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OPTIMAL SENSOR LOCATION FOR THE IDENTIFICATION OF MOVING HEAT SOURCES

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Abstract: For many thermal processes described by partial differential equations, parametric identification of the model can be performed by solving an inverse problem. Measurements noises occurring on one or several measured characteristic states have to be carefully taken into account in order to provide robust identification procedures. However, spatial inaccuracy of the sensor location is rarely investigated. In this paper, a thermal process is presented in order to study the effect of this class of measurement noises on identification robustness. Then the problem of optimal location of the sensor is stated. *Copyright © 2002 IFAC*

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1. INTRODUCTION

For many thermal processes, an accurate model, established from partial differential equations, can provide an efficient predictive tool and leads to the determination of optimal control procedures. The model structure (issued from an energy balance) is rarely questionable but when thermophysical characteristics of the material or heat exchanges are not well known, an identification problem has to be solved.

From temperature measurements, thermal evolution of the material is known and an inverse problem can be stated for which a cost function describing the error between the simulation and the observation has to be minimized. Measurements noises occurring on one or several sensors have to be carefully taken into account in order to provide robust identification procedures. However, spatial inaccuracy of the sensor location is rarely investigated and can sharply reduce the reliability of the results by introducing an important error.

In this paper, an experimental thermal process is presented in order to study the effect of this class of measurement noise on identification robustness. The thermal process is briefly exposed in the following paragraph. Then, the model describing the temperature evolution is written and leads to the formulation of a non linear partial differential equations system. Several numerical results are given in the case of stationary heat source and in the case of moving heat source. In both cases, thermophysical characteristics of the material are given and a direct problem is solved by a finite element method. In the fourth section, the determination of the unknown heating strength is considered. The inverse problem is solved by a least square estimation. The minimization of the quadratic criterion is achieved by a conjugate gradient algorithm. The sensitivity functions are estimated by solving the sensitivity equations and lead to the calculation of the descent depth of the minimization algorithm and to the determination of an optimal sensor location.

2. DESCRIPTION OF THE THERMAL PROCESS AND MODEL

The thermal process investigated in this paper (see figure 1) has been developed in the IMP-CNRS Institute. The circular heating source is made of inconel $\left(f = 210^{-2}\ m\right)$. The spatial uniformity of the heat source is controlled by mean of a water circulation around its support. This source can be moved in the horizontal plane, very closely to the underneath sheet of metal . The motion is obtained with two step by step motors which are supervised by with a great precision. Temperature obtained on the surface of the heating source is uniform and can not exceed 1000K. Several sheet of metal can be used in order to study various thermal behavior : steel, copper or coated materials ... Domain dimensions are given in the following paragraph. A large investigation domain is offered by this experimental process : tracking of the heating trajectory, detection of the heating source (fault detection and diagnosis), identification of the heating strength, optimal sensor location for parametric identification or closed-loop control …

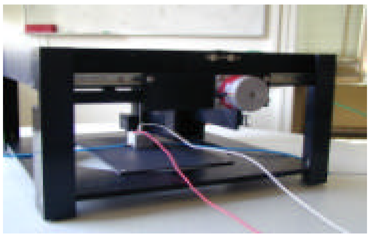

Fig. 1. Experimental apparatus.

The methodologies developed in this communication can be used for several industrial applications :

- optimal control for welding processes,
- hardening of steel due to application of high density of solar flux, see (Autrique *et al*., 2000),
- optimal experiment design for tribometer.

In order to describe the temperature evolution of the sheet of metal heated by the circular source, a model is established. Let us denote by :

- $x \in \Omega$, the space variable, where $\Omega \subset \mathbb{R}^3$ is the domain corresponding to the parallelepipedic sheet of metal. The surface of Ω is denoted by Γ.
- $t \in T = \left[0, t_f\right]$ is the time variable.
- $q(x,t)$ is the temperature and the initial temperature is constant : $q_0 = 293K$,
- $r(q)$ the mass density, $c_p(q)$ the specific heat, $l(q)$ the thermal conductivity,
- h the convective exchange coefficient, ε the emissivity and s the stefan constant,
- $w_s \subset \Gamma$ is the subdomain of Γ corresponding to the spatial support of the circular heating source, (w_s is time dependent for moving heat source),
- $j(t)$ the heat flux.

The thermal evolution of the material during the process is described by the following equations :

- state equation : $\forall (x,t) \in \Omega \times T$

$$r(q) c_p(q) \frac{\partial q}{\partial t} - div\left(l(q)\, \overrightarrow{grad}(q) \right) = 0 \qquad (1)$$

- initial condition : $\forall x \in \Omega$

$$q(x,0) = q_0 \qquad (2)$$

- heating condition : $\forall (x,t) \in w_s \times T$

$$-l(q) \frac{\partial q}{\partial \vec{n}} = -j(t) \qquad (3)$$

- heat exchange condition : $\forall (x,t) \in (\Gamma - w_s) \times T$

$$-l(q) \frac{\partial q}{\partial \vec{n}} = h(q - q_0) + es\left(q^4 - q_0^4\right) \qquad (4)$$

where $\vec{n}$ is the normal vector exterior to the surface.

3. DIRECT PROBLEM

According to the previous notations, direct problem can be formulated as follows :

<u>Problem</u> P_{dir} : find the temperature $q(x,t)$ solution of the non linear distributed parameter system (S) :

$$(S)\begin{cases} \text{state equation} & (1) \\ \text{initial condition} & (2) \\ \text{boundary condition} & (3)\quad (4) \end{cases}$$

Except for well-known problems (involving specific non linearities and boundary conditions), existence and uniqueness of the solution of problem P_{dir} can not be stated. Nevertheless, numerical method such as finite element method can lead to a numerical determination of state $q(x,t)$ in an adapted function space. The steel selected is a refractory NS30 steel, whose thermophysical properties are given in table 1.

<u>Table 1 Thermophysical characteristics of the steel</u>

mass density $\left(kg.m^{-3}\right)$	thermal conductivity $\left(W.m^{-1}.K^{-1}\right)$
$r(q) = -0.444q + 8121.3$	$l(q) = 0.0129q + 10.03$

specific heat $\left(Jkg^{-1}.K^{-1}\right)$
$c_p(q) = \begin{cases} 0.22q + 432.7 & if \quad 273 \le q \le 888 \\ 0.46q + 219.6 & if \quad 888 < q \le 1300 \end{cases}$

$\Omega \subset \mathbb{R}^3$ is the parallelepipedic domain (in meters) :

$$\Omega = \left\{ x = (x_1, x_2, x_3) \in [0;0.3] \times [0;0.2] \times \left[0;510^{-3}\right] \right\}$$

The time interval is $T = \left[0; t_f\right]$ where $t_f = 600s$ .The heating source is defined as a disk $D(I,r)$ (center I , $r = 10^{-2}m$ in radius). Then w_s can be formulated as follows :

- case 1 : non moving source

$$w_s = \left\{ x = \left(x_1, x_2, 510^{-3}\right) / (x_1, x_2) \in D\left((0.15;0.1), r\right) \right\}$$

• case 2 : moving source

$w_s = \left\{ x = (x_1, x_2, 510^{-3}) / (x_1, x_2) \in D\left((510^{-4}t; 0.1), r\right) \right\}$

Both cases are presented on figure 2.

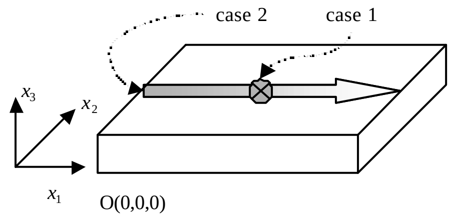

Fig. 2. non moving and moving source.

Emissivity e is considered equal to 1 (while the surface of the used sheet of steel is black painted) ; $s = 5.6710^{-8}$ $W.m^{-2}.K^{-4}$ is the Stefan constant. The heat exchanges are quite difficult to estimate. For natural convection phenomena, it is usual to take into account an exchange coefficient h which is often determined from thermal considerations. Several boundaries have to be considered :

• on the upper face, $x_3 = 510^{-3} m$, $h = 20\, W\, m^{-2}\, K^{-1}$,

• on the lower face, $x_3 = 0$, $h = 2\, W\, m^{-2}\, K^{-1}$,

• on the four lateral faces $h = 1\, W.m^{-2}.K^{-1}$.

These values are realistic but not accurate since the natural convection phenomena are complex to describe. Problem P_{dir} is solved by a finite element method in space (space step is about $2.510^{-3} m$) and finite differentiation in time (time step is $5\,s$) for a given heat flux (which can be time dependent) : $j(t) = 2.510^{5}$ $W m^{-2}$. On the following figures (fig. 3 and 4), the temperature evolution in the middle of the domain is presented for case 1 and case 2 at several depth :

• black line : point $(0.15, 0.1, 0)$ under the sheet,

• dashed line : point $(0.15, 0.1, 2.510^{-3})$ in the sheet,

• signs (+) : $(0.15, 0.1, 510^{-3})$ on the heated surface.

On figures 5 & 6, the temperature spatial distribution at the end of the heating cycle $t = 600s$ is presented for case 1 and case 2, for an arbitrary constant value of the heat flux. Figures 3 to 6 show how the source motion affects the temperature distribution.

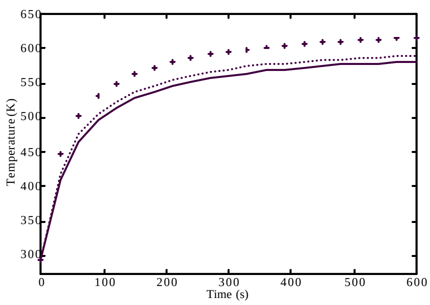

Fig. 3. Case 1 : temperature evolution

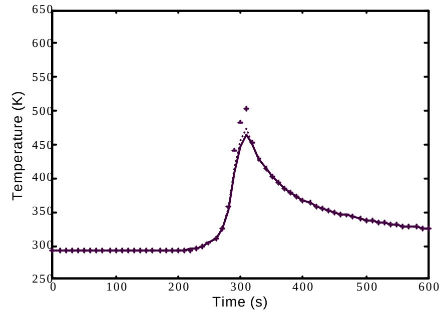

Fig. 4. Case 2 : temperature evolution

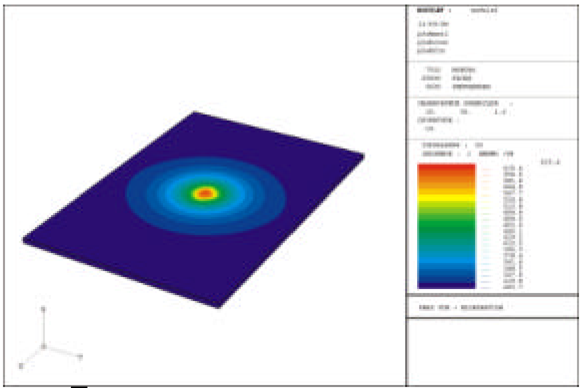

Fig 5. 1 : temperature spatial distribution at $t = 600s$

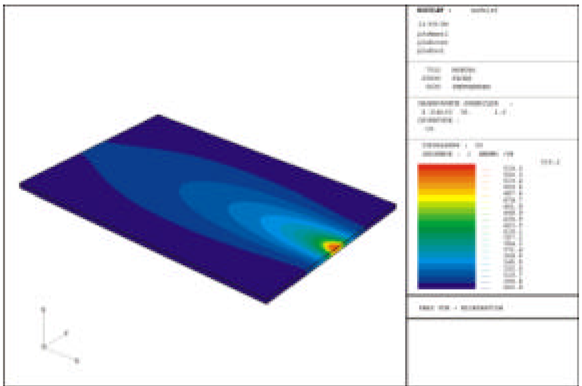

Fig 6. 2 : temperature spatial distribution at $t = 600s$

4. PROCESS PARAMETER IDENTIFICATION AND NON LINEAR OPTIMIZATION

In the following, while the trajectory and the speed of motion of the heat source is fixed, its magnitude has to be estimated. Heat flux is modeled as a time-varying function, by $j(t) = \sum_{i=1}^{N-1} j_i x_i(t)$. In the latter, x_i is a time dependent continuous piecewise linear function such that, for $t_j = \frac{600\, j}{N}$, $j = 1, \cdots, N$: $x_i(t_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$. According to this notation, $j(0) = j(600) = 0\ W.m^{-2}$, and $j(t)$ is known when $\bar{J} = (j_i)_{i=1,\cdots N-1}$ is identified. Then the following inverse problem is considered :

Problem P_{inv} :
find $\vec{j} \in \mathbb{R}^{N-1}$ which minimizes the cost function :

$$J(\vec{j}) = \frac{1}{2}\int_T \left(\sum_{j=1}^{n_{sens}} \left(q(x_j, t; \vec{j}) - \hat{q}_j(t) \right)^2 \right) dt \qquad (5)$$

where n_{sens} is the number of sensor, $\hat{q}_j$ is the temperature measured at sensor j located on point x_j ; with the constraint : $q(x,t)$ is solution of (S).

Hadamard (1923) introduced the notion of ill-posedness in the field of partial differential equations. A problem is *well-posed* when a solution *exists*, is *unique* and depends *continuously* on the initial data. It is *ill-posed* when it fails to satisfy at least one of these criteria. It is well known that most of inverse problem occuring in thermal situations are ill-posed since solution is strongly affeted by data errors : initial state, measurements bias, discrete approximation, … In order to identify a physical parameter occuring in a thermal model by solving an inverse problem, iterative regularization principle is often proposed in order to take into account ill-posed situation. In the following, the implementation of a conjugate gradient method is proposed ; such a method leads to iterative resolution of three well conditionned problem : direct problem, sensitivity problem, adjoint problem. The interest of this approach for the solution of non-linear, ill-posed problems such as P_{inv}, has been shown many times in computational experiments.

4.1 General conjugate gradient algorithm :

a) initialisation $k = 0$; let us denote by $\vec{j}^0$ the given initial approximation of $\vec{j}$ and $\vec{d}^0 = -\nabla J(\vec{j}^0) = -\left(\left(\frac{\partial J}{\partial j_i} \right)_{i=1,\cdots,N-1} \right)(\vec{j}^0)$ the initial descent direction,
b) at iteration k, from point $\vec{j}^k$, the next point is obtained : $\vec{j}^{k+1} = \vec{j}^k + g^k \vec{d}^k$ where $g^k = \underset{g\in\mathbb{R}}{\operatorname{argmin}} \left(J(\vec{j}^k + g\vec{d}^k) \right)$
c) the next direction is defined by : $\vec{d}^{k+1} = -\nabla J(\vec{j}^{k+1}) + b^k \vec{d}^k$ with : $b^k = \frac{\left\| \nabla J(\vec{j}^{k+1}) \right\|^2}{\left\| \nabla J(\vec{j}^k) \right\|^2}$.
d) stopping of the iterative process if $J(\vec{j}^{k+1})$ is close to zero or : $k \leftarrow k+1$ and go to (b).

4.2 Adjoint problem for the gradient calculation

Gradient $\nabla J(\vec{j})$ verifies : $dJ = \sum_{i=1}^{N-1} \left(\frac{\partial J(.)}{\partial j_i} dj_i \right)$.

Moreover, $dJ = J(\vec{j} + d\vec{j}) - J(\vec{j})$

$$\begin{aligned} dJ &= \int_T \left(\sum_{j=1}^{n_{sens}} \left(q(x_j, t; \vec{j}) - \hat{q}(t) \right) dq(x_j, t; \vec{j}) \right) dt \\ &= \int_T \int_\Omega \left(\sum_{j=1}^{n_{sens}} \left(q(x_j, t; \vec{j}) - \hat{q}_j(t) \right) z_j(.) \right) dq \, d\Omega dt \end{aligned}$$

where $z_j(x)$ is the Dirac distribution of sensor j.
Let $L(q, j, y)$ the Lagrangian associated to the optimization problem defined by equation (5) and constraints (S) :

$$L(q, j, y) = J(q, j) + \left\langle y, r(q)c(q)\frac{\partial q}{\partial t} - div\left(l(q)\overrightarrow{grad}q \right) \right\rangle$$

where $y(x,t)$ is a Lagrange multiplier and $\langle u, v \rangle$ is the scalar product in $L^2(T, L^2(\Omega))$. When y is fixed then : $dL = \frac{\partial L}{\partial q} dq + \frac{\partial L}{\partial j} dj$. The Lagrange multiplier $y(x,t)$ is fixed such that: $\frac{\partial L}{\partial q} dq = 0$, $\forall dq$. Then according to the expression developed in (Abou Khachfe and Jarny, 2001), $y(x,t)$ has to satisfy the following equations:

- state equation : $\forall (x,t) \in \Omega \times T$

$$\begin{aligned} -r(q)c(q)\frac{\partial y}{\partial t} - l(q)\Delta y &= \\ -\sum_{j=1}^{n_{sens}} \left(q(x_j, t; \vec{a}) - \hat{q}_j(t) \right) z_j(.) & \end{aligned} \qquad (6)$$

- final condition : $\forall x \in \Omega$

$$y(x, t_f) = 0 \qquad (7)$$

- boundary condition : $\forall (x,t) \in w_s \times T$

$$-l(q)dq\frac{\partial y}{\partial \vec{n}} = -y\,dj(t) \qquad (8)$$

- boundary condition : $\forall (x,t) \in (\Gamma - w_s) \times T$

$$-l(q)\frac{\partial y}{\partial \vec{n}} = y\left(h + es\,4q^3 \right) \qquad (9)$$

The following adjoint problem has to be solved in order to determine the Lagrangian multiplier $y(x,t)$:

Problem P_{lag} :
find the Lagrangian multiplier $y(x,t)$ solution of the system (S_{lag}) :

$$\begin{cases} \text{state equation} & (6) \\ \text{final condition} & (7) \\ \text{boundary condition} & (8) \quad (9) \end{cases}$$

The problem P_{lag} is solved by the same numerical method which is implemented for problem P_{dir} (for example, the mesh used in paragraph 4 is still considered). Considering $y(x,t)$ solution of $\{S_{lag}\}$ and $q(x,t)$ solution of $\{S\}$, it becomes : $dJ = dL$

$$\sum_{i=1}^{N-1}\left(\frac{\partial J(.)}{\partial j_i}dj_i\right)=-\int_T\int_{w_s} y\,dj\,dtd\Gamma$$

Thus : $\frac{\partial J(.)}{\partial j_i}=-\int_T y\frac{\partial j}{\partial j_i}dtd\Gamma$

4.3 Calculation of the descent depth

g^k is obtained by minimizing :

$$\min_{g\in\mathbb{R}}\left(\frac{1}{2}\int_T\left(\sum_{j=1}^{n_{sens}}\left(q\left(x_j,t;\bar{j}^k+g\,\bar{d}^k\right)-\hat{q}_j(t)\right)^2\right)dt\right) \qquad (10)$$

g^k solution of (10) is given by : :

$$g^k=\frac{\int_T\left(\sum_{j=1}^{n_{sens}}dq\left(x_j,t;\bar{j}^k\right)\left(q\left(x_j,t;\bar{j}^k\right)-\hat{q}_j(t)\right)\right)dt}{\int_T\left(\sum_{j=1}^{n_{sens}}\left(dq\left(x_j,t;\bar{j}^k\right)\right)^2\right)dt}$$

where $(dq)_j\left(x_j,t;\bar{j}^k\right)$ is the solution of the sensitivity problem P_{sens} in the direction $d\bar{j}=\bar{d}^k$, defined in the following.

4.4 Sensitivity problem

In this paragraph, sensitivity equations are presented according to the method developed in (Beck and Arnold, 1977). Sensitivity equations are written in order to determine a temperature variation resulting from a heat flux variation. Let us denote by $(dq)_{mdj}$ the temperature variation resulting from the heat flux variation mdj where : $dj=\sum_{i=1}^{N}\frac{\partial j}{\partial j_i}dj_i$.

$$dq_{mdj}(x,t)=q(x,t;j+mdj)-q(x,t;j)$$

The sensitivity function is defined as follows :

$$dq(x,t;j)=\lim_{m\to 0}\frac{q(x,t;a+mdj)-q(x,t;j)}{m}$$

Let us denote by : $q^+=q(x,t;j+mdj)$, $q=q(x,t;j)$ et $a(.)=r(.)c_p(.)$. Evolution of q^+ is described by the following equations:

- state equation : $\forall(x,t)\in\Omega\times T$

$$a\left(q^+\right)\frac{\partial q^+}{\partial t}-div\left(l\left(q^+\right)\overrightarrow{grad}\left(q^+\right)\right)=0 \qquad (11)$$

- initial condition : $\forall x\in\Omega$

$$q^+(x,0)=q_0 \qquad (12)$$

- heating condition : $\forall(x,t)\in w_s\times T$

$$-l\left(q^+\right)\frac{\partial q^+}{\partial\vec{n}}=-(j+mdj)(t) \qquad (13)$$

- heat exchange condition : $\forall(x,t)\in(\Gamma-w_s)\times T$

$$-l\left(q^+\right)\frac{\partial q^+}{\partial\vec{n}}=h\left(q^+-q_0\right)+es\left(q^{+4}-q_0^4\right) \qquad (14)$$

By comparison between equations (1-4) and equations (11-14), the following equations are obtained :

- state equation : $\forall(x,t)\in\Omega\times T$

$$\frac{\partial}{\partial t}\left(a(q)dq\right)-\Delta\left(l(q)dq\right)=0 \qquad (15)$$

- initial condition : $\forall x\in\Omega$

$$dq=0 \qquad (16)$$

- heating condition : $\forall(x,t)\in w_s\times T$

$$-\frac{\partial}{\partial\vec{n}}\left(l(q)dq\right)=-dj \qquad (17)$$

- heat exchange condition : $\forall(x,t)\in(\Gamma-w_s)\times T$

$$-\frac{\partial}{\partial\vec{n}}\left(l(q)dq\right)=hdq+4esq^3dq \qquad (18)$$

According to the previous notations, sensitivity problem can be formulated as follows :

> <u>Problem</u> P_{sens} :
>
> find the temperature varied $dq(x,t)$ solution of the linear distributed parameter system (S_{sens}):
>
> $$\begin{cases}\text{state equation} & (15)\\ \text{initial condition} & (16)\\ \text{boundary condition} & (17)\quad(18)\end{cases}$$
>
> with j , q and dj given.

It is important to note that the problem P_{sens} is solved by the same numerical method which is implemented for problem P_{dir} and problem P_{lag} .

Then the following algorithm is available for solving the inverse problem P_{inv}:

> <u>General conjugate gradient algorithm:</u>
>
> a) initialisation $k=0$; $\bar{j}^0$ the given initial approximation,
>
> b) solve the direct problem P_{dir} to compute $q\left(x,t;\bar{j}^k\right)$ and the cost function $J\left(\bar{j}^k\right)$,
>
> c) solve the adjoint problem P_{lag} to compute the components of the gradient in order to know the descent direction $\bar{d}^k$,
>
> d) solve the sensitivity problem P_{sens} to compute the descent depth g^k , and $\bar{j}^{k+1}$
>
> e) stopping of the iterative process if $J\left(\bar{j}^{k+1}\right)\le J_{stop}$ is close to zero or : $k\leftarrow k+1$ and go to (b).

Remarks : J_{stop} is a positive scalar which depends on the variance of the temperature measurement errors in order to avoid unstable solutions ; see the iterative regularizing principle (Alifanov, 1994).

5. OPTIMAL SENSOR LOCATION

Methods for optimum sensor locations often lay on classical optimal experiment design techniques. A commonly used criterion for optimal experiment design maximizes the amount of information in the collected data by maximizing the determinant of the Fisher Information Matrix, which hopefully minimizes the asymptotic confidence intervals of the identified parameters – D-optimality, see (Walter and Pronzato, 1997; Fadale, *et al.*, 1995; Emery and Fadale, 1997). As the Fisher Information Matrix is build using the sensitivity functions, such criteria often lead to maximizing the sensitivity function of model output to the identified parameters. Such a criterion has been used in a previous work for optimal sensor location, see (Autrique *et al*., 2000). Numerical results have been obtained with a constant uncertainty of $dj = 100\, W.m^{-2}$, and the shape of the sensitivity function $dq(x,t)$ distribution is quite similar to the one obtained in figures 3 to 6. By analyzing the evolution (in time and space) of the sensitivity function $dq(x,t)$, the optimal sensor location, taken as the maximum of the sensitivity function, was found as the center of the lower non heated side of the sheet of steel. In this work, a criterion previously proposed by Vande Wouver (2000), Point (1996), is adapted. The optimal location of *n* sensors are the one which maximizes the Gram determinant defined by :

$$g(\hat{x}^1,\ldots,\hat{x}^n) = \det[\mathbf{F}]$$

where $\hat{x}^i = (\hat{x}^i_1, \hat{x}^i_2, 0)$ is the location of the sensor i (on the lower non heated side of the sheet of steel) and the matrix $\mathbf{F}$ is defined by

$$\mathbf{F} = \int_0^{t_f} \mathbf{M}(\hat{x}^1,\ldots,\hat{x}^n,t)\mathbf{M}^T(\hat{x}^1,\ldots,\hat{x}^n,t)dt$$

In (Vande Wouver, 2000), the $\mathbf{M}$ vector is taken either as sensitivity vector in the context of parametric identification or as impulse responses in the context of state estimation. In the following, the sensitivity function $dq(x,t;j)$ (derived for a direction $dj = 1$) is considered. This criterion is used in the case of 1 sensor then in the case of 2 sensors.

6. UNCERTAIN LOCATION OF THE SENSORS

According to the previous paragraphs, from sensor measurements, an inverse problem is solved and leads to the identification of the unknown heating source while its position or its trajectory is known. A methodology based upon the Gram determinant has been exposed in order to locate one (or several) sensor on the lower non heated side of the sheet of steel. Then, several situations can be considered :

- how many sensors have to be implemented ?
- how to ensure an optimal location for the sensors if the inaccuracy of the location can not be neglected.

In order to estimate the effect of sensor location inaccuracy, a perturbation-kind technique will be used : a perturbed identification is undertaken while supposing an uncertainty in sensor location. The difference between the perturbed and the non-perturbed identifications will serve to deriving the sensitivity to uncertainty in location. This sensitivity may be used to study the robustness of the identification procedure.

7. PERSPECTIVES

As the continuation of this work, uncertainty on the sensor location might be taken into account as a *nuisance parameter*. Then optimality criterion can be defined in order to estimate the unknown heat flux without the determination of the real position of the sensor (for example, the D_S optimality criterion in (Walter and Pronzato, 1997). Such an approach, which seems to provide an attractive alternative, has to be carefully investigated.
The next issue is to find out what should be the sampling time interval. Its determination has to be connected to the speed of motion of the source.

REFERENCES

Abou Khachfe, R. and Y. Jarny (2001). Determination of heat sources and heat transfer coefficient for two dimensional heat flow – numerical and experimental study, *Int. Journal of Heat and Mass Transfer*, **Vol. 44**, pp. 1309-1322.

Alifanov, O.M. (1994). *Inverse Heat Transfer Problems*, pp. 348, Springer-Verlag.

Autrique L., C. Chaussavoine, J.P. Leyris and A. Ferrière (2000). Optimal sensor strategy for parametric identification of a thermal system, *In Proceedings of IFAC-SYSID 2000, Santa Barbara*, USA.

Beck, J.V. and K.J. Arnold (1977). *Parameter estimation in engineering and science*, pp. 348, Wiley, New-York, USA.

Emery, A.F. and T.D. Fadale (1997), The effect of imprecisions in thermal sensor location and boundary conditions on optimal sensor location and experimental accuracy, *Journal of Heat Transfer*, **Vol. 119**, pp. 661-665.

Fadale, T.D., A.V. Nenarokomov and A.F. Emery, (1995), Two approaches to optimal sensor locations, *Journal of Heat Transfer*, **Vol. 117**.

Point N., A. Vande Wouver and M. Remy (1996). Practical issues in distributed parameter estimation : gradient computation and optimal experiment design, *Control Engineering Practice*, **Vol.4-11**, pp.1553-1562.

Vande Wouver A. Point N., Porteman S. et Remy M. (2000) An approach to the selection of optimal locations in distributed parameters, *Journal of Process Control* **Vol. 10**, pp. 291-300, 2000.

Walter E. and L. Pronzato (1997). *Identification of parametric models from experimental datas*, pp. 413, Springer-Verlag, Berlin, Germany.