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A DECOUPLING GRINDING CIRCUIT CONTROL SYSTEM

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Abstract: The problem of simultaneous decoupling and pole placement without cancelling invariant zeros is important, especially in the case of unstable invariant zeros. An experimental model developed of a primary grinding circuit contains such zeros. The simultaneous decoupling and pole–placement problem without cancelling the unstable invariant zeros of the primary grinding circuit is approached by searching for solutions of the nonlinear system of equations composed of the characteristic equation and the decoupling conditions using ideas of an αBB global optimization approach. The simultaneous steady–state decoupling and pole placement problem is then solved for the primary grinding circuit without cancelling invariant zeros using an eigenvector based approach as well as the steady–state decoupling condition.

Keywords: Grinding circuit control, decoupling, global optimization

1. INTRODUCTION

The decoupling problem has been of interest for many years, as one of the important methods in the control of multiple-input multiple-output (MIMO) systems. The first methods essentially resulted in integrator decoupling, i.e., the resulting diagonal elements were integrators (Gilbert, 1969), (Furuta and Kamiyama, 1977). Those methods were later adapted to include pole–placement decoupling, wherein the diagonal elements contained poles not necessarily at the origin, thus allowing a wider range of dynamical responses to be designed for, see, e.g., (Furuta *et al.*, 1988).

Essentially, a feedforward gain matrix and state feedback are used in a state space representation to achieve the desired result in the classical decoupling methods. In general, state feedback can be used to place poles as well as to affect the element zeros of transfer function matrices in MIMO systems. The invariant zeros (Schrader and Sain, 1989), (Emami-Naeini and Dooren, 1982) of MIMO systems are, however, not affected by state feedback or feedforward gain. In the classical decoupling methods, the invariant zeros are in general cancelled by a number of the new system poles, thus, effectively leading to an overall reduced–order system.

In many cases, such a decoupled overall reduced–order system results in a first–order differential equation relating the decoupled inputs to the individual outputs, thus, somewhat limiting the dynamical response achievable by the pole placement. Often, this does not pose a major problem, as the first–order response can be shaped by an outer–loop controller, e.g., a PID controller, once the system is decoupled. The fact that the classical decoupling methods cancel all invariant zeros is a much more serious drawback, as in the case of unstable invariant zeros, those are cancelled by unstable controller poles, thus rendering such a controller useless in practice.

It is therefore of interest to explore the design of a decoupling pole–placement controller, that leaves invariant zeros intact and allows full pole placement. This is accomplished in the Faddev algorithm in (Gestsson

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and Hauksdóttir, 1995), by imposing the decoupling as well as pole–placement conditions iteratively, easily applicable to low–order systems. The simultaneous decoupling and pole placement problem is approached in (Hauksdóttir and Ierapetritou, 2001), by finding all solutions of the nonlinear system of equations composed of the characteristic equation and the decoupling conditions using the ideas of the αBB global optimization approach proposed by Maranas and Floudas (Maranas and Floudas, 1995).

In this paper, the simultaneous decoupling and pole–placement conditions are presented in Section 2. An experimental model developed of a primary grinding circuit (Jämsä *et al.*, 1983) containing unstable invariant zeros is described in Section 3. The simultaneous decoupling and pole–placement problem without cancelling the unstable invariant zeros of the primary grinding circuit is approached in Section 4 by searching for all solutions of the nonlinear system of equations composed of the characteristic equation and the decoupling conditions using the ideas of the αBB global optimization approach. Expanding the results of (Lohmann, 2000) for the case $D \neq 0$, the simultaneous steady–state decoupling and pole–placement problem is then solved in Section 5 for the primary grinding circuit by finding the state–feedback using an eigenvector based approach without cancelling invariant zeros, and using the steady–state decoupling condition for finding the static feedforward matrix. Conclusions and future studies are discussed in Section 6.

2. SIMULTANEOUS DECOUPLING AND POLE–PLACEMENT CONDITIONS

Consider a square system in a minimal form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad (1)$$

where $A \in {}^{n\times n}$, $B \in {}^{n\times m}$, $C \in {}^{m\times n}$ and $D \in {}^{m\times m}$. Feedforward and feedback of the form

$$u = Fx + Ev \qquad (2)$$

will be applied to decouple the system and to place it's poles. The resulting system is then,

$$\begin{aligned} \dot{x} &= [A + BF]x + BEv \\ &= A_{cl}x + BEv \\ y &= [C + DF]x + DEv. \end{aligned} \qquad (3)$$

The closed–loop transfer function matrix (TFM) is given by

$$\begin{aligned} G_{cl}(s) &= (C+DF)(sI - A_{cl})^{-1}BE + DE \\ &= \frac{1}{\alpha(s)}(C+DF)Adj(sI - A_{cl})BE + DE \end{aligned} \qquad (4)$$

where

$$\begin{aligned} \alpha(s) &= det(sI - A_{cl}) \\ &= s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1}s + \alpha_n \\ &= (s+\lambda_1)(s+\lambda_2)\cdots(s+\lambda_n) \end{aligned} \qquad (5)$$

denotes the system's desired characteristic equation.

The invariant zeros of a square system are given by

$$\det(CAdj(sI - A)B + Da(s)) = 0, \qquad (6)$$

where $a(s)$ is the original systems characteristic equation. Such invariant zeros are neither affected by feedback or feedforward gains, i.e.,

$$\begin{aligned} &\det(CAdj(sI-A)B + Da(s)) = \begin{vmatrix} sI - A & B \\ -C & D \end{vmatrix} \\ &= \begin{vmatrix} sI - A - BF & B \\ -C - DF & D \end{vmatrix} \\ &= \det\big((C+DF)Adj(sI - A_{cl})B + D\alpha(s)\big) \\ &= 0. \end{aligned} \qquad (7)$$

Further,

$$\begin{aligned} &\det\big(((C+DF)Adj(sI - A_{cl})B + D\alpha(s))E\big) \\ &= \det\big((C+DF)Adj(sI - A_{cl})B + D\alpha(s)\big)\det E \\ &= 0. \end{aligned} \qquad (8)$$

Expanding the adjoint in the numerator part of the closed-loop TFM results in

$$\begin{aligned} &(C+DF)Adj(sI - A_{cl})BE + DE\alpha(s) \\ &= s^n DE + s^{n-1}((C+DF)BE + \alpha_1 DE) \\ &+ s^{n-2}((C+DF)A_{cl}BE + \alpha_1(C+DF)BE + \alpha_2 DE) \\ &+ \ldots + (C+DF)A_{cl}^{n-1}BE + \alpha_1(C+DF)A_{cl}^{n-2}BE \\ &+ \ldots + \alpha_{n-1}(C+DF)BE + \alpha_n DE. \end{aligned} \qquad (9)$$

Thus, the conditions for decoupling are

$$DE = diag\{\gamma_1^0, \gamma_2^0, \ldots, \gamma_m^0\} \qquad (10)$$

and

$$(C+DF)A_{cl}^{(k-1)}BE = diag\left\{\gamma_1^k, \gamma_2^k, \ldots, \gamma_m^k\right\} \qquad (11)$$

for $k = 1, \ldots, n$. It was furthermore shown, that the solution of the simultaneous decoupling pole–placement problem for the case $D = 0$ in (Hauksdóttir and Ierapetritou, 2001) leads to the standard solution wherein all invariant zeros are cancelled if and only if

$$\gamma_i^k = (-\lambda_i)^{(k-1)} \qquad (12)$$

for $k = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m$, where λ_i are m of the closed–loop eigenvalues.

The simultaneous decoupling and pole–placement problem can now be stated as follows: Simultaneously solve

$$\begin{aligned} \alpha(s) &= det(sI - A_{cl}) \\ &= s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1}s + \alpha_n, \end{aligned} \qquad (13)$$

$$D_{i\cdot}E_{\cdot j} = 0 \tag{14}$$

and

$$(C + DF)_{i\cdot} A_{cl}^{(k-1)} B E_{\cdot j} = 0 \tag{15}$$

for $k = 1, \ldots, n$ and for $i \neq j$.

As invariant zeros are not affected by feedforward or feedback, they will appear in the numerator of the diagonal elements of a decoupled system. Further, it is obvious that if some invariant zeros are to be retained, they must not be a factor of $\alpha(s)$, as the factors of $\alpha(s)$ appear in the denominator of the different diagonal elements of a decoupled system.

3. MODELING OF THE PRIMARY GRINDING CIRCUIT

The primary grinding stage in the Vuonos concentrator comprises a rod mill and a pebble mill with a hydrocyclone classifier (Jämsä *et al.*, 1983). The instrumentation of the grinding circuit is shown in Fig. 1. The crushed ore feed to the rod mill is measured by a belt-weigher and controlled by a belt-feeder. The pulp density in the rod mill is stabilized by proportioning the waterfeed at the mill inlet to the crushed ore feed. The slurry level in the pump sump is measured by a pressure transducer and controlled by changing the speed of the direct–current drive pump. The electric powerdraw of the pebble mill is regulated to a certain setpoint by controlling the pebble feed to the mill. The densities of the cyclone feed and overflow are measured by radioisotope density gauges and the flow rates by electromagnetic flow–meters. The operation of the cyclone is also checked by measuring the pressure of the cyclone feed and the massflow of the cyclone underflow. The particle size of the product of the grinding circuit is measured by an ultrasonic particle size meter located at the cyclone overflow.

A dynamic process model was developed with the aid of step responses. According to the present control strategy the process inputs are the crushed ore feed ($U_1(s)$) and the water feed to the cyclone pump sump ($U_2(s)$) and the process outputs are the cyclone overflow particle size ($Y_1(s)$) and the cyclone feed density ($Y_2(s)$). The transfer function matrix of the process is given by (Jämsä *et al.*, 1983):

$$Y(s) = G(s)U(s) \tag{16}$$

or

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{-2e^{-8s}}{6s+1} & \frac{0.004(24.5s+1)e^{-1.87s}}{(3.1s+1)^2} \\ \frac{10e^{-8s}}{5s+1} & \frac{-0.15e^{-0.93s}}{s+1} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}.$$

Here all time constants are given in minutes.

As seen from the model strong interactions exist in the process, in particular U_1 (the crushed ore feed)

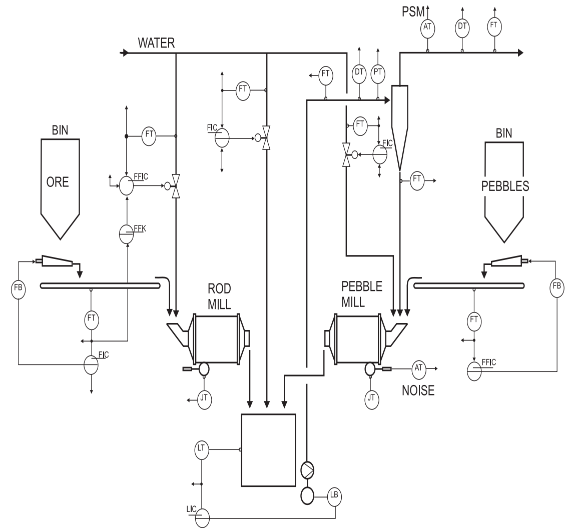

Fig. 1. A flowsheet of the Vuonos grinding circuit.

has a very strong effect on both outputs, i.e., both the cyclone overflow particle size and the cyclone feed density. On the other hand U_2 (the water feed to the cyclone pump sump) has a minimal effect on both outputs. Therefore the transfer functions from U_2 are simplified such that they both have the average delay of 1.4*min*. Further, the transfer function from U_2 to Y_1 is represented as a first–order transfer function, i.e.,

$$G(s) = \begin{bmatrix} \frac{-2e^{-8s}}{6s+1} & \frac{0.004e^{-1.4s}}{3.1s+1} \\ \frac{10e^{-8s}}{5s+1} & \frac{-0.15e^{-1.4s}}{s+1} \end{bmatrix}. \tag{17}$$

Approximating the delay of α seconds with a first–order Taylor series

$$e^{(-\alpha s)} = 1 - \alpha s \tag{18}$$

results in

$$G(s) = \begin{bmatrix} 2.6667\frac{s-0.125}{s+0.1667} & -0.0018\frac{s-0.7143}{s+0.3226} \\ -16\frac{s-0.125}{s+0.2} & 0.21\frac{s-0.7143}{s+1} \end{bmatrix}. \tag{19}$$

Arranging these in a minimal state space form of Eq. (1) gives: where

$$A = \begin{bmatrix} -0.1667 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0 \\ 0 & 0 & -0.3226 & 0 \\ 0 & 0 & 0 & -1.0 \end{bmatrix}, \tag{20}$$

$$B = \begin{bmatrix} 0.8819 & 0 \\ 2.2804 & 0 \\ 0 & 0.0433 \\ 0 & 0.6 \end{bmatrix}, \tag{21}$$

$$C = \begin{bmatrix} -0.8819 & 0 & 0.0433 & 0 \\ 0 & 2.2804 & 0 & -0.6 \end{bmatrix} \tag{22}$$

and

$$D = \begin{bmatrix} 2.6667 & -0.0018 \\ -16.0 & 0.21 \end{bmatrix}. \qquad (23)$$

This system has invariant zeros at 0.7143, −0.2222, −0.2653 and 0.1250 i.e., two unstable ones both due to the input time delays. Assuming the new eigenvalues are selected as −0.7143, −0.2222, −0.2653 and −0.1250, i.e, as the stable invariant zeros which will then be cancelled in the decoupled system and the rest as the negative of the unstable invariant zeros, gives the new characteristic equation

$$\begin{aligned} &\alpha(s) = det(sI - A_{cl}) = det(sI - A - BF) \\ &= s^4 + 1.3268s^3 + 0.5574s + 0.0930s + 0.0053 \quad (24) \\ &= 0. \end{aligned}$$

4. SIMULTANEOUS DECOUPLING AND POLE PLACEMENT FOR GRINDING CIRCUIT CONTROL

The primary aim of the controller design is to make the two control loops as independent as possible. This means that changes in the setpoint of the particle size ($Y_1(s)$) do not cause strong effects on the density of the cyclone feed ($Y_2(s)$) and vice versa. Grinding circuit control has been studied e.g. in (Jämsä *et al.*, 1983) and (Niemi *et al.*, 1997).

Assuming an observer has been build for the necessary state estimates and applying state feedback and feedforward for full decoupling, the off-diagonal elements of the Markow parameters must be zero, i.e.,

$$\begin{aligned} D_{1.}E_{.2} &= 0 \\ D_{2.}E_{.1} &= 0 \\ (C+DF)_{1.}(A+BF)^k BE_{.2} &= 0 \\ (C+DF)_{2.}(A+BF)^k BE_{.1} &= 0 \\ \text{for } k = 0,\ldots,3 & \end{aligned} \qquad (25)$$

Using $E = (D)^{-1}$ and deriving the characteristic equation coefficients as functions of the F-matrix, results in a total of three linear and nine nonlinear equations to be solved simultaneously for the elements of F.

No solutions to the above system of equations were found using an algorithm based on the ideas of an αBB global optimization approach. One solution, however, was obtained, resulting in close to lower diagonal decoupling and appropriate pole locations, with F given by

$$F = \begin{bmatrix} -0.0411 & 0.3663 & 0.0030 & -0.0451 \\ -2.8597 & 15.3960 & 0.2218 & -0.5768 \end{bmatrix}, \qquad (26)$$

and the corresponding step response shown in Fig. 2.

Another solution was obtained, resulting in close to steady–state decoupling and appropriate pole locations, with F given by

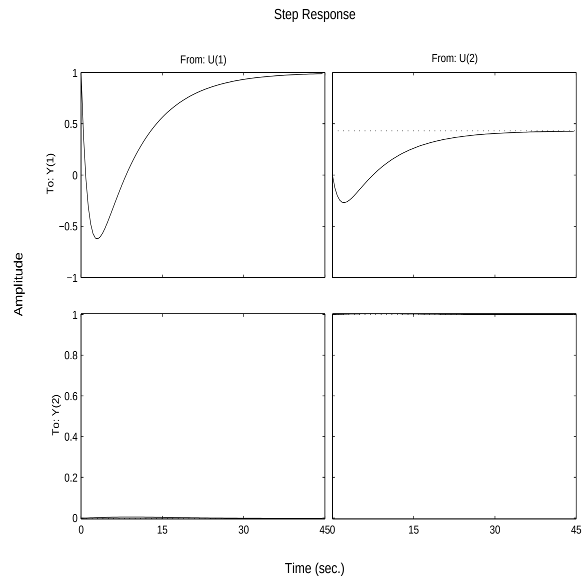

Fig. 2. A step response of the resulting lower diagonal decoupled system with simplified input delays.

$$F = \begin{bmatrix} 0.0909 & -0.0223 & -0.0072 & 0.0035 \\ -12.7465 & 5.0123 & 0.6138 & 0.5105 \end{bmatrix}, \qquad (27)$$

resulting in the corresponding step response shown in Fig. 3.

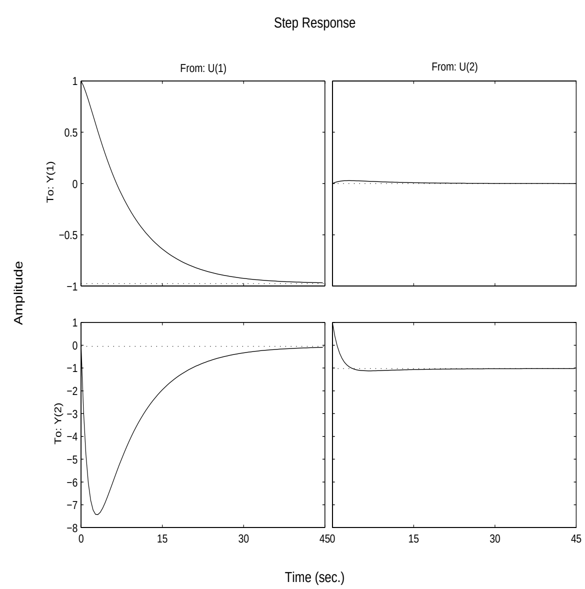

Fig. 3. A step response of the resulting steady–state decoupled system with simplified input delays.

5. SIMULTANEOUS STEADY–STATE DECOUPLING AND POLE PLACEMENT FOR GRINDING CIRCUIT CONTROL

In cases where full decoupling and pole placement is not achievable without the cancellation of invariant zeros, steady–state decoupling and pole placement without cancellation of invariant zeros may be possible. In fact, both solutions obtained by the global optimization approach for the grinding circuit control, were close to such steady–state decoupling.

Consider again the closed–loop system of Eq. (3). In the pole–placement problem, the closed–loop eigenvectors r_i are given by

$$(A - \lambda_i I + BF) r_i = 0 \tag{28}$$

or with slight abuse of notation

$$(A - \lambda I + BF) R = 0 \tag{29}$$

where R contains the closed–loop eigenvectors r_i as columns. Then, in order to cancel invariant zeros, the obtained eigenvectors must be orthogonal to the closed–loop output matrix, i.e.,

$$(C + DF) R = 0. \tag{30}$$

Likewise, in order to avoid cancellation of invariant zeros, the obtained eigenvectors must not be orthogonal to the closed–loop output matrix, i.e.,

$$(C + DF) R = [\, I \;\; 0 \,]. \tag{31}$$

Then, one may express both of the above combined as

$$\begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix} \begin{bmatrix} R \\ FR \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} \tag{32}$$

or

$$\begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix} \begin{bmatrix} \Psi_u \\ \Psi_l \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}. \tag{33}$$

Then, by solving the combined eigenvectorproblem, subsequently F can be solved for by

$$F = \Psi_l R^{-1} = \Psi_l \Psi_u^{-1}. \tag{34}$$

Finally, E is solved for such that the steady state TFM is the identity matrix, i.e.,

$$E = ((C + DF)(-A - BF)^{-1} B + D)^{-1}, \tag{35}$$

thus, assuring steady–state decoupling. A result for the case $D = 0$ was derived along similar lines in (Lohmann, 2000).

Returning back to the grinding problem, but this time without simplifying the input delays, i.e., applying the Taylor series expansion on the delays of the model

$$Y(s) = G(s) U(s) \tag{36}$$

or

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{-2e^{-8s}}{6s+1} & \frac{0.004e^{-1.87s}}{3.1s+1} \\ \frac{10e^{-8s}}{5s+1} & \frac{-0.15e^{-0.93s}}{s+1} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}. \tag{37}$$

Several solutions were found to the steady–state decoupling and pole placement without cancellation of invariant zeros problem, two of them which are reported here. The first one essentially corresponds to complete lower diagonal decoupling (a special case of steady–state decoupling) maintaining all invariant zeros and is given by

$$F = \begin{bmatrix} -0.0220 & 0.2525 & 0.0018 & -0.0369 \\ -2.5241 & 12.6163 & 0.2077 & -0.3780 \end{bmatrix} \tag{38}$$

and

$$E = \begin{bmatrix} 0.2632 & -0.1238 \\ 30.1934 & -7.0287 \end{bmatrix}. \tag{39}$$

The corresponding step response is given in Fig. 4.

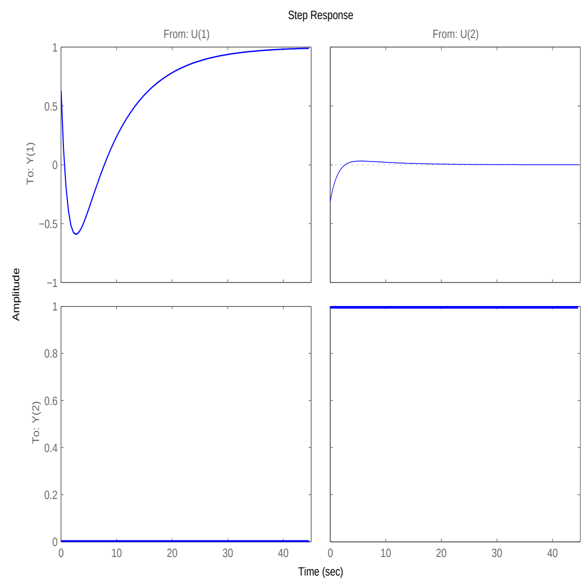

Fig. 4. A step response of the resulting lower diagonal decoupled system.

The second solution is complete steady–state decoupling, again maintaining all invariant zeros and is given by

$$F = \begin{bmatrix} 0.0357 & 0.0242 & -0.0093 & 0.0006 \\ -2.546 & 8.63 & -0.360 & 0.543 \end{bmatrix} \tag{40}$$

and

$$E = \begin{bmatrix} -0.347 & -0.0158 \\ 19.90 & -4.530 \end{bmatrix}. \tag{41}$$

The corresponding step response is given in Fig. 5.

6. CONCLUSIONS AND FUTURE STUDIES

It is known that the general problem of decoupling and pole placement without cancelling the invariants zeros can be solved for some examples, while in other cases no solution exists. In this paper, it was attempted to solve this problem for an experimental model developed of a primary grinding circuit containing two unstable invariant zeros due to input time delays, by

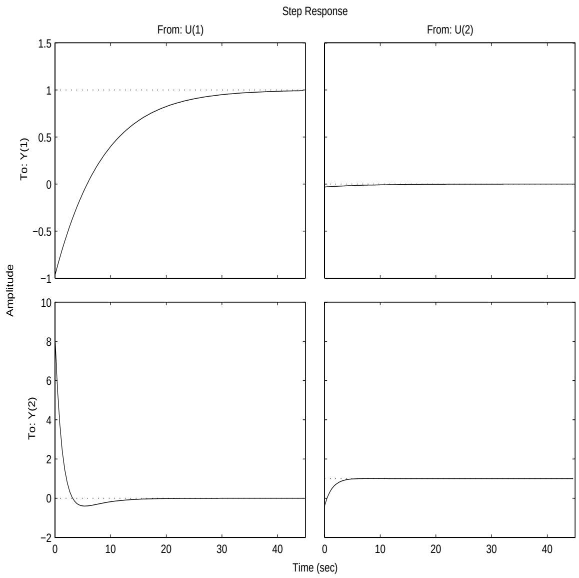

Fig. 5. A step response of the resulting steady–state decoupled system.

placing the system poles such as to cancel stable invariant zeros, but leaving unstable invariant zeros in tact. This was done by searching for all solutions of the nonlinear system of equations composed of the characteristic equation and the decoupling conditions based on the ideas of the global optimization algorithm proposed by Maranas and Floudas (Maranas and Floudas, 1995). No solutions were found to the complete problem, however, solutions were found for the close to lower diagonal decoupling problem as well as the steady–state decoupling problem.

Expanding the results of (Lohmann, 2000) for the case $D \neq 0$, the simultaneous steady–state decoupling and pole–placement problem was solved for the primary grinding circuit by finding the state–feedback using an eigenvector based approach without cancelling invariant zeros, and using the steady–state decoupling condition for finding the static feedforward matrix.

It is of interest to consider other related MIMO problems, such as more general eigenstructure placement problems. It is of particular interest to explore the solution of such control problems using the global optimization approach developed in (Hauksdóttir and Ierapetritou, 2001).

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