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A PERTURBATION ESTIMATION TO SOLVE THE FUNDAMENTAL PROBLEM OF RESIDUAL GENERATION

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Abstract: This paper deals with fault detection and more particularly with the Fundamental Problem of Residual Generation (FPRG). In former works, conditions, based on the properties of invariant distributions involving the disturbance vector field, were given to solve this problem. The aim of the present paper is to provide, when those conditions are not fulfilled, an alternative solution to the FPRG, based on a perturbation estimator.

Keywords: Fault Detection, differential geometric methods, sliding mode, triangular form, nonlinear observers

1. INTRODUCTION

The problem of fault detection and isolation has been widely investigated (Frank, 1990; Frank *et al.*, 1999; Staroswiecki *et al.*, 1993).... The main requirement of this problem is the residual generation which allows to detect and isolate the fault by an appropriate evaluation. The detection of faults is established by a logic decision based on the residual which is an output signal generated by one or many observers. The Fundamental Problem of Residual Generation (FPRG) was first studied for linear systems with one fault signal where the residual is required to recognize the fault signal without confusing it with the disturbance (Massoumnia *et al.*, 1989). Most of the time, in the nonlinear case, the design methods and the diagnostic observers for detection are based on the hypothesis that the system evolves in the neighborhood of an operating point, and the linearization method is used. The disadvantage of this method is that the observation error based on the linearized system can be misinterpreted as faults by the detection algorithm and hence lead to false alarm. One of the nonlinear method is based on the disturbance decoupling approach (de Persis and Isidori, 1999), (Seliger and Frank, 1991). Nevertheless, some conditions may appear too restrictive or not be satisfied in the considered problem. Here we suggest a way of investigation for solving the FPRG when those conditions are not fulfilled. For a sake of simplicity, the case of only one fault signal is taken into account and an example is given throughout the paper as a way of illustration. It is assumed henceforth that the reader is familiar with the basic concepts and tools of the differential geometric approach (Isidori, 1995) and the sliding mode theory (Utkin, 1992), (Perruquetti and Barbot, 2002).

2. PROBLEM STATEMENT

To approach the (FPRG), let us consider the nonlinear system

$$\dot{x} = f(x,u) + l(x)m + p(x)\omega \qquad (1)$$

$$y = h(x) \qquad (2)$$

where $x(t) \in \mathcal{X} := \mathbb{R}^n$, $u(t) \in \mathcal{U} := \mathbb{R}$, $y(t) \in \mathcal{Y} := \mathbb{R}^p$, $m(.) : [0,+\infty) \to \mathcal{M} := \mathbb{R}$ is an unknown input (supposed to be piecewise constant), $\omega(t)$ is an unknown disturbance. The system is supposed to be perfectly known when $m(.) = w(.) = 0$. $f(x,u), h(x), l(x), p(x)$ are smooth vector fields. Without loss of generality it is supposed that $x_0 = 0$ is an equilibrium point for the system (1-2). The main problem of the FPRG is to design a filter, or an observer, modelled by equations of the form (Massoumnia *et al.*, 1989):

$$\dot{\zeta} = f_r(\zeta(t), y(t), u(t)) \quad (3)$$
$$r(t) = h_r(\zeta(t), y(t), u(t)) \quad (4)$$

where h_r, f_r are $\mathcal{C}^\infty$ and $r(t)$ represents the residual which is a valued signal containing informations on the time and the location of the occurrence of the fault. In the ideal case, $r(t)$ should be near zero when there is no fault and deviate from zero when a fault occurs. This is, however, impossible in practical case, because of noise and modelling errors and a filter has been used to obtain the residual $r(t)$. Moreover, to detect the fault, the following conditions are required:

- The observable states of the system are bounded,
- $r(t)$ is not affected by $\omega(.)$ and $u(.)$,
- $r(t)$ is affected by $m(.)$.

In this case, and if the filter (3-4) exists, a solution to the FPRG exists. Hereafter is recalled a method based on differential geometric considerations (de Persis and Isidori, 1999). The first stage of designing a residual generator consists in decoupling the fault with respect to the perturbation. To this end, one usually considers Δ_p, the smallest involutive distribution containing p and invariant with respect to f (the construction of such distributions can be found in the book of Isidori (Isidori, 1995)). If $\dim \Delta_p(x_0) = d < n$, it is known that there exists a diffeomorphism $z = \Theta(x)$, with $\Theta(0) = 0$, defined in U^0, a neighborhood of x_0 such that, in the new coordinates, the system (1-2) is represented by:

$$\dot{z}_1 = \varphi_1(z, y, u) + l_1(z)m + \tilde{p}(z)\omega \quad (5)$$
$$\dot{z}_2 = \varphi_2(z_2, y, u) + l_2(z)m \quad (6)$$
$$y_1 = \tilde{h}_1(z) \quad (7)$$
$$\tilde{y}_2 = \tilde{h}_2(z_2) \quad (8)$$

where $z_1 = (z_{11}, ..., z_{1d})^T$, $z_2 = (z_{2,d+1}, ..., z_{2n})^T$. The subsystem (6-8) is assumed to be observable. The fault must act on the z_2 dynamics, that is to say that the condition $l(x) \notin \Delta_p$ must be satisfied. In order to design a residual which is decoupled from the input ω, it is necessary to get an output $\tilde{y}_2$, only function of the original outputs y_1, y_2, and on which the perturbation does not act: that is to say a function $\psi(y_1, y_2)$ such that $d\psi \in \Delta_p^\perp$ ($\Delta_p^\perp$ being the annihilator of Δ_p) and $L_l L_g^i \psi \neq 0$ for at least one $i \in \{0, .., n\}$.

Example As a way of illustration, we will consider throughout the paper the following nonlinear system (without loss of generality, the system can be supposed autonomous):

$$\begin{cases} \dot{x}_1 = x_1x_3 + x_2 + \alpha\omega + x_1 m \\ \dot{x}_2 = x_3 + m \\ \dot{x}_3 = x_4 - x_2x_3 \\ \dot{x}_4 = x_3^2 + x_2(x_4 - x_2x_3) + \delta\omega + x_3 m \end{cases} \quad (9)$$

with the outputs

$$\begin{cases} y_1 = x_1 \\ y_2 = x_3 \end{cases}.$$

The classical method (de Persis and Isidori, 1999) can not be applied here since

$$\Delta_p = \text{span}\left\{p, [f,p], ad_f^2 p, ad_f^3 p\right\}$$

with $p = (\alpha, 0, 0, \delta)^T$, $[f,p] = -(\alpha x_3, 0, \delta, \delta x_2)^T$, $ad_f^2 p = \left(\alpha(x_3^2 + x_2x_3 - x_4) + \delta x_1, \delta, 0, \delta x_3\right)^T$, $ad_f^3 p = \left(\alpha x_3(3(x_4 - x_2x_3) - x_3^2) + \delta(x_2 - 1), 0, 0, 0\right)^T$ and it is easily checked that $\dim \Delta_p = 4$.

In a recent paper, Djemaï et al. (2000) considered the particular case when $\dim \Delta_p = n$. Still to have a residual completely independent from the perturbation, they proposed to use an output injection $\rho(y)$ in order to rewrite the system in the form

$$\dot{x} = \bar{f}(x) + \rho(y) + p(x)\omega(t) + l(x)m \quad (10)$$

and so that the dimension of $\bar{\Delta}_p$, the smallest involutive distribution containing p and invariant with respect to $\bar{f}(x) = f(x) - \rho(y)$, is $\bar{d} < n$.

However this method can neither be applied here. Indeed, let us rewrite the system (9) in the following form

$$\begin{cases} \dot{x}_1 = x_2 + \rho_1(y) + \alpha\omega + x_1 m \\ \dot{x}_2 = \rho_2(y) + m \\ \dot{x}_3 = x_4 - x_2x_3 + \rho_3(y) \\ \dot{x}_4 = x_2(x_4 - x_2x_3) + \rho_4(y) + \delta\omega + x_3 m \end{cases}$$

where $\rho(y) = (\rho_1(y), \rho_2(y), \rho_3(y), \rho_4(y))^T = \left(y_1y_2, y_2, 0, y_2^2\right)^T$ is an output injection. Then one can compute the distribution $\bar{\Delta}_p$:

$$\bar{\Delta}_p = \text{span}\left\{p, \left[\bar{f}, p\right], ad_{\bar{f}}^2 p, ad_{\bar{f}}^3 p\right\}$$

where $\bar{f} = (x_2, 0, x_4 - x_2x_3, x_2(x_4 - x_2x_3))^T$. One gets

$$\bar{\Delta}_p = \text{span}\left\{ \begin{pmatrix} \alpha \\ 0 \\ 0 \\ \delta \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \delta \\ \delta x_2 \end{pmatrix} \right\}.$$

Since $\dim \bar{\Delta}_p = 2$, it is possible to define a change of coordinates to decouple the fault signal from the other inputs. However, one can not find a new output $\psi(y_1, y_2)$ such that $d\psi \in \bar{\Delta}_p^\perp$ and thus one can not generate a residual not affected by ω.

Motivated by this, the problem addressed in this paper is to provide an other solution for the FPRG. The idea is to get an estimation of the disturbance input ω and then to use it to design a filter to detect the fault.

3. DISTURBANCE AND DEFECT DECOUPLING

To solve the FPRG, the first stage is now to decouple the disturbance effects from the fault signal. For this, let us consider Δ_l the smallest involutive distribution containing $l(x)$ and invariant with respect to f and p. Assume that:

(1) $\dim \Delta_l = d_l < n$,
(2) there exists a function $\tilde{y}_2 \in \operatorname{span}\{h_1, \ldots, h_p\}$ such that $d\tilde{y}_2 \in \Delta_l^{\perp}$,
(3) $p(x) \notin \Delta_l$.

Then one can find $z = \Theta_{\Delta_l}(x)$, a diffeomorphism based on the vector fields belonging to Δ_l and its annihilator, such that the system is locally transformed into (Isidori, 1995):

$$\dot{z}_1 = \tilde{\varphi}_1(z_1, z_2, y_1, y_2, u) + \tilde{l}(z)m + \tilde{p}_1(z)\omega \quad (11)$$

$$y_1 = \tilde{h}_1(z_1, z_2) \quad (12)$$

$$\dot{z}_2 = \tilde{\varphi}_2(z_2, y_1, y_2, u) + \tilde{p}_2(z_2)\omega \quad (13)$$

$$\tilde{y}_2 = \tilde{h}_2(z_2) \quad (14)$$

where $z_1 = (z_{11}, ..., z_{1d_l})^T$, $z_2 = (z_{2d_l+1}, ..., z_{2n})^T$. The subsystem (13-14) is assumed to be locally observable (Hermann and Krener, 1977).

Remark 1. The hypothesis (2) means that, in order to design an observer for the perturbation, the output of (13-14) must be decoupled from the fault signal.

Remark 2. The assumption that Δ_l is invariant w.r.t. p ensures that $\tilde{p}_2$ is only a function of z_2.

Example Let us compute Δ_l for the system (9)

$$\Delta_l = \operatorname{span}\left\{l, [f,l], [p,l], ad_f^2 l, [f,[p,l]], ad_p^2 l, [p,[f,l]]\right\}$$
$$= \operatorname{span}\left\{ \begin{pmatrix} x_1 \\ 1 \\ 0 \\ x_3 \end{pmatrix}, \begin{pmatrix} \alpha \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

This distribution can be used to define the following change of coordinates

$$z = \Phi(x) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 - x_2 x_3 \end{pmatrix}$$

so that the system is now given by

$$\begin{cases} \dot{z}_1 = z_1 z_3 + z_2 + \alpha\omega + z_1 m \\ \dot{z}_2 = z_3 + m \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = \delta\omega \end{cases}$$

with the following outputs

$$\begin{cases} y_1 = z_1 \\ y_2 = z_3 \end{cases}.$$

The disturbance and the defect are perfectly decoupled since $dy_2 \in \Delta_l^{\perp}$. Considering the subsystem

$$\begin{cases} \dot{z}_3 = z_4 \\ \dot{z}_4 = \delta\omega \\ y_2 = z_3 \end{cases} \quad (15)$$

it is now possible to design an estimator of the perturbation. Once the disturbance is estimated, it will be reinjected in the system (11-12) for which a filter is designed to solve the FPRG.

4. AN ESTIMATION OF THE PERTURBATION'S PROJECTION

Let us consider the subsystem (13-14), which is not influenced by the fault signal, and write henceforth

$$\tilde{\varphi}_2(z_2, y, u) = \tilde{f}_2(z_2, y) + \tilde{g}_2(z_2, y, u).$$

It is aimed here to design a sliding mode observer for this system in order to estimate the perturbation. Because of the nonlinearities and due to the fact that some unmeasurable states appear in the dynamics, an appropriate method for the design of the observer relies on the so-called observable triangular form (Barbot *et al.*, 1996). For this, it is assumed that the following conditions are satisfied:

(1) *$\tilde{f}_2, \tilde{g}_2$ and $\tilde{h}_2$ are analytic vector functions.*
(2) *The system (13-14) is BIBS: Bounded Input Bounded State, in finite time.*
(3)
$$rank \begin{pmatrix} d\tilde{h}_2 \\ dL_{\tilde{f}_2}\tilde{h}_2 \\ \vdots \\ dL_{\tilde{f}_2}^{n-d_l-1}\tilde{h}_2 \end{pmatrix} = n - d_l$$
where $L_{\tilde{f}_2}\tilde{h}_2 = \frac{\partial \tilde{h}_2}{\partial x}\tilde{f}_2$ is the classical Lie derivative.
(4) *$\tilde{g}_2$ verify for all $u \in U \subset \mathbb{R}$, where U is the set of admissible inputs,*
$$dL_{\tilde{g}_2} L_{\tilde{f}_2}^i \tilde{h}_2 \in \Omega^i \qquad \forall i \in \{0, ..., n - d_l - 1\}$$
with $\Omega^i = \operatorname{span}\{d\tilde{h}_2, dL_{\tilde{f}_2}\tilde{h}_2, ..., dL_{\tilde{f}_2}^i \tilde{h}_2\}$.
(5) *Denoting the distribution*
$$\Omega = \operatorname{span}\left\{d\tilde{h}_2, dL_{\tilde{\varphi}_2}\tilde{h}_2, \ldots, dL_{\tilde{\varphi}_2}^{n-d_l-2}\tilde{h}_2\right\},$$
it is assumed that $\tilde{p}_2(z_2) \in \Omega^{\perp}$.
(6) *The term $\tilde{p}_2(z_2)\omega$ is bounded and $\tilde{p}_2(z_2) \neq 0$ for all z_2.*

The condition (3) is a "degenerate" weak observability rank condition (Hermann and Krener, 1977). Conditions (5) and (6) allow to estimate the perturbation.

Proposition 3. (Marino and Tomei, 1992) Under the above conditions, the system (13-14) can be transformed into a triangular observable form, by using the diffeomorphism

$$\xi \triangleq \Theta(z_2) = \left[\tilde{h}_2(z_2), \ldots, L_{\tilde{f}_2}^{n-d_l-1}\tilde{h}_2(z_2)\right]^T,$$

defined on $\mathcal{U}$, a neighborhood of z_2:

$$\frac{d\xi}{dt} = \bar{A}_2\xi + \bar{g}_2(\xi, u) + \bar{P}\omega \tag{16}$$

$$\tilde{y}_2 = [1, 0, ..., 0]\,\xi = \xi_1 \tag{17}$$

$$\bar{A}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \bar{P} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \bar{p}_2(z_2) \end{pmatrix}$$

$$\bar{g}_2(\xi, u) = \begin{pmatrix} \bar{g}_{21}(\xi_1, u) \\ \bar{g}_{22}(\xi_1, \xi_2, u) \\ \vdots \\ \bar{g}_{2,n-d_l-1}(\xi_1, ..., \xi_{n-d_l-1}, u) \\ \bar{f}_2(\xi) + \bar{g}_{2,n-d_l}(\xi, u) \end{pmatrix}$$

$$\xi = \left[\, \xi_1 \;\, \xi_2 \;\, \cdots \;\, \xi_{n-d_l} \,\right]^T.$$

From the works of Boukhobza et al. (1996) and Drakunov and Utkin (1995), the following type of triangular sliding mode observer for the system (16-17) is designed:

$$\begin{cases} \dfrac{d\hat{\xi}_1}{dt} = \hat{\xi}_2 + \bar{g}_{21}(\xi_1, u) + \lambda_1 sign_1(\xi_1 - \hat{\xi}_1) \\ \dfrac{d\hat{\xi}_2}{dt} = \hat{\xi}_3 + \bar{g}_{22}(\xi_1, \tilde{\xi}_2, u) + \lambda_2 sign_2(\tilde{\xi}_2 - \hat{\xi}_2) \\ \vdots \\ \dfrac{d\hat{\xi}_{n-d_l-1}}{dt} = \hat{\xi}_{n-d_l} + \bar{g}_{2,n-d_l-1}(\xi_1, ..., \tilde{\xi}_{n-d_l-1}, u) \\ \qquad +\lambda_{n-d_l-1} sign_{n-d_l-1}(\tilde{\xi}_{n-d_l-1} - \hat{\xi}_{n-d_l-1}) \\ \dfrac{d\hat{\xi}_{n-d_l}}{dt} = \bar{f}_2(\xi_1, ..., \tilde{\xi}_{n_l-1}) + \bar{g}_{2,n-d_l}(\xi_1, ..., \tilde{\xi}_{n-d_l}, u) \\ \qquad +\lambda_{n-d_l} sign_{n-d_l}(\tilde{\xi}_{n-d_l} - \hat{\xi}_{n-d_l}) \\ \hat{y}_2 = \hat{\xi}_1 \end{cases} \tag{18}$$

where $\tilde{\xi}_i = \hat{\xi}_i + \lambda_{i-1} sign_{i-1}(\xi_{i-1} - \hat{\xi}_{i-1})$ for $i = 2, ..., n - d_l$. The $sign_i(\xi)$ function denotes the usual sign function but with a low pass filter of the ξ variable (Drakunov and Utkin, 1995) and an anti-peaking structure (Khalil, 1996). This anti-peaking structure comes from the idea that we do not inject the observation error information before reaching the sliding manifold linked with this information. Moreover, thanks to the particular *sign* function, the manifolds are reached one by one. Thus a subdynamics of dimension one is obtained and consequently no peaking phenomena appear (Sussmann and Kokotovic, 1991). More precisely $sign_i(.)$ is equal to zero if there exists $j \in \{1, ..., i\}$ such that $\tilde{\xi}_j - \hat{\xi}_j \neq 0$ (by definition $\tilde{\xi}_1 = \xi_1$, $sign_1 = sign$), else $sign_i(.)$ is equal to the usual $sign(.)$ function. Thus, $\tilde{\xi}_i - \hat{\xi}_i$ converges to zero if all the $\tilde{\xi}_j - \hat{\xi}_j$ with $j < i$ have converged to zero before.

Theorem 4. Considering the BIBS system (16-17) and the observer (18), for any initial state $\xi(0)$, $\hat{\xi}(0)$ and any bounded input u and $\bar{p}_2(z_2)\omega$, there exists a choice of λ_i allowing, in finite time, to recover an estimation of the perturbation term.

Proof: The whole proof of the theorem can be found in (Barbot *et al.*, 1996). Let us show that the use of this observer allows to get an estimation of the disturbance's projection $\bar{p}_2(z_2)\omega$. Denote the error observation $e = \xi - \hat{\xi}$. As this observer is characterized by a step by step convergence, at the d_l^{th} step, the observation errors are given by:

$$\begin{aligned} &\dot{e}_1 = 0 \\ &\dot{e}_2 = 0 \\ &\vdots \\ &\dot{e}_{d_l-1} = 0 \\ &\dot{e}_{d_l} = -\lambda_{d_l} sign(e_{d_l}) + \bar{p}_2(z_2)\omega \end{aligned} \tag{19}$$

Under considerations about the sliding mode theory (Utkin, 1992), e_{d_l} tends to zero in finite time if

$$\lambda_{d_l} > \|\bar{p}_2(z_2)\omega\|_{\max}.$$

Then the resulting equivalent dynamics on the sliding surface $e_{d_l} = 0$ provides an estimation of the perturbation

$$\bar{p}_2(z_2)\omega = \lambda_{d_l} sign_{eq}(e_{d_l})$$

where, according to the sliding mode theory, $sign_{eq}(e_{d_l})$ is the mean value of the function $sign(e_{d_l})$. The value of $sign_{eq}(e_{d_l})$ is obtained with a low pass filter. It can generate some approximations that are negligible if the bandwidth of the filter is well chosen.

Remark 5. The condition on the perturbation might appear quite restrictive but this approach could be proved interesting as far as the problem of isolation is concerned.

Example Let us take again the example (9) for which the disturbance has been decoupled from the fault. Note that the system (15) is already in the observable triangular form. Let us define the following observer

$$\begin{cases} \dfrac{d\hat{\xi}_3}{dt} = \hat{\xi}_4 + \lambda_1 sign_1(z_3 - \hat{\xi}_3) \\ \dfrac{d\hat{\xi}_4}{dt} = \lambda_2 sign_2(\tilde{\xi}_4 - \hat{\xi}_4) \end{cases} \tag{20}$$

where $\tilde{\xi}_4 = \hat{\xi}_4 + \lambda_1 sign_1(z_3 - \hat{\xi}_3)$. The dynamics of the observation error $e = z - \hat{\xi}$ is given by

$$\begin{cases} \dot{e}_3 = e_4 - \lambda_1 sign_1(e_3) \\ \dot{e}_4 = \delta\omega - \lambda_2 sign_2(\tilde{\xi}_4 - \hat{\xi}_4) \end{cases} \tag{21}$$

Provided that $\lambda_1 > \|e_4\|_{\max}$, one gets after a finite time $e_4 = \lambda_1 sign(e_3)$ and $\tilde{\xi}_4 = \hat{\xi}_4 + e_4 = z_4$ so that the equivalent dynamics for the last equation of (21) is

$$\dot{e}_4 = \delta\omega - \lambda_2 sign_2(e_4)$$

and, choosing the observation gain such that $\lambda_2 > \|\delta\omega\|_{\max}$, a sliding mode occurs on $e_4 = 0$. This implies that

$$\delta\omega = \lambda_2 sign_{eq}(e_4)$$

which gives us an estimation of the perturbation ω.

5. FAULT DETECTION

Once an estimation of the perturbation has been obtained, the system can be rewritten as

$$\begin{aligned}
\dot{z}_1 &= \tilde{\varphi}_1(z, y, u) + \tilde{l}(z)m + \frac{\tilde{p}_1(z)}{\bar{p}_2(z_2)}\lambda_{d_l} sign_{eq}(e_{d_l}) \\
\dot{z}_2 &= \tilde{\varphi}_2(z_2, y_1, y_2, u) + \frac{\tilde{p}_2(z_2)}{\bar{p}_2(z_2)}\lambda_{d_l} sign_{eq}(e_{d_l}) \\
y_1 &= \tilde{h}_1(z_1, z_2) \\
\tilde{y}_2 &= \tilde{h}_2(z_2)
\end{aligned}$$

If the system is a linear one, it is then possible to use the classical methods proposed in the literature (see e.g. (Frank, 1990)). If there exist some nonlinearities, the fault can be detected by applying the algorithm proposed in (Djemai *et al.*, 2000). A sliding mode observer is designed to cancel the unknown nonlinearities and then a high gain observer allows to detect the fault.

Example In our example, the fault can be detected thanks to the following step-by-step observer. For this, let us define the change of coordinates

$$\begin{cases} \chi_1 = z_1 \\ \chi_2 = z_2 + z_1 m \end{cases}$$

so that the subsystem

$$\begin{cases} \dot{z}_1 = z_1 z_3 + z_2 + \alpha\omega + z_1 m \\ \dot{z}_2 = z_3 + m \end{cases}$$

is transformed into

$$\begin{cases} \dot{\chi}_1 = z_1 z_3 + \chi_2 + \alpha\omega \\ \dot{\chi}_2 = z_3 + \mu(m) \end{cases}$$

where $\mu(m) = m(1 + z_1 z_3 + \chi_2 + \alpha\omega)$ ($\dot{m} = 0$ since m is assumed to be piecewise constant). The chosen filter used to detect the fault signal m is

$$\begin{aligned}
\dot{\hat{\chi}}_1 &= z_1 z_3 + \hat{\chi}_2 + \frac{\alpha}{\delta}\lambda_2 sign_{eq}(e_4) + \eta_1 sign(\chi_1 - \hat{\chi}_1) \\
\dot{\hat{\chi}}_2 &= z_3 + \eta_2(\tilde{\chi}_2 - \hat{\chi}_2) \\
r(\hat{\chi}, y_1) &= (\tilde{\chi}_2 - \hat{\chi}_2)
\end{aligned}$$

where $\tilde{\chi}_2 = \hat{\chi}_2 + \eta_1 sign(\chi_1 - \hat{\chi}_1)$. The observation error ($\varepsilon = \chi - \hat{\chi}$) dynamics is

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 - \eta_1 sign(\varepsilon_1) \\ \dot{\varepsilon}_2 = \mu(m) - \eta_2(\tilde{\chi}_2 - \hat{\chi}_2) \end{cases}$$

After a finite time, choosing $\eta_1 > \|\varepsilon_2\|_{\max}$, a sliding mode occurs on the manifold $\varepsilon_1 = 0$ and the equivalent dynamics is given by

$$\varepsilon_2 = \eta_1 sign(\varepsilon_1)$$

so that $\tilde{\chi}_2 = \chi_2$. Then $\dot{\varepsilon}_2 = \mu(m) - \eta_2\varepsilon_2$. On the manifold $\varepsilon_1 = 0$, the chosen residual is ε_2 which is null if no fault occurs and different from zero when a fault appears.

Remark 6. Note that $\varepsilon_2 = 0$ if $m = 0$ or $\bar{\mu}(m) = (1 + z_1 z_3 + \chi_2 + \alpha\omega) = 0$. This singularity relies on the structure of the system. Indeed, let us consider the extended system

$$\begin{cases} \dot{z}_1 = z_1 z_3 + z_2 + \alpha\omega + z_1 m \\ \dot{z}_2 = z_3 + m \\ \dot{m} = 0 \end{cases} \tag{22}$$

with the output $y = z_1$. Computing the observability codistribution $d\mathcal{O}$:

$$d\mathcal{O} = \begin{pmatrix} 1 & 0 & 0 \\ z_3 + m & 1 & z_1 \\ (z_3 + m)^2 & (z_3 + m) & z_1(z_3 + m) + \bar{\mu}(m) \end{pmatrix},$$

the weak observability condition (restricted to the third order) implies that the system is not observable on the manifold $\bar{\mu}(m) = 0$ (rank $d\mathcal{O} = 0$). However, this problem can easily be overcome since it is possible to evaluate $\bar{\mu}(m)$.

The Figure 1 shows a simulation where a failure is assumed to occur at $t = 0.5sec$. Observe that the fault can not be detected from the outputs of the system but that the output of the residual generator clearly indicates the failure.

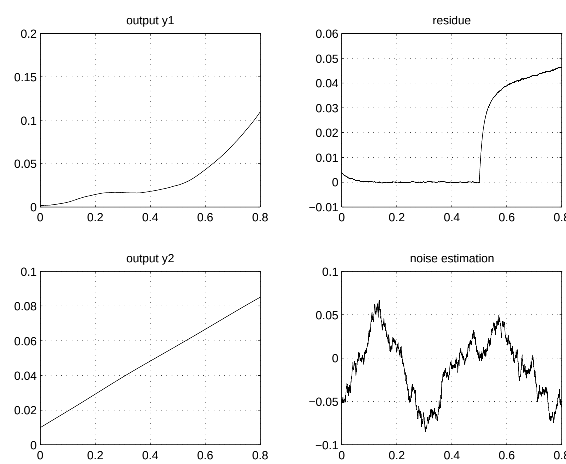

Fig. 1. The residual generation

6. CONCLUSION

In this paper, the problem of fault detection for a nonlinear system has been studied. Particularly, we proposed an alternative solution to the FPRG when some known methods can not be applied.

To this end, conditions have been given for the existence of a fault-decoupled and observable subsystem which is affected by the disturbance. Then the disturbance inputs have been estimated by the used of a step-by-step sliding mode observer. The perturbations being known, it has been shown that a residual generator could be designed to detect the fault signal. Some simulations has been given to show the efficiency of the proposed method. In further works, this approach will be extended to the case of the detection and the isolation of one or more faults.

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