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IMPROVED MODEL PREDICTIVE CONTROL AND ITS APPLICATION

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Abstract: This paper presents a modified form of model based predictive control that exploits the concept of non-trivial terminal state weighting in the cost function. While maintaining simplicity of implementation, the performance of this algorithm is not sacrificed and the computational burden compared with traditional approaches is greatly reduced. This paper describes the proposed algorithm in detail and through application to several systems, including a benchmark fluidised catalytic cracker unit, demonstrates its advantages over traditional model based predictive control algorithms.

Keywords: Model based predictive control, fluidised catalytic cracker unit

1. INTRODUCTION

Control of unconstrained linear systems is an established branch of automatic control engineering that has seen significant theoretical advances in recent years. Despite these advances the practical application of this theory to systems that are subject to constraints on both states and manipulated variables still presents a significant challenge. One of the most popular methods for dealing with the constrained regulation of linear systems is Model Predictive Control (MPC), a detailed survey of which can be found in Mayne *et al.* (1998). In this approach control moves that minimise a time-domain performance function, evaluated over a prediction horizon, subject to system dynamics and constraints on state and manipulated variables are applied to the process.

Traditionally, the issue of solving the constrained infinite-horizon MPC (CIHMPC) control problem, which would automatically guarantee nominal stability and feasibility assuming the existence of constrained optimum, has not been successfully addressed due to the inherent fact that the resulting optimisation problem is infinite-dimensional and hence insoluble. The solution had, therefore, to be obtained through recasting the infinite-horizon into the finite-horizon and repetitive solution of the corresponding finite-dimensional optimisation problem at each time instant while only the first element of the optimal control sequence is implemented. In this approach the performance function has been defined over the restrictive subset of the time-domain, which is of comparable size to the most dominant transient characteristics of the system, such as rise time or settling time.

Even though the constrained infinite-horizon MPC control problem has been solved recently, it may still require extensive computational burden in order to accommodate for conditions imposed on its implementation. Therefore, it becomes important to address the practical issues as well as theoretical ones when introducing the new control algorithm, such as CIHMPC.

In this paper, the MPC controller that has been proposed by Scokaert and Rawlings (1998) as an intermediate step towards developing CIHMPC controller has been implemented on a number of systems, including the benchmark simulation of a fluid catalytic cracking unit taken from MacFarlane *et al.* (1993). The fundamental objective of this paper being to demonstrate the capability of this algorithm to satisfy performance requirements while keeping the computational burden low. The presented controller is compared with the nominal MPC controller and the CIHMPC from (Scokaert and Rawlings, 1998; Chmielewski and Manousiouthakis, 1996) and shown to satisfy the expectations.

The paper is organised as follows. In the second section, the general state- space system is introduced together with the mathematical description of the constraints. This is followed with a description of nominal MPC, standard CIHMPC and the modified form of MPC. Section 5 details application of the modified MPC to the control of a double integrator, non- minimum phase system and the simulation of the fluid catalytic cracking unit (FCCU) plant and

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compared modified MPC with both nominal MPC and the CIHMPC in the more practical context. Finally, the paper provides several conclusions from the work.

2. CONSTRAINED OPTIMAL CONTROL FORMULATIONS

2.1 System Description

Consider a discrete linear time- invariant system of the following form:

$$x_{k+1} = Ax_k + Bu_k \tag{2.1}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control vector, $A \in \Re^{n \times n}$ is the state transition matrix, $B \in \Re^{n \times m}$ is the control (input) distribution matrix and $k \in Z^+$ is a nonnegative integer denoting the time instant.

In most practical situations, states and/or the manipulated inputs are bounded in magnitude. These constraints on the states and inputs can be incorporated into the mathematical model of the system by the following relations:

$$Du_k \le d \qquad k \in Z^+ \tag{2.2}$$

$$Hx_k \le h \qquad k \in Z^+ \tag{2.3}$$

where D and H represent constraint distribution matrices while d and h are the corresponding constraint levels for state and control vectors, respectively.

2.2 Constrained Optimal Control

Several constrained optimal control regulation problems are now presented.

(P1): Constrained Finite- Horizon Optimal Control Problem

Given an initial state x_0, the task is to find the optimal control sequence:
$\mathbf{u}^N = \{u_k\}_{k=0}^{N-1} = \{u_0, u_1, \ldots, u_{N-1}\}$, such that the constraints on states and/ or inputs (controls) are not violated while x_0 is regulated to the origin of the state-space in optimal manner, defined by the following optimisation problem:

$$J_N(x_0, \mathbf{u}^N) = \min_{\mathbf{u}^N} \left[\sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k) + x_N^T P_0 x_N \right] \tag{2.4}$$

subject to:

$$x_{k+1} = Ax_k + Bu_k, \qquad k = 0,1,\ldots, N-1$$

$$Du_k \le d, \qquad k = 0,1,\ldots, N-1$$

$$Hx_k \le h, \qquad k = 0,1,\ldots, N$$

where Q and R are weighting matrices and P_0 is the terminal weight, which, in this case, is pragmatically taken to be equal to the nominal state weight, i.e. $P_0 = Q$.

Note that no information concerning the system's evolution for $k \ge N+1$, is encrypted into the algorithm. Hence there is a requirement for the prediction horizon, N, to be large enough to ensure stability, feasibility and satisfactory performance requirements. For the remainder of this paper, (P1) will be termed the nominal MPC scheme.

The on-line implementation of the general finite-horizon MPC algorithms is of receding- horizon character wherein at every sampling instant, the entire control sequence is calculated but only the first control move is implemented. In this way, the MPC controller attains the feedback structure, taking into account the plant-model mismatch and external disturbances. Note that significant computational constraints are imposed onto the overall control system as quadratic programming program needs to be resolved at each time instant. It is for this particular reason, that MPC is mainly applied to linear systems where relatively infrequent sampling allows for extensive computation.

CIHMPC, has been proposed in (Scokaert and Rawlings, 1998; Chmielewski and Manousiouthakis, 1996). The major motivation for its development is the guarantee of nominal stability that the algorithm offers. By using this approach, the solution is stabilizing, under the assumptions on the feasibility, stabilizability and detectability. It is also noted that if there is no feasible solution to the CIHMPC then there is no solution for the given control problem setting as stated in (Scokaert and Rawlings, 1998; Chmielewski and Manousiouthakis, 1996).

(P2) CIHMPC Formulation:

$$J_N(x_0, \mathbf{u}^N) = \min_{\mathbf{u}} \left[\sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k) + x_N^T P_0 x_N \right] \tag{2.7}$$

subject to:

$$x_{k+1} = Ax_k + Bu_k \qquad k \ge 0$$

$$Hx_k \le h \qquad N \ge k \ge 0$$

$$Du_k \le d \qquad N-1 \ge k \ge 0$$

$$x_N \in X_{K_{LQ}}$$

where $P_0 = S = S^T > 0$ is taken to be the solution of the Discrete Algebraic Riccati Equation (DARE). $X_{K_{LQ}}$ is defined as the set which is invariant under the application of the unconstrained LQR control law for present and all future time, and contained within the state/ control constraints:

$$X_{K_{LQ}} = \left\{ \begin{array}{l} x_0 \in \Re^n \mid H(A+BK_{LQ})^k x_0 \le h, \\ \text{and } DK_{LQ}(A+BK_{LQ})^k x_0 \le d \,\forall k \in Z^+ \end{array} \right\} \tag{2.8}$$

where K_{LQ} is the unconstrained LQR gain, obtained from solving the DARE. By incorporating the terminal weight to be equal to the solution of the DARE and imposing the terminal state inclusion condition, $x_N \in X_{K_{LQ}}$, it is ensured that from $k = N$ up to $k = \infty$, unconstrained LQR regulates a system without violating the constraints. Hence, by using the principle of optimality, the finite- horizon MPC controller, which solves (P2), is found to be equivalent to CIHMPC.

The main limitation with the solutions to both (P1) and (P2) is the computational burden that is imposed through repetitive solving of the corresponding quadratic program. This computational burden is introduced, in the case of CIHMPC, with the large prediction horizon that may be required to guarantee that the terminal state belongs to $X_{K_{LQ}}$.

Finally the modified MPC algorithm, proposed by Scokaert and Rawlings (1998), as an intermediate step from nominal MPC to the CIHMPC, is presented next:

(P3): Modified Constrained Finite-Horizon Optimal Control Problem

$$J_N(x_0, \mathbf{u}^N) = \min_{\mathbf{u}} \left[\sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k) + x_N^T P_0 x_N \right] \tag{2.12}$$

subject to:

$$x_{k+1} = Ax_k + Bu_k \qquad k \ge 0$$
$$Hx_k \le h \qquad N \ge k \ge 0$$
$$Du_k \le d \qquad N-1 \ge k \ge 0$$

and where $P_0 = S = S^T > 0$ is taken to be the solution of the DARE.

Similarly to the CIHMPC, it is assumed through the relation $P_0 = S$ that from $k = N$ up to $k = \infty$, the unconstrained LQR control law regulates the plant. However, in this modified MPC algorithm there is no assurance that all the inequality constraints, imposed on the inputs and states, are inactive from $k = N$ up to $k = \infty$, since the terminal state inclusion condition is not imposed. As a consequence, modified MPC is not equivalent to the CIHMPC. However, by keeping the terminal weight equal to the solution of DARE, some information, concerning the future evolution of the system, is present during the subsequent optimisation. Furthermore, the assumption of inactive constraints at the end of the prediction horizon may not induce large deviation from the actual situation and hence performance of the modified MPC may not greatly vary from the CIHMPC controller, as demonstrated in the following section.

Note that the key difference between (P2) and (P3) lies in relaxation of $x_N \in X_{K_{LQ}}$ condition in the case of (P3). Consequently, complexity of the controller corresponding to (P3) is greatly reduced since the length of the prediction horizon does not need to comply with the terminal state inclusion condition. This is a very important aspect in any real-time control application due to the requirement for repetitive solving of the corresponding quadratic program. It is demonstrated later in this paper that the performance of the modified MPC controller is comparable to that obtained with the MPC algorithm proposed by (Scokaert and Rawlings, 1998; Chmielewski and Manousiouthakis, 1996) with much longer prediction horizons. In this way, the computational burden is reduced without sacrificing the performance of the resulting control system.

However, the absence of infinite- horizon guarantee disallows any theoretical properties, concerning CIHLQR, to be replicated in the case of (P3). On the other hand, it can be argued that issues such as nominal stability are usually satisfied and, in fact, through the simulation results in this paper it is argued that in many circumstances satisfactory performance results even if strict theoretical properties of a control algorithm are not fully established.

In the following section, it will be shown that in many situations, absence of terminal state inclusion condition minimises the computational burden, by keeping the length of the corresponding prediction horizon small, while the performance remains similar if not identical to that of the CIHMPC. Additionally, it will be demonstrated that the modified MPC outperforms the nominal MPC with the same length of the prediction horizon and the same choice of weighting matrices.

3. SIMULATION RESULTS

3.1 Introduction

In this section a number of simulation experiments, implemented in MATLAB, demonstrate the efficiency of the modified MPC controller when compared with CIHMPC, and also its superiority over the equivalent nominal MPC. Equivalence here is in terms of the length of the prediction horizon and weighting matrices.

The control problems of state regulation as well as set- point tracking have been considered. Note, however, that the reference signal tracking can be also implemented in straightforward fashion.

3.2 Double Integrator

The first system to be implemented is the double integrator with the following discrete- time transfer function, assuming a sampling period of 1 second:

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \cdot u_k$$

The control input is assumed to be bounded by $-0.5 \le u \le 0.5$ and the states are assumed to be unbounded. The weighting matrices, used in the cost function of the MPC controller are chosen to be equal to:

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}; \qquad R = 1$$

The initial states, used in this experiment, are 121 points in the state- space sub- region, defined by:

$$\begin{bmatrix} -5 \\ -5 \end{bmatrix} \le x_0 \le \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

The value of the prediction horizon, N^* required to solve the CIHMPC problem was determined for each of the initial states.

The modified MPC controller was also implemented with the length of the prediction horizon, denoted by $\hat{N}$, chosen to be one with such a property that the resulting control move converges to the control input of the standard CIHMPC formulation. This modified MPC formulation has been named the least- conservative CIHMPC.

In Figure 1, a frequency histogram of the length of the prediction horizon for both the standard CIHMPC and the least- conservative CIHMPC is shown, taken over the 121 data points, i.e. initial states. In other words, the number of initial states for which a particular length of the prediction horizon is required to guarantee solution to (P2) and the number of initial states for which (P3) with a particular length of a prediction horizon would result in the same initial control move as the solution to (P2) are plotted as functions of the length of the prediction horizon.

The mean and standard deviations of $\hat{N}$ is shown to be much smaller than the mean and standard deviation of N^*:

$$\mu_{\hat{N}} = 1.15, \sigma_{\hat{N}} = 0.60$$
$$\mu_{N^*} = 13.4, \sigma_{N^*} = 8.0$$

In this experiment, it has been demonstrated that for a large number of states, the CIHMPC optimal and least- conservative, in the sense of computational burden, solution is given simply by the modified MPC with the length of the prediction horizon equal to 1 while the standard CIHMPC requires much larger lengths of prediction horizon to satisfy the terminal state inclusion condition. As expected, prediction horizon of

CIHMPC was found to show a strong dependence on the initial state while this was not the case for the modified MPC controller, listed in Figure 1 as least- conservative CIHMPC.

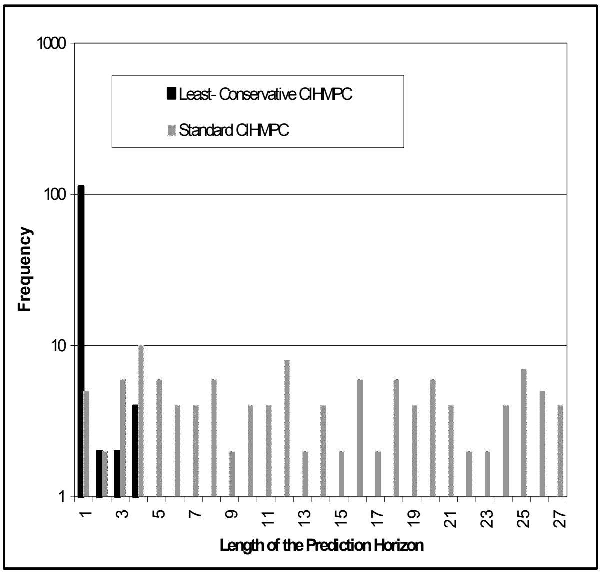

Figure 1: Frequency Histogram of Prediction Horizon

3.3 Non- Minimum Phase System

The second system that has been considered is the following non- minimum phase system:

$$G(s) = \frac{-4s+1}{s^2(s+1)}$$

Its discrete- time state- space matrices, assuming sampling interval of 1 second, are given as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 3.0806 \\ 0 & 1 & 0 & -4.1612 \\ 0 & 0 & 1 & 3.0806 \end{bmatrix}, \; B = \begin{bmatrix} 0.6744 \\ 2.1946 \\ -1.3558 \\ -0.5938 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

The control input is assumed to be bounded by $-0.5 \le u \le 0.5$ and the output is assumed to be bounded by $-12 \le y \le 12$. The weighting matrices, used in the cost function of the MPC controller are chosen to be equal to $Q = diag(1,1,1,10)$ and $R = 1$ with initial output set to $y_0 = x_{4,0} = 1$.

In this case, all three controllers, discussed in the previous section, have been implemented.

In order for stability and feasibility to be satisfied, prediction horizon length of the nominal MPC had to be kept above or equal to 10. On the other hand, modified MPC with prediction horizon greater or equal to 2 produced satisfactory, stable and feasible response. However, more importantly, the modified MPC with prediction horizon length of 2 has been found to be equivalent, in terms of control inputs computed by the quadratic program, to the CIHMPC for which the initial length of the prediction horizon, required to satisfy the terminal state inclusion constraint, has been found to be equal to 8. The results of this experiment are shown below in Figures 2 and 3.

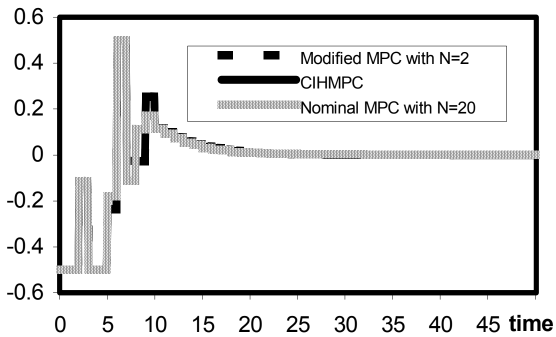

Figure 2: Control Input

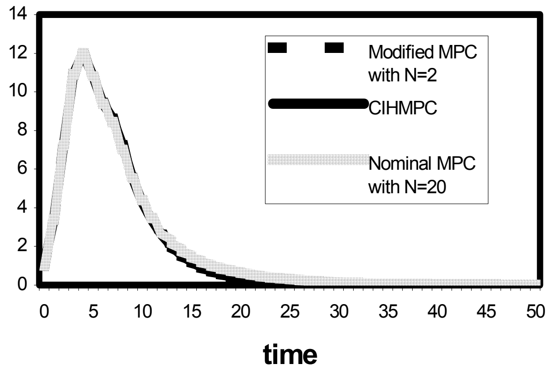

Figure 3: Output Response

3.4 FCCU Plant

Fluidised catalytic cracking units are complex chemical plants within which several feeds consisting of high boiling point components from several refinery process units are cracked into lighter and more valuable components. After further processing, the product streams of the FCCU are blended with streams from other refinery units to produce a number of products, such as distillate and various grades of gasoline.

The model that is used for emulation of real FCCU has been developed by MacFarlane *et al.* (1993).

In this work, the control of a subsystem of the FCCU has been implemented with the following inputs:

$F_1 =$ set- point for the flow of wash oil into the reactor riser (lb/s)

$F_3 =$ the flowrate of fresh feed to reactor riser (lb/s)

$F_T =$ the total airflow (into regenerator) controller (lb/s)

and the following outputs:

$T_r =$ Riser temperature ($^\circ F$)

$T_{reg} =$ Regenerator temperature ($^\circ F$)

$C_{O_2,sg} =$ Concentration of oxygen in the stack gas

$V_{11} =$ Wet gas compressor suction valve position

An incremental 12th order linear state-space model was obtained for the system using standard system identification techniques. The states used in the model were chosen to be past output measurements. The set- points used were as follows:

$T_r^{SET} = 990^\circ F$ $\qquad T_{reg}^{SET} = 1270^\circ F$

$C_{O_2,sg}^{SET} = 1.7\%$ $\qquad V_{11}^{SET} = 0.85$

An unmeasured disturbance, which is coke formatting factor, modelling the unknown composition of the fresh feed, was introduced to upset the overall system and test the controller.

The weightings on the states, in matrix Q, are given as follows:

Weighting on $T_r = 300$;

Weighting on $T_{reg} = 100$;

Weighting on $C_{O_2,sg} = 400$;

Weighting on $V_{11} = 900$

The control inputs have been weighted equally by $R = 200 \cdot I^{3\times3}$.

Note that these weights were chosen through trial and error. The constraints on the manipulated variables in this system are as follows:

$$0 \le F_1 \le 17 \qquad 0 \le F_3 \le 144$$
$$0 \le F_T \le 100$$

$$-0.1 \le \Delta F_1 \le 0.1 \qquad -0.2 \le \Delta F_3 \le 0.2$$
$$-0.5 \le \Delta F_T \le 0.5$$

Figures 4 and 5 compare the response of the regenerator temperature and the concentration of the oxygen in the stack gas with the system under modified MPC (MMPC) and nominal MPC (NMPC) with varying prediction horizons.

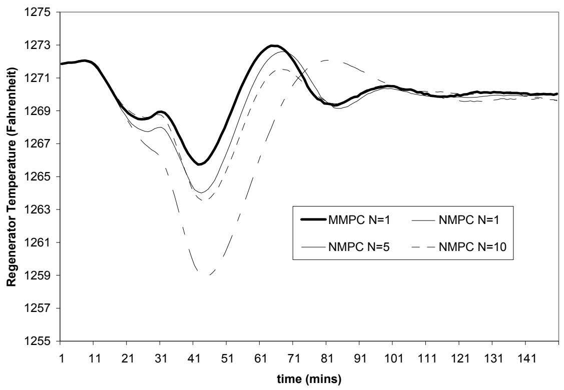

Figure 4: Regenerator Temperature

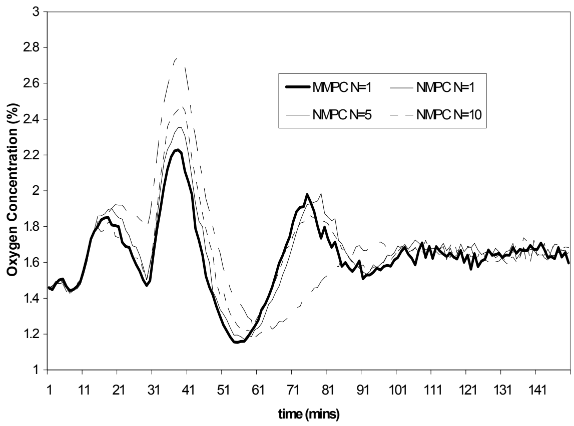

Figure 5: Oxygen Concentration

These results demonstrate how modified MPC with a prediction horizon of 1 is capable of regulating regenerator temperature and oxygen concentration with greater accuracy than nominal MPC with the same prediction horizon. It was found that increasing the prediction horizon for modified MPC produced very little improvement and as the prediction horizon was increased for nominal MPC, it began to approach the control performance of modified MPC with a prediction horizon of 1. This result confirms the expectation that modified MPC should produce good control performance with a small length of the prediction horizon, thus significantly reducing the computation burden while maintaining satisfactory performance.

4. CONCLUSION

In this paper, the issue of modifying the traditional finite-horizon MPC has been explored in order to improve performance of the resulting control system. These modifications are closely linked with constrained infinite-horizon MPC. Through applications to a simple double integrator, non-minimum phase system and to a benchmark simulation of a FCCU plant it is shown that the modified MPC algorithm produces satisfactory control performance with a significantly reduced prediction horizon, when compared to nominal MPC as well as standard CIHMPC controller, as proposed by (Scokaert and Rawlings, 1998; Chmielewski and Manousiouthakis, 1996). This reduction in the prediction horizon length means that the computation burden of the algorithm is reduced which is an important result since it shows that the application area for MPC may be extended to dynamic systems that require rapid sampling rates.

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