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FUZZY OBJECTIVE FUNCTIONS IN MULTIVARIABLE PREDICTIVE CONTROL

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Abstract: In order to incorporate fuzzy goals and constraints in model predictive control, this control technique have recently been integrated with fuzzy decision making. The goals and the constraints of the control problem are combined by using a decision function from the theory of fuzzy sets. This technique have been studied for single-input single-output processes. This paper extends this approach for multivariable processes. The simulation of a gantry crane system is used as case study. The results show clearly the advantage of using fuzzy predictive control in multivariable systems. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Fuzzy logic is widely used in control, by describing the control law by if–then rules. However, human expertise can be used to define the design specifications. These specifications are translated to performance criteria using fuzzy sets, by defining the (fuzzy) goals and the (fuzzy) constraints for the system under control.

This procedure is a particular approach of fuzzy model-based control, following closely the classical model predictive control (MPC) design approach, but it makes use of the fuzzy sets theory in a higher level than in standard fuzzy logic control, where the fuzzy rules to control the system are given directly from expert knowledge. In the approach presented in this paper, the appropriate control actions are obtained by means of a multistage fuzzy decision making algorithm, as introduced by Bellman and Zadeh (1970).

Some examples of this high-level approaches can be found: The first application in this field was in the automatic train operation using a linguistic description of the system (Yasunobu and Miyamoto 1985). A survey on model-based approach to fuzzy control and decision making is presented by Kacprzyk (1997). More recently, satisficing decisions have also been used in a similar setting to design controllers (Goodrich *et al.* 1998).

A detailed study of model predictive control using fuzzy decision functions is presented by Sousa and Kaymak (2001). This paper generalizes this last approach to multivariable systems. An illustrative example, the predictive control of a container gantry crane, is used to show the necessary steps to go from single to multivariable fuzzy predictive control.

This paper begins by describing the application of fuzzy criteria to predictive control in Section 2. The possible types of fuzzy objective functions for predictive control are briefly presented in Section 3, where the operators to aggregate fuzzy criteria are also briefly addressed. The generalization of the fuzzy goals and constraints to multivariable systems is illustrated by an example, consisting of the simulation of a container gantry crane in Section 4. Finally, Section 5 concludes the paper.

2. FUZZY PREDICTIVE CONTROL

Multistage decision making has been applied to control by several authors (Bellman and Zadeh 1970, Kacprzyk 1997). When multistage decision making is translated to the control environment, the set of alternatives constitute the different *control actions*, the *system* under control is a relationship between the system inputs and outputs (or causes and effects), and the mapping relating the inputs to the outputs of the system under control is referred to as the *model*. Moreover, fuzzy constraints are defined for several variables presented in the system, which can be 'hard' or 'soft' constraints, and the decision criteria (fuzzy goals and constraints) are the translation of the control performance criteria to the decision making setting.

One of the main issues in model predictive control is the type of model of the system under control (Richalet 1993). Another important issue in fuzzy decision making (FDM) applied to control is the termination time, which can be seen as a generalization of the prediction horizon defined for model predictive control (Soeterboek 1992). Multistage decision making in a fuzzy environment considering any type of model in closed-loop control have been introduced by Sousa and Kaymak (2001). This paper assumes that the termination time is the prediction horizon, which is shifted when time evolves. This condition is necessary to allow the application of multistage fuzzy decision making to predictive control.

Let G_i, with $i = 1,\ldots,q$, be a fuzzy goal characterized by its membership function μ_{G_i}, which is a mapping from the space of the goal G_i to the interval $[0,1]$. Let also C_l, $l = 1,\ldots,r$ be a fuzzy constraint characterized by its membership function μ_{C_l}, mapping the space of the constraint C_l to the same interval $[0,1]$. The fuzzy goals G_i and the fuzzy constraints C_l can be defined for the domain of the control actions, system's outputs, state variables or for any other convenient domain. Note that fuzzy constraints are usually defined in the domain of the control actions, and fuzzy goals are usually defined in the domain of the state space variables. A fuzzy set in the appropriate domain characterizes both the fuzzy goals and the fuzzy constraints. The goals and constraints are defined on relevant system variables. For example, a common control goal G_i is the minimization of the output error. The satisfaction of this goal is represented by a membership function, which is defined on the space (universe of discourse) of the output error.

Each fuzzy goal G_i and each fuzzy constraint C_l constitute a *decision criterion* ζ_j, $j = 1,\ldots,T$, where $T = q+r$ is the total number of goals and constraints. Each criterion is defined in the domain Φ_j, $j = 1,\ldots,T$, which can be any of the various domains used in control. In order to solve the optimization problem in low computational time, the optimization problem is defined in a discrete control space with a finite number of control alternatives. This limitation to digital control is however not too severe, and this methodology can still be applied to a large number of control problems. Therefore, the confluence of goals and constraints is defined in the following for discrete alternatives.

The fuzzy criteria must be aggregated in the control environment. Assume that a policy π is defined as a sequence of control actions for the entire prediction horizon in MPC, H_p:

$$\pi = \mathbf{u}(k),\ldots,\mathbf{u}(k+H_p-1), \quad \pi \in \Omega, \qquad (1)$$

where the control actions belong to a set of alternatives Ω. In the general case, all the criteria must be applied at each time step i, with $i = 1,\ldots,H_p$. Thus a criterion ζ_{ij} denotes that the criterion j is considered at time step $k+i$, with $i = 1,\ldots,H_p$ and $j = 1,\ldots,T$. Further, let $\mu_{\zeta_{ij}}$ denote the membership value that represents the satisfaction of the decision criteria after applying the control actions $\mathbf{u}(k+i)$. The total number of decision criteria for the decision problem is thus given by $\tilde{T} = T \cdot H_p$. The confluence of goals and constraints can be done by aggregating the membership values $\mu_{\zeta_{ij}}$. The membership value μ_π for the control sequence π is obtained using the aggregation operator $\circledast$ to combine the decision criteria, i.e.

$$\begin{aligned}\mu_\pi = \mu_{\zeta_{11}} \circledast \ldots \circledast \mu_{\zeta_{1q}} \circledast \\ \mu_{\zeta_{1(q+1)}} \circledast \ldots \circledast \mu_{\zeta_{1T}} \circledast \\ \vdots \\ \mu_{\zeta_{H_p(q+1)}} \circledast \ldots \circledast \mu_{\zeta_{H_pT}} .\end{aligned} \qquad (2)$$

In this equation the aggregation operator $\circledast$ combines the goals and the constraints. Various types of aggregation operations can be used as decision functions for expressing different decision strategies using the well-known properties of these operators (Kaymak 1998). Parametric triangular norms can generalize a large number of t-norms, and can control the degree of compensation between the different goals and constraints. Usually, parametric t-norms depend only on one parameter, which makes them much easier to tune when compared to weighted t-norms. On the other hand, they are not so general as the weighted approaches (Kaymak and Sousa 2002). The translation of each goal and each constraint for a given policy π to a membership value as in (3) avoids the specification of the criteria in a large dimensional space. The combination of criteria in different domains is done for a set of discrete alternatives, which corresponds to different policies π that can be applied to find the optimal control policy. The decision criteria in (3) should be satisfied as much as possible, which corresponds to the maximal value of the overall decision. Thus, the optimal sequence of control actions π^* is found by the maximization of μ_π:

$$\pi^* = \underset{\mathbf{u}(k),\dots,\mathbf{u}(k+H_{p-1})}{\arg\max} \mu_\pi. \quad (3)$$

Because the membership functions for the fuzzy criteria can have an arbitrary shape, and because of the nonlinearity of the decision function, the optimization problem (3) is usually non-convex. To deal with the increasing complexity of the optimization problem, a proper optimization algorithm must be chosen. One possibility is to use, for instance, the branch-and-bound method, as presented in (Sousa 2000).

The definition of fuzzy goals and constraints must be given by an operator or design engineer. Therefore, when FDM in control is considered, human knowledge is involved in specifying the control objectives and constraints, rather than the control protocol itself (Goodrich *et al.* 1999, Meiritz *et al.* 1995). Using a process model, a fuzzy decision making algorithm selects the control actions that best meet the specifications. Hence, a control strategy can be obtained that is able to push the process closer to the constraints, and that is able to force the process to a better performance based on the goals and the constraints set by the operator together with the known conditions provided by the system's designers. Note that this approach is closely related to model predictive control. The formulation of the control problem as a confluence of fuzzy goals and fuzzy constraints leads to a generalization of the objective function used in MPC. In this control environment, a policy π with the possible control actions $\mathbf{u}(k),\dots,\mathbf{u}(k+H_p-1)$ can be defined as in (1). The objective function using fuzzy criteria was defined in (3). The closed-loop control configuration is now discussed in more detail, in aspects concerning the criteria and the aggregation operator(s) used to combine them.

3. FUZZY CRITERIA IN PREDICTIVE CONTROL

Fuzzy criteria play a main role in fuzzy decision making. When FDM is applied to control, the fuzzy goals and the fuzzy constraints must be a translation of the (fuzzy) performance criteria defined for the system. The definition of performance criteria in the time domain has shown to be quite powerful in the model predictive control framework (Camacho and Bordons 1995). This section briefly presents the use of fuzzy performance criteria in predictive control, as introduced by Sousa and Kaymak (2001), and generalizes the criteria for multivariable control.

When a control system is designed, performance criteria must be specified. In the time domain, these criteria are usually defined in terms of a desired steady-state error between the reference and the output, rise time, overshoot, settling time, etc., representing the goals of the control system. In MPC, these goals must be translated into an objective function. This function is normally minimized (or maximized) over the prediction horizon, given the desired control actions. The translation of the (fuzzy) goals into an objective function can be done in two different ways.

- The control goals are explicitly expressed in the objective function, leading usually to long term predictions of the behavior of the system, and large prediction horizons H_p. This method requires an accurate process model and large computational effort.
- Only short-term predictions (a few steps ahead) are used in the objective function. This method is usually applied in predictive control when the available model of the system is not very accurate. However, it still can lead to high performance control, when the control goals can be translated to short-term goals, which are then represented in the objective function. This method is especially suitable for nonlinear systems, where a compromise between computational time to derive the control actions and accuracy of the predictions must be made.

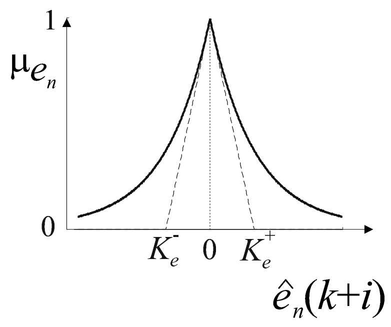
3.1 *Classical objective functions*

Conventional MPC mainly utilizes sum-quadratic functions as the objective function (Soeterboek 1992). In predictive control of multivariable systems, the output values $\hat{\mathbf{y}}(k+i)$, $i = 1,\dots,H_p$ depend on the states of the process at the current time k and on the future control signals $\mathbf{u}(k+j), j = 1,\dots,H_c$, where H_c is the *control horizon*. Let the overall control goals for the time domain be stated as achieving a fast system response while reducing the overshoot and the control effort. For multivariable systems these goals can be represented by the objective function

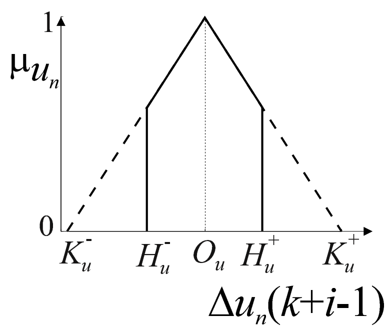
$$J = \mathbf{e}^T R\mathbf{e} + \Delta\mathbf{u}^T Q\Delta\mathbf{u} \quad (4)$$

where the first term of (4) accounts for the minimization of the outputs errors, the second term represents the minimization of the control effort, and R and Q are weighting matrices. Note that these parameters have two functions: they normalize the different outputs and inputs of the system, and they vary the importance of the two different terms in the objective function (4) over the time steps. If this is not the case, the optimization automatically weights different variables, which is not desirable, and it leads to poor control performance.

The objective function (4) can be interpreted as follows. The term containing the predicted errors indicates that these should be minimized, while the term containing the change in the control actions indicates that the control effort should be reduced. The matrices containing the weights, R and Q, can be changed so that the objective function is modified in order to lead to a desired system's response.



(a) Membership functions for the error e_n



(b) Membership functions for the control action Δu_m

Fig. 1. Membership functions for the criteria considered in fuzzy predictive control.

3.2 *Fuzzy objective functions*

When fuzzy multicriteria decision making is applied to determine the objective function, additional flexibility is introduced. Each criterion ζ_{ij} is described by a fuzzy set, where $i = 1,\dots,H_p$, stands for the time step $k+i$, and $j = 1,\dots,T$ are the different criteria defined for the considered variables at the same time step. Fuzzy criteria can be described in different ways. The most straightforward and easy way is just to adapt the classical criteria in MPC. This generalization have been done for SISO systems in (Sousa and Kaymak 2001). This paper extends the fuzzy criteria for multivariable systems.

Let the system under control have control actions $\mathbf{u}(k)$ and outputs $\mathbf{y}(k)$. Figure 1 shows examples of general membership functions that can be used for one of the errors $e_n(k+i) = r_n(k+i) - y_n(k+i)$, with $i = 1,\dots,H_p$ and $n = 1,\dots,n_y$, and n_y is the number of outputs, and for the change in the control action $\Delta u_m(k+i-1)$, with $m = 1,\dots,n_u$, and n_u is the number of inputs.

In this example, the minimization of the output error $\mu_{e_n}(e_n(k+i))$, with $n = 1,\dots,n_y$ is represented by an exponential membership function. This well-known function has the nice property of being tangent to the triangular membership function defined using the parameters $K_{e_n}^-$ and $K_{e_n}^+$, see Fig. 1. Another interesting feature of this exponential membership function is that it never reaches the value zero. Therefore, this criterion is considered to be a fuzzy goal.

The control effort $\mu_{u_m}(\Delta u_m(k+i-1))$, with $m = 1,\dots,n_u$ is, in this case, represented by a triangular membership function around zero, which is considered to be a fuzzy constraint. The crisp rate constraints on Δu_m representing the maximum and the minimum allowed in the system are given by $H_{u_m}^-$ and $H_{u_m}^+$, respectively. These constraints are related to physical limitations of the system. The membership degree should be zero outside the interval $[H_{u_m}^-, H_{u_m}^+]$. The parameters defining the range of the triangular membership function are $K_{u_m}^-$ and $K_{u_m}^+$. Note that membership function μ_{u_m} does not have to be symmetrical. Further, μ_{u_m} can also be defined as a trapezoidal membership function.

In principle, different criteria can be defined at each time instant $k+i$, $i = 1,\dots,H_p$. This example has $T = n_y + n_u$ decision criteria and the total number of criteria in a fuzzy MPC problem is thus given by $(n_y + n_u) \cdot H_p$. However, it is much simpler to consider the same membership functions μ_{e_n} and μ_{u_m} for all time steps $k+i$. Some tuning guidelines for these parameters are going to be described in Section 4. Note that in the FDM formulation it is no necessary to scale the several parameters R and Q, as in (4), because the use of membership functions introduce directly the normalization required. After the membership functions have been defined, they are combined by using a decision function, such as a parametric aggregation operator from the fuzzy sets theory, as the Yager t-norm (Yager 1980).

4. APPLICATION EXAMPLE

The approach presented in this paper is applied to the control of a simulated gantry crane, which is shown in Fig. 2. A container gantry crane consists of a bridge girder on portal legs from which a trolley system is suspended. The trolley can travel along the bridge girder that stretches over the container ship and part of the quay for loading and unloading the ship. A hoisting mechanism consisting of a spreader suspended from the trolley by means of hoisting cables is used for grabbing and hoisting the container. The control goal is the position of the trolley at a desired horizontal location x, with a rope length h, while the swing θ of the load is damped so that the container can be positioned accurately (see Fig. 2). The predictive control structure applied includes measurement noise and system disturbances, which have values similar to the ones in the real system. With classical objective functions the controller uses a simplification of the cost function (4), considering only the error. This cost function revealed to be sufficient to control the system.

The simulation model of the gantry crane is implemented using the Lagrangian of the system, consider-

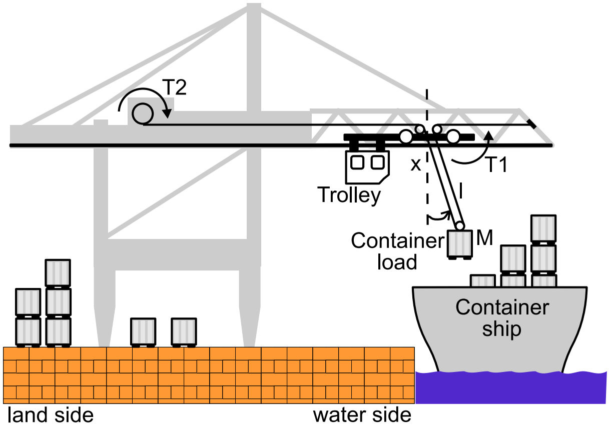


Fig. 2. Schematic picture of the container gantry crane

Table 1. Errors results for conventional model predictive control and FDM.

	h	x	θ
Gen.mean	1	1	1
Yager t − norm	1.21	1.28	0.91
Convent.MPC	1.64	0.63	1.16

ing also the models of the electric motors, and the viscous friction. The parameters of the model are taken from a real crane at the port of Rotterdam. The input of the crane motors is in the interval $[-200 \quad 200]$ V. The trolley can reach a maximum velocity of 3.2 ms^{-1} for a maximum load of 53 ton. The crane construction is assumed to be stiff, and the maximum acceleration is 0.8 ms^{-2}.

The vector of the errors is given $e^T = [e_1 \ e_2 \ e_3]$, where e_1, e_2 and e_3 are the errors between the references and the the three controlled variables, which are respectively the horizontal position x, the length of the rope h and the swing angle θ. In conventional MPC the matrix R in (4) is the weight matrix given by:

$$R = \text{diag}(1,1,1)$$

This weights mimic the objective of following with the same importance the rope and position references, without neglecting the swing angle.

Fuzzy predictive control is applied using the generalized mean (Dyckhoff and Pedrycz 1984) and the Yager t-norm (Yager 1980), as aggregation operators in (3). A parameter must be tuned for both these operators. The best values are found to be 3.5 for the generalized mean and 2.5 for the Yager t-norm. The control and prediction horizons are set respectively to $H_c = 3$ and $H_p = 5$.

The simulations are made using MatLab on a 200 MHz Pentium PC running *Windows 95*. The B&B method has been applied as optimization algorithm to both the classical and the fuzzy predictive controllers. This algorithm revealed to be the best for situations as described in this paper (Sousa *et al.* 1997, Sousa 2000, Sousa and Setnes 1999).

Table 1 shows sum squared errors obtained for various simulations. The control variable is discretized in 3

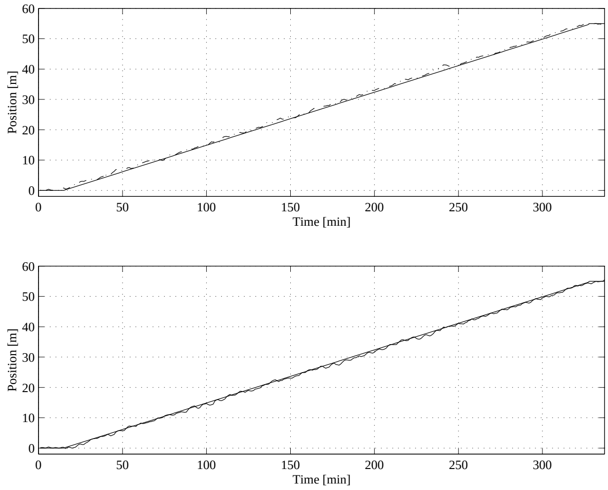


Fig. 3. Position with conventional model predictive control (solid) and with generalized mean (dashed-dotted).

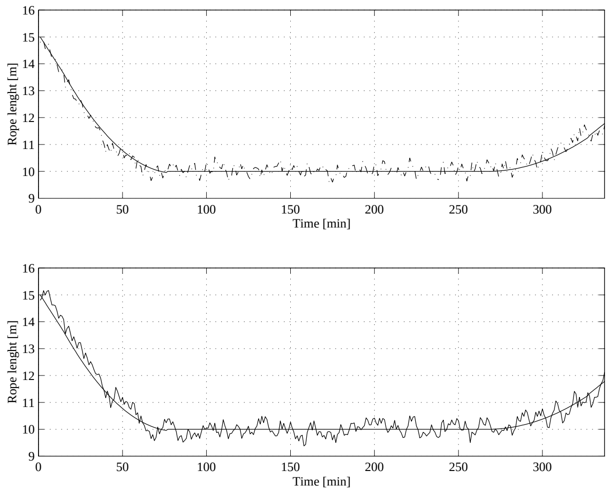


Fig. 4. Rope length with conventional model predictive control (solid) and with generalized mean. (dashed-dotted).

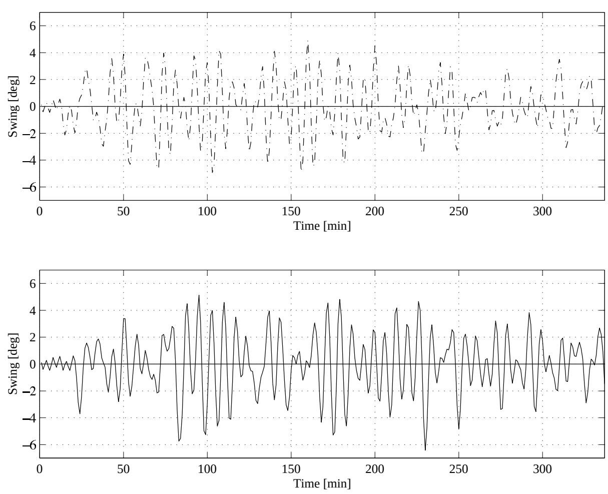


Fig. 5. Swing with conventional model predictive control (solid) and with generalized mean. (dashed-dotted).

discretizations. The error using the generalized mean operator is taken as 1 (100%), i.e. it serves as the normalization to be compared with the errors using other methods.

Simulations using conventional model predictive control and generalized mean are depicted for the container gantry crane position in Fig. 3, for the rope length in Fig. 4 and for the swing angle in Fig. 5. Note that it is not possible to distinguish clearly, just by examining these figures, which controller is the best. Therefore, it is important to present the errors as in Table 1. However, it is clear that the fuzzy predictive controller has a better behavior in terms of swing angle, i.e. it presents smaller swing as normally desired.

All the controllers present good performance for both the position and rope length, which are followed by the controller with small position error. The conventional controller is slightly better in terms of controlling the position x. The error for the swing angle is much larger with conventional model predictive control when compared to other fuzzy methods. The maximum absolute value for the swing angle is thus 6.4° using conventional model predictive control, 4.9° using the generalized mean and 4.3° using the Yager t-norm. Therefore, we can conclude that in general the fuzzy predictive controllers present better performance, especially in terms of reducing the swing angle, which is a crucial variable in the crane system.

5. CONCLUSIONS

This paper generalizes the application of fuzzy decision making to predictive control for multivariable systems. The problem of choosing fuzzy criteria in a multivariable MPC framework is addressed. The generalization of classical objective functions to fuzzy objective functions in multivariable MPC is presented. This generalization brings additional flexibility to the definition of the objective functions, as shown by one simulation example, where the improvements of the controller response by using fuzzy objective functions in MPC is clear. The proposed method presents good performance, and obtained faster responses with smaller overshoots than the classical MPC. Future research must consider the generalization of the fuzzy objective function in order to include weights and hierarchical fuzzy criteria.

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