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A ROBUST CONTROL DESIGN TO ACHIEVE A REQUIRED LOOP SHAPE [1]

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Abstract: A robust control design method which realizes a required loop shape formed by nominal performance and robust stability specifications is proposed. An identification based method is given to design the pre-filter required to obtain the desired loop shape. Then a $\mathcal{H}_\infty/\mu$ method is used in order to handle both the parametric uncertainties and the unmodelled dynamics together. Thus the robust performance requirement is also met. The method is demonstrated in an active suspension design.

Keywords: robust control, optimal control, identification algorithms.

1. INTRODUCTION AND MOTIVATION

The main objectives of the control design are to reduce the effect of the unmeasured disturbances and that of the system uncertainties. In the loop shaping context it means that a compensator which meets the nominal performance and robust stability requirements is to be developed. These methods are close to the designer's point of view since the performance specifications can be defined directly for the loop shape. One of their important advantages is that the desired controlled system is achieved by the manipulation of the open-loop gains. A common lack of these type of methods that they can not guarantee robustness of the performance against model uncertainties.

A direct loop-shaping method that incorporates the characteristics of both loop-shaping and $\mathcal{H}_\infty$ design was proposed by McFarlane and Glover (1992). First, an open loop pre-compensator and a post-compensator are selected in such a way that the magnitude of the so-called shaped model satisfies the performance requirements. Second, an $\mathcal{H}_\infty$ synthesis is performed to satisfy the robust stability requirement. The first difficulty of this method is the selection of the pre-compensator and post-compensator, which can be done in an ad-hoc manner. The second difficulty is that the performance requirements can not always be met since the design methodology guarantees only robust stability and the effect of the uncertainties are not taken into consideration from the performance point of view.

A procedure for the designing of pre- and–post compensators has been developed by Hyde (1991) by reordering the inputs and outputs of the nominal plant and the pre-compensator was selected for low frequency dynamics and the post-compensator was selected for noise rejection. A systematic procedure for designing the compensators has been developed by Papageorgiou and Glover (1997) by using a kind of an approximative inner-outer factorization method. A method has been developed by Vinnicombe (1993) for designing a pre-compensator to make the transfer matrix from references to outputs equal to some reference model.

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The second step of the loop-shaping method proposed by McFarlane and Glover (1992) consists in designing a compensator that guarantees robust stability against model uncertainties. This step works also as a validation step for the desired loop shape. However by closing the loop with an acceptable compensator computed in this second step, a possible performance degradation will be obtained in comparison with the performance requirements set by the designer in choosing the desired loop shape. Moreover little is known about the robustness of the achieved performance.

In an earlier result of our project a new pre- or post-compensator choosing algorithm was proposed. To achieve the required loop transfer function the weighting functions were designed by an identification procedure. A frequency-weighted GOBF (Generalized Orthonormal Basis Functions) method is applied to approximate the required loop shape with higher accuracy and to emphasize the important frequency domains, (Gáspár *et al.*, 2000).

In this paper, some new results of our project are presented. The weighting functions for achieving a required loop shape are designed by using an identification procedure for the outer part of the model. Then a $\mathcal{H}_\infty/\mu$ synthesis is applied, in which the performance requirements, the real parametric uncertainties and the unmodelled dynamics can be handled together. Thus, the designed compensator guarantees not only nominal performance and robust stability but also robust performance. Comparing our method with the McFarlane and Glover method, the main differences are that the required loop shape is approximated by using an identification step instead of selecting pre- and post-compensators and that the designed compensator meets robust performance requirements. These modifications are motivated by different engineering problems. In practice, one of the most important difficulties in the design is that the model contains a large number of components, the behavior and properties of which are unknown, uncertain, or varying during operation. In this paper the method is illustrated in an active suspension control problem.

The organization of the paper is as follows. Section 2 presents the steps of the robust control synthesis. Section 3 discusses the design of the weighting functions. Section 4 discusses the design of the robust compensator. Section 5 demonstrates the algorithm in an active suspension design problem. Finally, Section 6 presents concluding remarks.

2. A LOOP-SHAPING DESIGN PROCEDURE

The specifications on nominal performance and robust stability are assumed to be available as a required loop shape, $\sigma(L)$. It means that the singular value functions of the loop are defined taking into consideration the disturbance attenuation performance and plant perturbation specifications.

The steps of the proposed design procedure are as follows:

- Step 1: Identify a SISO loop transfer function, ℓ, for the frequency response of the loop-shaping defined by the designer using the nominal performance and robust stability specifications. The model must be identified as accurately as possible in the frequency domains that are relevant in terms of the controlled system.
- Step 2: Design weighting functions K_F and W_2, for the plant model, G, in such a way that the singular values of the loop transfer function $W_2 G K_F$ tends toward the required loop shape. Thus, the transfer function W_2 and K_F is determined so that
$$\sigma(\ell)(\omega) = \sigma(W_2 G K_F)(\omega).$$
- Step 3: Design a robust compensator K_R using the $\mathcal{H}_\infty/\mu$ synthesis, in which both the parametric uncertainties and the neglected dynamics are taken into consideration.
- Step 4: Construct the compensator using its components in the following way:
$$K = K_F K_R W_2.$$

In the next sections the main steps of the loop-shaping design are presented.

3. THE DESIGN OF A WEIGHTING FUNCTION TO ACHIEVE A REQUIRED LOOP SHAPE

The strategy for the computation of the weights for loop shaping is the following: first, a SISO minimum phase transfer function ℓ is identified corresponding to the desired loop shape. The first step can be performed by a subspace type algorithm described in Van Overschee and De Moor (1996). A time domain method is proposed by McKelvey and Helmersson (1996).

First an inner–outer factorization is performed corresponding to the plant. If the plant is square invertible, i.e., $G = G_o G_i$, then the weight function is computed as $K_F = \ell G_o^{-1}$, the realized loop being $L = \ell G_i$.

Since the Ricatti equation based inner-outer factorization methods can be computational intensive for high complexity models, a method based on Zhang and Freundenberg (1993) is proposed. A continuous time version is described in Zhang and Freundenberg (1993) and Shaked (1989). The main idea is to determine the inner factor by computing the transmission zeros, the state and output zero directions of the system. For a MIMO system of dimension $n \times n$ with state space realization (A, B, C, D) the transmission zeros are defined as the set of complex numbers that satisfy

$$rank \begin{bmatrix} z\mathbb{I} - A & -B \\ -C & -D \end{bmatrix} < 2n,$$

the multiplicity of z being equal to the algebraic multiplicity. For a transmission zero z, the left output zero direction η is defined as

$$\eta^* G(z) = 0,$$

such that $\eta^*\eta = 1$.

Let us consider the set $\{z_1, z_2, \cdots, z_l\}$, including multiplicities, of nonminimum-phase zeros of the transfer function G and the corresponding zero directions $\{\eta_1, \cdots, \eta_l\}$. The inner factor G_i can be written as

$$G_i(z) = \prod_{i=1}^{l} B_{z_i}(z),$$

where

$$B_{z_i}(z) = \mathbb{I} - P_i + \frac{z - z_i}{1 - \overline{z}_i z} P_i, \quad P_i = \eta_i \eta_i^*.$$

The state space realization for the outer factor G_o is given by (A, B, C_o, D_o) The method in Zhang and Freundenberg (1993) uses a step by step computational process to determine the state matrix C_o of the outer factor. However, it can be more efficient to perform an identification step for $G_o = GG_i^*$ in order to determine C_o and D_o, using a subspace identification method or a generalized orthogonal basis function expansion method, if the transfer function G was already parametrized in these terms.

One can always assume that the plant is proper, otherwise one can multiply the transfer function by $z\mathbb{I}$ till this requirement is satisfied.

If the original plant G with state space realization (A, B, C, D) was not square and invertible then one can start from a singular value decomposition of D of the form

$$D = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^*.$$

By partitioning the matrices $U^*C = [\, C_1 \; C_2 \,]$ and $BV = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ corresponding to this splitting, one has

$$G = U \begin{bmatrix} C_1(z\mathbb{I} - A)^{-1}B_1 + \Sigma & C_1(z\mathbb{I} - A)^{-1}B_2 \\ C_2(z\mathbb{I} - A)^{-1}B_1 & C_2(z\mathbb{I} - A)^{-1}B_2 \end{bmatrix} V^*,$$

i.e., by using the weights

$$W_1 = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix} U^*, \quad \text{and} \quad W_2 = V \begin{bmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{bmatrix},$$

one has to deal with the problem of shaping the square invertible system

$$G_{11} = C_1(z\mathbb{I} - A)^{-1}B_1 + \Sigma.$$

It follows that the plant will be shaped with the weights

$$K_F = \begin{bmatrix} \ell G_{11,o}^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*,$$

and W_2 the achieved loop shape being

$$L = \begin{bmatrix} \ell G_{11,i} & 0 \\ 0 & 0 \end{bmatrix}.$$

4. A ROBUST CONTROL DESIGN BASED ON THE $\mathcal{H}_\infty/\mu$ METHOD

Consider the closed-loop system in Figure 1, which includes the feedback structure of the required loop shape, $L = W_2 G K_F$, the robust compensator K_R, and the components associated with the uncertainty models and performance objectives. Here, u is the control input, y is the measured output, w is the disturbance, n is the measurement noise, and z represents the performance outputs. The uncertainties comprise two parts: the parametric uncertainties are represented by the Δ_r block, whose input and output are u_δ, and y_δ, and the unmodelled dynamics is represented by Δ_m block, whose input and output are e, and d. The weighting function W_r reflects the model uncertainties, and Δ_m is assumed to be stable with the norm condition, $\|\Delta_m\|_\infty < 1$. The W_p represents the weighting functions for the performance signals. The weighting function W_n and W_w represent the impact of the sensor noise n and disturbance w, respectively. Note, that the weighting functions W_2, K_F appear in the scheme as known components, therefore the new control input will be $\tilde{u}$, and the new output will be $\tilde{y}$. In the augmented plant structure the realized loop transfer function $W_2 G K_F$ will take place of the nominal plant model G.

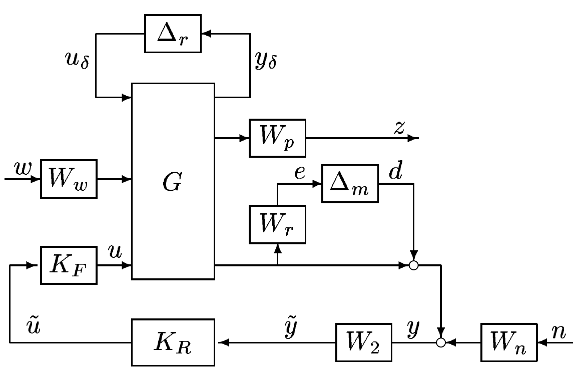

Fig. 1. Closed-loop interconnection structure

The proposed design problem can be formulated as a standard $P - K - \Delta$ structure as illustrated in Figure 2. By applying the weighting functions, the augmented plant P is as follows:

$$P = \left[\begin{array}{cc|cc|c} 0 & W_R G_{eu_\delta} & W_R G_{ew} W_w & 0 & W_R G_{eu} \\ 0 & G_{y_\delta u_\delta} & G_{y_\delta w} W_w & 0 & G_{y_\delta u} \\ \hline 0 & W_p G_{zu_\delta} & W_p G_{z_w} W_w & 0 & W_p G_{zu} \\ \hline I & G_{yu_\delta} & G_{yw} W_w & W_n & G_{yu} \end{array}\right]$$

The augmented plant $\tilde{P}$ for the shaped plant is given by:

$$\tilde{P} = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & W_2 \end{bmatrix} P \begin{bmatrix} \mathbb{I} & 0 \\ 0 & K_F \end{bmatrix}.$$

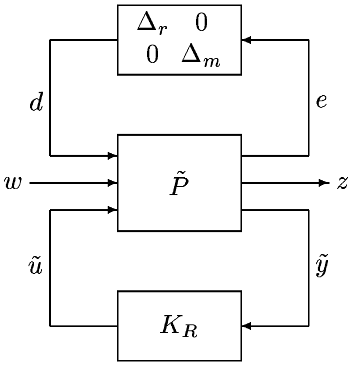

Fig. 2. P-K structure with uncertainties

The closed-loop system is formulated by the lower linear fractional transformation using the feedback law: $\tilde{u} = K_R \tilde{y}$, in the following way:

$$\tilde{M} = F_l(\tilde{P}, K_R) = \tilde{P}_{11} + \tilde{P}_{12} K_R (I - \tilde{P}_{22} K_R)^{-1} \tilde{P}_{21}.$$

The system $\tilde{M}$ is a 2×2 block-structured transfer function matrix: $\begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix}$. Applying this equation, the analysis and the synthesis of the robust control problem can be formalized, (Stein and Doyle, 1991).

The goal of the $\mathcal{H}_\infty/\mu$ synthesis is to minimize over all stabilizing controllers K, the peak value $\mu_\Delta()$ of the closed–loop transfer function $F_l(\tilde{P}, K_R)$. The formula is as follows:

$$\min_{K_R} \sup_{\omega} \mu_\Delta [F_l(\tilde{P}, K_R)(j\omega)],$$

where the admissible set of uncertainties $\tilde{\Delta}$ is defined as follows:

$$\tilde{\Delta} = \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & \Delta_m & 0 \\ 0 & 0 & \Delta_p \end{bmatrix}$$

where Δ_r represents the parametric uncertainties, Δ_m corresponds to the neglected dynamics, and Δ_p is a fictitious uncertainty block, which is used to incorporate the $\mathcal{H}_\infty$ nominal performance objective into the $\mathcal{H}_\infty/\mu$ framework. The optimization is an NP hard problem, and an iterative scheme is used to solve it, (Balas *et al.*, 1991).

After performing the D or D, G steps in the so called complex μ or real μ algorithms, respectively, one has to solve the minimization problem $\min_K ||\mathcal{F}_l(\hat{P}, K)||_\infty$, where $\hat{P}$ is the modified augmented plant given by the formulae of type:

$$\hat{P} = \begin{bmatrix} D_{min} & 0 \\ 0 & \mathbb{I} \end{bmatrix} P \begin{bmatrix} D_{min}^{-1} & 0 \\ 0 & \mathbb{I} \end{bmatrix}$$

or

$$\hat{P} = \begin{bmatrix} D_{min} & 0 \\ 0 & \mathbb{I} \end{bmatrix} P \begin{bmatrix} D_{min}^{-1} G_M & 0 \\ 0 & \mathbb{I} \end{bmatrix} - \begin{bmatrix} G_N & 0 \\ 0 & 0 \end{bmatrix},$$

respectively, see (Zhou and Doyle, 1998). This is the so called K step of the iteration.

One can see that loop shaping does not modify the part of the augmented plant affected by the D or D, G step and it influences the K step only. Therefore by starting the iteration from an already shaped plant one may "precondition" the convergence of the iterative scheme. This is the fact that motivates our approach in performing a loop shaping step before setting the $\mathcal{H}_\infty/\mu$ problem. Since the chosen weight is invertible by construction we do not alter the possibility of finding a solution, if there is any.

5. DEMONSTRATION EXAMPLE: CONTROL DESIGN FOR ACTIVE SUSPENSION SYSTEMS

In the demonstration example the motivation of this method is highlighted through an active suspension design problem. The main objectives are to provide good road handling and to improve passenger comfort while decreasing the harmful vibrations caused by road irregularities. The difficulties are that the performance requirements are usually in conflict and that the model to be used in the control design contains uncertainty components. Several methods have been proposed for this problem, e.g. (Hrovat, 1990), (Yamashita *et al.*, 1994), (Gáspár *et al.*, 2001). In these methods the model is augmented with appropriate weighting functions for performances and uncertainties, and then the loop shape designed is tested in a verification step.

The quarter-car vehicle model, which is shown in Figure 3, is used for active suspension designs. Let the sprung mass and the unsprung mass be denoted by m_s, m_u, the suspension stiffness and the tire stiffness are denoted by k_s, k_t, and suspension damping is denoted by b_s. The nominal parameters are the following:
$m_s = 365kg, m_u = 40kg, k_s = 18000N/m, k_t = 175500N/m, b_s = 950N/m/s$.

The quarter-car model is a two degrees-of-freedom system: the vertical displacement of the sprung mass and the vertical displacement of the unsprung mass, x_1 and x_2. The disturbance, w, is caused by road irregularities. The input signal, f is generated by the actuator, which is modelled as $G_a(s) = \frac{1}{1/75s+1}$. The measured output is the acceleration of the sprung mass. The performance functions are the acceleration of the sprung mass, the suspension deflection, the wheel travel, and the control input.

In the example the loop-shape is defined by the designer in such a way that the performances are better than the performances from other traditional methods and that the robust stability requirement is satisfied. The result of a traditional LQG method is illustrated by the dashed line and the required loop shape by the solid line as illustrated in Figure 4. The first peak corresponds to the eigen-frequency of the sprung mass. Since the tire-hop frequency is an invariant point, the second peak cannot be reduced by a feedback.

In order to simplify the illustration of the design procedures, it is assumed that the required loop shape is approximated. It means, that in our method a weighting function is identified, and in McFarlane and Glover procedure appropriate pre- and post-compensators are selected. In this section the robust control design step is emphasized.

In order to perform the $\mathcal{H}_\infty/\mu$ synthesis, weighting functions are selected both for performances and uncertainties. The purpose of weighting functions is to keep the vertical acceleration and the suspension deflection small in the desired frequency range. These are illustrated in the left hand side of the Figure 5. The control force is limited by the weighting function $4 \cdot 10^{-3}$. A typical weighting function of the unmodelled dynamics is

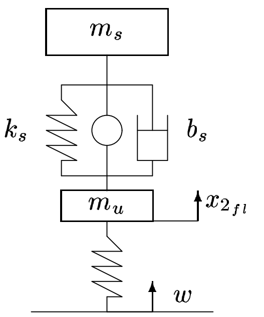

Fig. 3. Quarter-car model

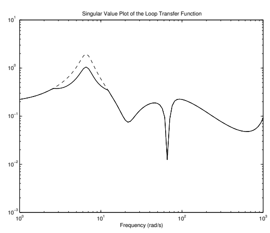

Fig. 4. Required loop-shape (solid) and an LQG loop-shape (dashed)

is illustrated in the right hand side of the Figure 5.

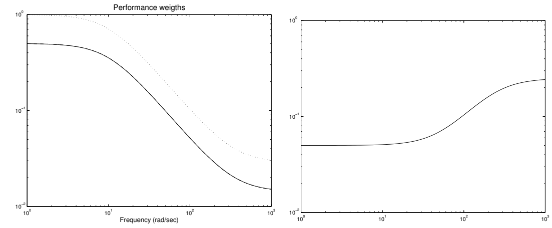

Fig. 5. Weighting functions for performances and robustness

In the $\mathcal{H}_\infty/\mu$ synthesis, the control design is performed by using the $D-K$ iteration method. After the Step 2, the peak μ-value is 0.915 and the compensator order is selected 16. It results in a compensator in which all the nominal performance, the robust stability, and the robust performance are achieved. At the same time, an optimal controller is designed by using the McFarlane and Glover method. The value of the stability margin as an indication robustness to unstructured perturbations is 0.70, which indicates a sufficient robustness stability margin.

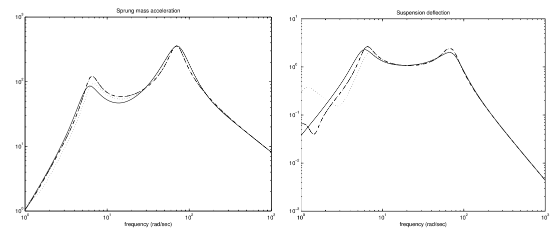

Fig. 6. Frequency responses of the controlled system

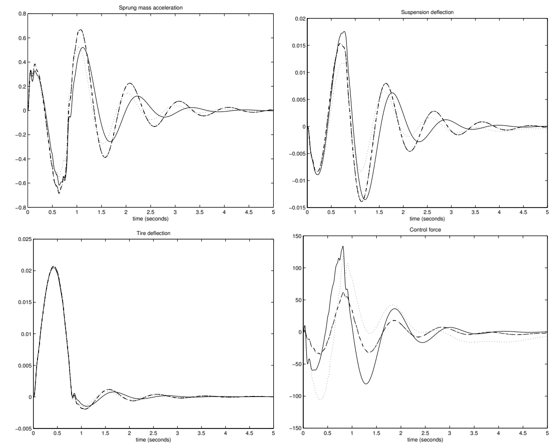

Fig. 7. Time responses of the controlled system

The frequency responses of the controlled system, i.e. the vertical acceleration, the suspension deflection are illustrated in Figure 6. The solid line corresponds to the $\mathcal{H}_\infty/\mu$ synthesis, the dashed line to the McFarlane method, and the dotted line to the result of the required system. The first amplitude peak, which corresponds to the eigenfrequency of the sprung mass is the smallest in the $\mathcal{H}_\infty/\mu$ design.

The designed controllers are verified in a real situation, in which the input signal is simulated as a bump with 0.02m maximal value. The time responses, i.e. the sprung mass acceleration, the suspension deflection, the tire deflection, and the input force, respectively, are illustrated in Figure 7. The solid line corresponds to the $\mathcal{H}_\infty/\mu$ synthesis, the dashed line to the McFarlane method. The effect of the disturbance on the sprung mass acceleration is the best in the case of $\mathcal{H}_\infty/\mu$ method, while the suspension deflection is acceptable in all cases.

6. CONCLUSIONS

In the paper a loop shaping based control design method has been proposed in order to achieve a compensator that meets both the robust performance and the robust stability requirements. The proposed method combines loop shaping procedure and the $\mathcal{H}_\infty/\mu$ synthesis in order to fulfill the robust performance requirement.

The weighting function for achieving a required loop shape is designed by using an identification procedure for the outer part of the model. In the augmented plant structure the realized loop transfer function takes place of the nominal plant model. Since the shaped plant realizes a desired performance criteria, by setting the $\mathcal{H}_\infty/\mu$ synthesis problem in this context one can obtain a "preconditioning" in the solution algorithm.

The proposed method is illustrated by a case study from the vehicle industry.

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