bar{V}C_2 x_n(t) + \bar{V}F_{f2}f(t) + \bar{V}F_{d2}d(t) \end{aligned} \tag{24}$$

where $\bar{L}$ and $\bar{V}$ are defined as in (17).

Let $e(t) = x(t) - \hat{x}(t)$. Considering (9)-(12), the transfer function from the external input $\eta = [\, f^T \ d^T \ v^T \,]^T$ to ξ can be expressed into the lower linear fractional transformation of an extended plant $P_p(s)$ and the post filter $R(s)$, i.e. $T_{\xi\eta}(s) = F_l(P_p(s), R(s))$, where

$$\begin{aligned} \dot{X}_p(t) &= A_p X_p(s) + B_{1p}\eta(t) + B_{2p}r(t) \\ \xi(t) &= C_{1p}X_p(s) + D_{11p}\eta(t) + D_{12p}r(t) \\ r_b(t) &= C_{2p}X_p(s) + D_{21p}\eta(t) + D_{22p}r(t) \end{aligned} \tag{25}$$

with $X_p = [\, x^T \ e^T \ x_n^T \,]^T$ and

$$A_p = A_{p0} + M_p \Sigma N_p$$

$$A_{p0} = \begin{bmatrix} A - BK & BK + BHC_2 & 0 \\ 0 & A - LC_2 & 0 \\ 0 & 0 & A - \bar{L}C_2 \end{bmatrix}$$

$$M_p = [\, M^T \ M^T \ 0 \,]^T, \ N_p = [\, N \ 0 \ 0 \,]$$

$$B_{1p} = \begin{bmatrix} E_f + BHF_{f2} & E_d + BHF_{d2} & B \\ E_f - LF_{f2} & E_d - LF_{d2} & 0 \\ E_f - \bar{L}F_{f2} & E_d - \bar{L}F_{d2} & 0 \end{bmatrix}$$

$$B_{2p} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \ C_{1p} = \begin{bmatrix} 0 & 0 & -\bar{V}C_2 \end{bmatrix}$$

$$C_{2p} = \begin{bmatrix} 0 & C_2 & 0 \end{bmatrix}, \ D_{21p} = \begin{bmatrix} F_{f2} & F_{d2} & 0 \end{bmatrix}$$

$$D_{11p} = \begin{bmatrix} -\bar{V}F_{f2} & -\bar{V}F_{d2} & 0 \end{bmatrix}, \ D_{12p} = I, \ D_{22p} = 0$$

Define

$$\Theta_R = \begin{bmatrix} A_R & B_R \\ C_R & D_R \end{bmatrix} \tag{26}$$

The closed-loop transfer function from η to ξ is

$$T_{\xi\eta}(s) = C_c(sI - A_c)^{-1}B_c + D_c \tag{27}$$

where

$$A_c = A_{c0} + M_c \Sigma N_c \tag{28}$$

$$A_{c0} = A_o + \tilde{B}\Theta_R\tilde{C}, \ B_c = B_o + \tilde{B}\Theta_R\tilde{D}_{21}$$

$$C_c = C_o + \tilde{D}_{12}\Theta_R\tilde{C}, \ D_c = D_{11p} + \tilde{D}_{12}\Theta_R\tilde{D}_{21}$$

$$A_o = \begin{bmatrix} A_{p0} & 0 \\ 0 & 0 \end{bmatrix}, \ B_o = \begin{bmatrix} B_{1p} \\ 0 \end{bmatrix}, \ C_o = \begin{bmatrix} C_{1p} & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 & B_{2p} \\ I & 0 \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} 0 & I \\ C_{2p} & 0 \end{bmatrix}, \ \tilde{D}_{21} = \begin{bmatrix} 0 \\ D_{21p} \end{bmatrix}$$

$$\tilde{D}_{12} = \begin{bmatrix} 0 & D_{12p} \end{bmatrix}, \ M_c = \begin{bmatrix} M_p \\ 0 \end{bmatrix}, \ N_c = \begin{bmatrix} N_p & 0 \end{bmatrix}$$

The optimization problem is thus re-formulated as to find an optimal parameter set Θ_R, so that the closed-loop system is stable and

$$\min_{\Theta_R} \gamma \tag{29}$$

$$\|T_{\xi\eta}(s)\|_\infty < \gamma \tag{30}$$

According to the well-known Bounded Real Lemma, (30) holds if and only if there exists a positive definite matrix P such that

$$\begin{aligned} &P(A_{c0} + M_c\Sigma N_c) + (A_{c0} + M_c\Sigma N_c)^T P + C_c^T C_c \\ &+ (PB_c + C_c^T D_c)(\gamma^2 I - D_c^T D_c)^{-1}(B_c^T P + D_c^T C_c) < 0 \end{aligned}$$

Because for any real number $\varepsilon > 0$, there is

$$\begin{aligned} &PM_c\Sigma N_c + (M_c\Sigma N_c)^T P \\ &\qquad \leqslant \varepsilon N_c^T N_c + \varepsilon^{-1} P M_c M_c^T P \end{aligned}$$

Thus if there exists $P > 0, \varepsilon > 0$ such that

$$\begin{bmatrix} PA_{c0} + A_{c0}^T P + \varepsilon N_c^T N_c & PB_c & C_c^T & PM_c \\ B_c^T P & -\gamma I & D_c^T & 0 \\ C_c & D_c & -\gamma I & 0 \\ M_c^T P & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

then (30) holds. Considering (28), the above LMI can be re-written as

$$\Pi + P_o \Lambda^T \Theta_R \Gamma + \Gamma^T \Theta_R^T \Lambda P_o < 0 \tag{31}$$

$$\Gamma = \begin{bmatrix} \tilde{C} & \tilde{D}_{21} & 0 & 0 \end{bmatrix}$$

$$\Lambda = [\, \tilde{B}^T \ 0 \ \tilde{D}_{12}^T \ 0 \,], \ P_o = diag\{P, I, I, I\}$$

$$\Pi = \begin{bmatrix} PA_o + A_o^T P + \varepsilon N_c^T N_c & PB_o & C_o^T & PM_c \\ B_o^T P & -\gamma I & D_{11p}^T & 0 \\ C_o & D_{11p} & -\gamma I & 0 \\ M_c^T P & 0 & 0 & -\varepsilon I \end{bmatrix}$$

According to Skelton *et al.* (1998), (31) is solvable for some Θ_R if and only if

$$\Gamma_\perp^T \Pi \Gamma_\perp < 0, \ \Lambda_\perp^T P_o^{-1} \Pi P_o^{-1} \Lambda_\perp < 0 \tag{32}$$

where $\Gamma_\perp$, $\Lambda_\perp$denote the bases of null spaces of Γ, Λ respectively.

Partition P and P^{-1} as

$$P = \begin{bmatrix} S & Y \\ Y^T & * \end{bmatrix}, \; P^{-1} = \begin{bmatrix} Q^{-1} & X \\ X^T & * \end{bmatrix} \tag{33}$$

By substituting (28) and (33) into (32) and taking into account that $B_{2p} = [\,0\ 0\ 0\,]^T$, $D_{12p} = I$, the following theorem is obtained.

Theorem 3. Consider system (27) and let $\bar{\Gamma}_\perp$ denote the base of null space of $[\,C_{2p}\ D_{21p}\,]$. There exists a post filter $R(s)$ of order k_r so that A_c is stable and (30) holds, if there exist matrices $S > 0, Q > 0$ and a real number $\varepsilon > 0$ satisfying

$$\bar{\Phi} < 0,\; W^T \bar{\Pi} W < 0 \tag{34}$$

$$S \geqslant Q,\; Rank(Q - S) \leqslant k_r \tag{35}$$

where $W = diag\{\bar{\Gamma}_\perp, I, I\}$ and

$$\bar{\Phi} = \begin{bmatrix} QA_{p0} + A_{p0}^T Q + \varepsilon N_p^T N_p & QB_{1p} & QM_p \\ B_{1p}^T Q & -\gamma I & 0 \\ M_p^T Q & 0 & -\varepsilon I \end{bmatrix}$$

$$\bar{\Pi} = \begin{bmatrix} SA_{p0} + A_{p0}^T S + \varepsilon N_p^T N_p & SB_{1p} & C_{1p}^T & SM_p \\ B_{1p}^T S & -\gamma I & D_{11p}^T & 0 \\ C_{1p} & D_{11p} & -\gamma I & 0 \\ M_p^T S & 0 & 0 & -\varepsilon I \end{bmatrix}$$

In conclusion, the robust optimal design of post filter $R(s)$ is summarized as follows:

- Compute the nominal optimal residual dynamics $\bar{r}(t)$ according to (24).
- Construct the matrices of $P_p(s)$ by (25).
- Solve iteratively (34)-(35) using alternating projection algorithm (Skelton *et al.*, 1998) to get the minimal γ and matrices $S > 0, Q > 0$.
- Compute two full column rank matrices X, Y such that $XY^T = I - Q^{-1}S$.
- Solve the linear equation
$$\begin{bmatrix} S & I \\ Y^T & 0 \end{bmatrix} = P \begin{bmatrix} I & Q^{-1} \\ 0 & X^T \end{bmatrix} \tag{36}$$
and obtain the unique solution P.
- Substitute P into the LMI (31) and get Θ_R.
- Partition Θ_R as (26).

In the fault-free case

$$\begin{aligned} \|r\|_2 &= \|T_{rd}(s)d(s) + T_{rv}(s)v(s)\|_2 \\ &\leqslant \|T_{rd}(s)\|_\infty \|d\|_2 + \|T_{rv}(s)\|_\infty \|v\|_2 \end{aligned}$$

Therefore, the threshold is set to be

$$J_{th} = \|T_{rd}(s)\|_\infty \Delta_d + \|T_{rv}(s)\|_\infty \|v\|_2 \tag{37}$$

which suggests that the threshold adapts to the reference input signal v.

7. CONCLUSION

In this paper, an FTC strategy is proposed for linear systems. Key is to apply the robust observer-based FDI system to the reconfiguration of controller parameters and thus to improve the control performance in every operating conditions of the system. Structurally the controller and the FDI system use the same observer while their designs are carried out successively. The proposed FTC system has an explicit physical structure and ensures good control and FDI performance simultaneously. The basic idea of the proposed FTC strategy can also be extended to nonlinear processes. Due to the essential complexity and variety of nonlinear processes, the design of an optimally sensitive and robust observer-based FDI system is still under research.

REFERENCES

Blanke, M., C.W. Frei, F. Kraus, R.J. Patton and M. Staroswiecki (2000). What is fault-tolerant control?. In: *Proc. IFAC Symp. SAFEPROCESS 2000*. pp. 40–51.

Chen, J. and R. Patton (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems.* Kluwer Academic Publishers. Boston.

Ding, S.X., T. Jeinsch, P.M. Frank and E.L. Ding (2000). A unified approach to the optimization of fault detection systems. *International Journal of Adaptive Control and Signal Processing* **14**, 725–745.

Ding, X. and L. Guo (1997). On observer based fault detection. In: *Proc. SAFEPROCESS '97*. pp. 112–120.

Francis, B.A. (1987). *A Course in H_∞ Theory.* Springer-Verlag. Berlin.

Frank, P.M. and X. Ding (1997). Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *J. of Process Control* **7**, 403–424.

Gertler, J.J. (1998). *Fault Detection and Diagnosis in Engineering Systems.* Marcel Dekker. New York.

Isermann, R., R. Schwarz and S. Stoelzl (2000). Fault-tolerant drive-by-wire systems - concepts and realizations. pp. 1–15.

Patton, R.J. (1997). Fault-tolerant control: The 1997 situation. In: *Proc. IFAC Symp. SAFEPROCESS '97*. Hull. pp. 1033–1055.

Skelton, R.E., T. Iwasaki and K.M. Grigoriadis (1998). *A Unified Algebraic Approach to Linear Control Design.* Taylor and Francis. London.

Wang, Y.J. and L.S. Shieh (1992). Observer-based robust h_∞ laws for uncertain linear systems. *J. Guidance, Control, Dyn.* **15**, 1125–1133.

Wu, N.E. and T.J. Chen (1996). Feedback design in control reconfiguration systems. *Int. J. of Robust and Nonlinear Control* **6**, 560–570.

Zhang, Y.M. and J. Jiang (1999). Design of integrated fault detection, diagnosis and reconfigurable control systems. In: *Proc. The 38th IEEE CDC*. pp. 3587–3592.