

## FEEDING-SEQUENCE SELECTION USING LINEAR DISCRIMINANTS IN A PARALLEL-MACHINE MANUFACTURING CELL<sup>1</sup>

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Abstract: The paper deals with the problem of finding the optimum feeding sequence in manufacturing cells with machines fed by robots. A discriminant function to select the best feeding sequence between two preselected ones was determined for the case of a real cell with four machines working on two pallets each one, fed by one robot and with random assistance requirements. The cell was modelled and simulation results for different working times were used to obtain a linear discriminant between a fixed and a variable sequence. *Copyright © 2002 IFAC*

Keywords: Manufacturing, Robotics, Scheduling algorithms, Sequences, Simulation

### 1. INTRODUCTION

#### 1.1 Definition of the problem

The paper deals with the problem of finding the optimum feeding sequence in manufacturing cells with machines fed by robots. A solution for a real cell that produces parts for the car manufacturing industry at the plant of Metaldyne International Spain SL (formerly R.J. Simpson International SL) located in Gavà, close to Barcelona (Spain), was stated. The cell has four machines,  $m_i$ , with  $i = 1, \dots, 4$ , working on two pallets each one (called pallets A and B) during a time  $t_{wa}$  and  $t_{wb}$  respectively; the machines are fed by one robot and they have random assistance requirements (the cell is described in detail below).

Each machine in the cell has some unproductive time when it is waiting for the robot to load it. This unproductive time depends on the different parameters of the task (i.e. the time needed by each activity in the cell) and on the sequence that the robot follows

to feed the machines. Differences in the unproductive time of the machines can arrive up to two orders of magnitude (in a particular given case, with real conditions, the unproductive times of the machines for two different feeding strategies were 0.045% and 5.72% respectively).

The working times  $t_{wa}$  and  $t_{wb}$  of the machines on each pallet depend on the part to be processed (also, variations can appear for the same part if different machining tools are used). Then, it is necessary to determine: "In which sequence should the robot feed the machines in order to optimize the productivity of the cell?" The authors are not aware of a general solution published up to now for the particular features of this cell (typically, robotized manufacturing cells are considered as the problem of feeding two or three machines with parts that have to pass through all of them, see, for instance, the works of Crama (1997), Hall, Kamoun and Sriskandarajah (1997), Stei et al. (1992)). On the other hand, discrete-event simulation provides with the methodology to analyze and compare the system performance with different feeding sequences (Law and Kelton, 1991). Petri Nets were

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<sup>1</sup> This work was partially supported by the CICYT projects DPI2001-2202 and TAP99-0839, and Metaldyne International SL.

also used for the simulation and analysis of manufacturing cells of similar type (see for instance the work of Lee and Dicesare (1994)). Then, the objective of this work was the search of a function that, given the machine working time on each pallet, returns the best robot loading sequence for the cell.

## 1.2 Detailed description of the Cell

Figure 1 shows a layout of the manufacturing cell. The cell is composed of four machines ( $m_i$ , with  $i = 1, \dots, 4$ ), all of them of the same type. Each machine operates alternatively over two pallets, A and B. The robot loads a part into one pallet while the machine is working on the other pallet.

Each part to be manufactured must be first loaded into pallet A of any machine, where a set of operations leave it in a medium-processed state. Then, the part must be removed from that pallet and loaded into pallet B of any machine (it can be the same machine), where another set of operations leave it in the final state (unprocessed and medium-processed parts are called type A and type B parts respectively). It is necessary to remove the parts from the pallets A of the machines and then reload them in the pallets B because the parts are positioned in a different orientation in each pallet so that all the tools of the machines can work on the proper side of the parts. Pallets A and B have different bridles to fix the parts in different positions.

The parts are loaded into the pallets and unloaded from them by a 6 d.o.f. robot that uses a rail to move from one machine to another. Once a part has been loaded in a pallet, a set of bridles must be closed to fix the part before the pallets turn to put the loaded pallet in the working side of the machine. On the other hand, after the turn of the pallets, the robot cannot recover the part in the pallet until the bridles are opened. During regular activity, any load operation implies a previous unload operation of the pallet, therefore the time considered for loading a machine includes the corresponding unload action.

There is an auxiliary storage line where the robot can put the parts unloaded from the machines and recover them when necessary. Completely unprocessed parts are automatically supplied to this storage line, and therefore the robot always has direct access to either unprocessed parts as well as to medium-processed parts. This is not a constraint in the system.

Each machine uses about 25 different tools for the operations on both pallet A and pallet B, and these tools have to be replaced after a number of operations. Since the life of each tool is different, the result is that the machine needs assistance from a human operator after a period of time that randomly varies within a given range. In this situation the machine stops working until a human operator replaces the

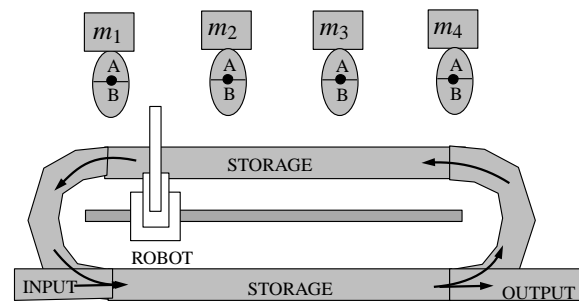


Fig. 1. Layout of the manufacturing cell.

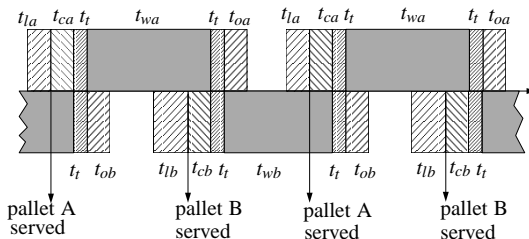


Fig. 2. Example of a machine cycle.

corresponding tools and put it on-line again. During the first cycle after the assistance for tool replacement, the operator has to check the machine performance with the new tools, and this produces a checking cycle that lasts more than the regular one.

System parameters:

$t_{wa}, t_{wb}$ : machine working times on pallets A and B.

$t_{r1}, t_{r2}, t_{r3}$ : times for the displacements of the robot to a next, second and third machine, respectively, from the current one.

$t_{la}, t_{lb}$ : loading times for pallets A and B (include unloading the parts already processed in the pallet).

$t_{ca}, t_{cb}$ : times for closing the bridles in pallets A and B.

$t_{oa}, t_{ob}$ : times for opening the bridles in pallets A and B.

$t_t$ : pallet turning time (changing positions between pallets A and B).

$t_s$ : machine working time before assistance request.

$t_{a1}$ : assistance time by the operator.

$t_{a2}$ : checking time by the operator in the machine cycle after the assistance.

The cell fixed times are:  $t_{r1} = 6.5''$ ,  $t_{r2} = 10,25''$ ,  $t_{r3} = 14''$ ,  $t_{la} = 47''$ ,  $t_{lb} = 55''$ ,  $t_{ca} = 28''$ ,  $t_{cb} = 30''$ ,  $t_{oa} = 12''$ ,  $t_{ob} = 12''$ ,  $t_t = 14''$ ,  $30' \leq t_s \leq 150'$ ,  $t_{a1} = 2'$ , and  $t_{a2} = 1'$ .

The variable times are:  $200'' < t_{wa} < 260''$  and  $140'' < t_{wb} < 220''$ . These ranges indicate the typical values and they were used to obtain the results presented in this work.

Figure 2 shows an example of a working cycle of one machine.

## 2. DEVELOPED SOLUTION

Since there is no general theoretical solution for the mentioned type of cell under the real condition of random assistance requirements by the machines, the problem was addressed by, first, using a simulator of the cell to analyze the system productivity for different sequences with different working times and, second, using pattern recognition techniques to identify the domain in which a given feeding sequence is better than another. Several different strategies were initially proposed, both fixed and variable, and from them, the following two were finally considered:

- 1) A fixed machine sequence (FM). The robot feeds the machines in a fixed sequence ( $m_1 - m_2 - m_3 - m_4$ ), feeding either pallet A or B depending on the needs at the loading time. A machine is skipped if it is being assisted by the operator.
- 2) A variable sequence First-In-First-Out (FIFO). Each time a machine has finished the work on a part, the pallet has turned, and the bridles have been opened, the machine is added to a queue. The robot feeds always the first machine in that queue.

Both strategies, FM and FIFO, require only binary signals available at the moment of the decision. If this is not a constraint, sequences based on the optimization of a cost function can be used, like, for instance, the following one. Once the robot has loaded a machine and have to choose the next one, the following cost function is computed for each machine: the time that the robot needs to arrive to the machine plus the time that the machine needs to be ready for a new load. The machine with the minimum cost is selected. The finish-time for each machine is obtained as the sum of the present time at the end of the loading operation plus the time for closing the bridles, turn the pallets, process the part, turn the pallets again and open the bridles. With this strategy, if two machines need the robot serving at the same time the robot will go to the closest one, and between two machines at the same distance the robot will go to the one that will first need to be loaded. Although very good results were obtained in simulations with this strategy, it was not considered for application in the real cell because the computation of the cost function requires a clock and numerical operations, which means some changes in the hardware of the cell while the strategies based only on binary signals were of direct application.

The work was done according to the following steps:

- 1) Implementation of a cell simulator capable of running simulations of the cell performance with different parameters and with different feeding sequences.
- 2) Execution of cell simulations for the two considered sequences with different working times on pallets A and B, and computation of the machine unproductive times due to the waiting time for the robot (indicated as percentage of the total absolute time).

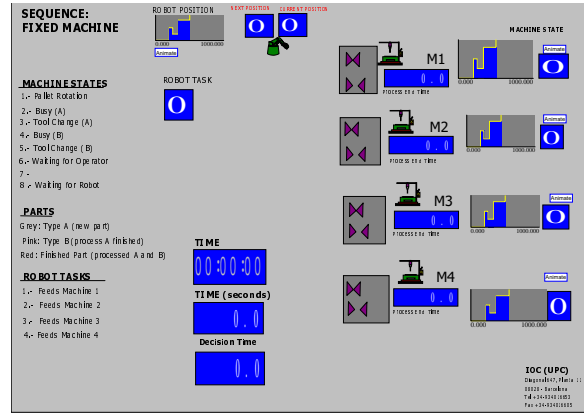


Fig. 3. Appearance of the graphic interface of the simulator.

- 3) Computation of a linear discriminant function using the set of labeled samples obtained from the previous step. The function indicates the best strategy for some given working times  $t_{wa}$  and  $t_{wb}$ .

Once the linear discriminant function is determined, the work to be done by the cell operators for each new set of working times on pallets A and B is just the evaluation of a simple linear function (the discriminant). The sign of the numerical result indicates the best feeding strategy. Details about each of the three steps are given in the following Subsections.

### 2.1 Cell simulator

A cell simulator was implemented using Arena 3.51 from Rockwell Software Inc. The simulator has been designed taking full advantage of the flexibility of this simulation software that allows to customize both the input and the output data. Figure 3 shows the graphic interface of the cell simulator for the fixed machine sequence, where the evolution of the status of each machine and the processed parts as well as the robot position can be checked during the simulation.

An input text file with a list of pairs of working times (i.e. pairs  $[t_{wa}, t_{wb}]$ ) are provided to the simulator. On the run command, the simulator reads the input file and makes a number of replications for each pair  $[t_{wa}, t_{wb}]$  in order to obtain the required precision in the statistical results. In this way, the cell simulator is able to automatically analyze the cell performance for a set of working times of a given family of products to be manufactured.

Arena incorporates Visual Basic for Applications (VBA) that allows to integrate Arena with other programs that support the Microsoft ActiveX<sup>TM</sup> Automation programming interface. Using this utility, the output data of the cell simulator is downloaded to a Microsoft Excel worksheet, where an output analysis is automatically performed. The following results are obtained for each pair  $[t_{wa}, t_{wb}]$  in the input data:

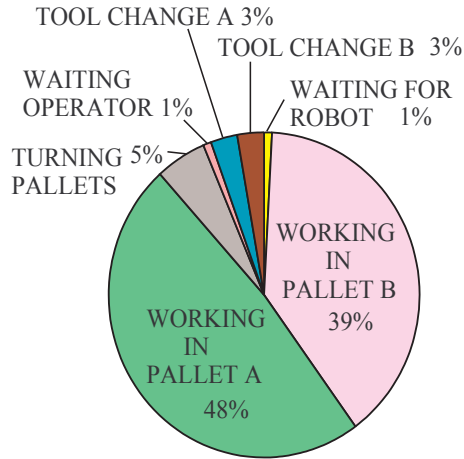


Fig. 4. Example of the percentage of time devoted to each activity (fixed sequence,  $t_{wa}=260''$  and  $t_{wb}=140''$ ).

- Mean and 95% confidence interval of the unproductive times (i.e machines waiting for the robot to load them).
- Statistics of the machine states for each machine.
- Mean and 95% confidence interval of the number of parts produced by the cell.

The total simulation time for a pair of working times  $[t_{wa}, t_{wb}]$  is approximately 45 minutes in a PC Pentium MMX 200.

## 2.2 Simulation results

With the simulator described in the previous Section, a number of simulation were done for the two considered sequences, FM and FIFO, and for several samples of  $t_{wa}$  and  $t_{wb}$  within their usual ranges ( $200'' < t_{wa} < 260''$  and  $140'' < t_{wb} < 220''$ ).

The simulator automatically perform 10 replications for each given pair  $[t_{wa}, t_{wb}]$ , ensuring in this way that the statistical influence of the random assistance to the machines is captured with a 95% confidence interval for the given least significant digit of each mean value. In this work, for each pair of working times on pallets A and B the simulations were done considering the cell working full day during a month. Statistical results for the considered strategies and different working times are shown in Table 1 and 2 respectively, and Figure 4 shows the percentage of time devoted to each activity for a particular case.

The average unproductive times obtained with both strategies were interpolated in order to represent them as two surfaces (Figure 5), so the 3-dimensional graphical representation can provide an intuitive view of the relation between each other and an approximation of their intersection (black line in the figure), whose projection on the base plane determines the dis-

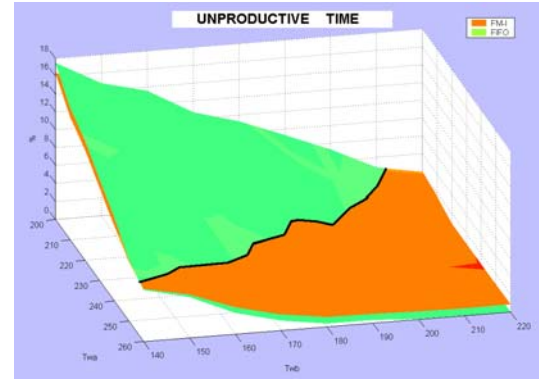


Fig. 5. Representation 3D of the unproductive time as a function of  $t_{wa}$  and  $t_{wb}$ .

criminant between the two regions where one strategy is clearly better than the other.

A set of labeled samples is obtained by associating to each pair  $[t_{wa}, t_{wb}]$  the feeding sequence with lower unproductive time.

## 2.3 Linear discriminant

A Linear Discriminant Function (LDF) is a linear function that divides the 2-dimensional workspace defined by  $\begin{pmatrix} t_{wa} \\ t_{wb} \end{pmatrix}$  into two regions, such that in each of these regions one the feeding strategies is better than the other. Then, the answer to the question "which feeding strategy will work better for a given pair of working times on pallets A and B?" can be easily answered just by evaluating the LDF for the given values  $t_{wa}$  and  $t_{wb}$ , and the sign of the result indicates the best feeding sequence (the sign actually indicates one of the half-spaces defined by the LDF, and each of the half-spaces corresponds to one of the feeding sequences).

The LDF is computed using the set of labeled samples obtained by simulations of the cell working with the two considered strategies on different pairs of working times  $[t_{wa}, t_{wb}]$  (Section 2.2). For the automatic determination of the LDF the lost function to be minimized was the distance from the discriminant to the labeled samples that are misclassified. A dead zone of width  $2d$  is considered in order to emphasize the influence of samples close to the linear discriminant. Figure 6 illustrates the followed approach.

The linear discriminant is given by (Meisel, 1972)

$$\text{LDF}(t_{wa}, t_{wb}) = w_a t_{wa} + w_b t_{wb} + w_c = 0$$

where  $w_a$ ,  $w_b$  and  $w_c$  are the parameters that define the linear discriminant (straight line) in the 2-dimensional space of the working times  $\begin{pmatrix} t_{wa} \\ t_{wb} \end{pmatrix}$ .

The LDF can also be expressed as

$$\text{LDF}(T_W) = W \cdot T_W = 0$$

Table 1. Percentage of absolute time that the machines were unproductive because they were waiting for the robot, for a fixed machine sequence (FM).

| FM                       | working time on pallet B |       |       |      |      |      |       |      |      |
|--------------------------|--------------------------|-------|-------|------|------|------|-------|------|------|
|                          | 140                      | 150   | 160   | 170  | 180  | 190  | 200   | 210  | 220  |
| 200                      | 16.35                    |       | 12.32 |      | 8.36 |      | 4.73  |      | 1.94 |
| 210                      |                          | 12.32 |       | 8.33 |      |      |       | 1.88 |      |
| working time on pallet A | 220                      | 12.34 |       |      | 3.24 |      |       |      | 0.75 |
|                          | 230                      |       | 6.53  |      | 3.44 |      | 0.95  |      |      |
|                          | 240                      | 8.51  |       | 3.65 | 2.25 | 1.11 | 0.79  | 0.78 | 0.80 |
|                          | 250                      |       | 5.58  |      | 1.43 | 0.89 | 0.829 | 0.83 |      |
|                          | 260                      | 5.87  |       | 3.01 | 1.79 | 1.08 | 0.88  |      | 0.88 |

Table 2. Percentage of absolute time that the machines were unproductive because they were waiting for the robot, for a variable sequence (FIFO).

| FIFO                     | working time on pallet B |       |       |      |      |      |       |       |      |
|--------------------------|--------------------------|-------|-------|------|------|------|-------|-------|------|
|                          | 140                      | 150   | 160   | 170  | 180  | 190  | 200   | 210   | 220  |
| 200                      | 17.48                    |       | 13.47 |      | 9.25 |      | 5.29  |       | 1.72 |
| 210                      |                          | 13.37 |       | 9.32 |      |      |       | 1.71  |      |
| working time on pallet A | 220                      | 13.35 |       |      |      | 3.42 |       |       | 0.15 |
|                          | 230                      |       | 7.43  |      | 3.43 |      | 0.577 |       |      |
|                          | 240                      | 9.39  |       | 3.59 | 1.93 | 0.71 | 0.18  | 0.048 | 0.02 |
|                          | 250                      |       | 5.60  |      | 0.87 | 0.26 | 0.07  | 0.02  |      |
|                          | 260                      | 5.73  |       | 2.38 | 1.08 | 0.36 | 0.05  |       | 0.01 |

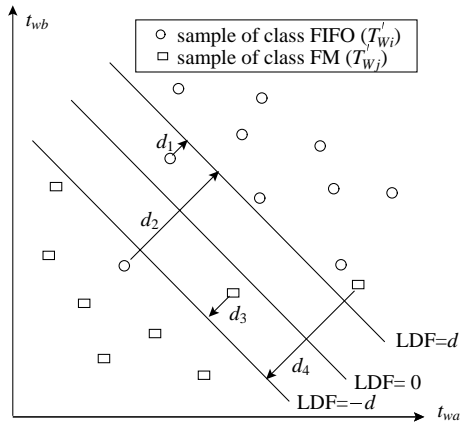


Fig. 6. Illustration of the LDF and the lost function considered in the work.

where

$$W = \begin{pmatrix} w_a \\ w_b \\ w_c \end{pmatrix} \quad T_W = \begin{pmatrix} t_{wa} \\ t_{wb} \\ 1 \end{pmatrix}$$

then, the following nomenclature will be used:

- $T_W$ : a sample of working times  $(t_{wa}, t_{wb}, 1)^T$ .
- $T'_W$ : a labeled sample, i.e. a vector  $(t_{wa}, t_{wb}, 1)^T$  for which the best feeding strategy is known.
- $T'_{Wi}$ : a labeled sample for which the best feeding strategy is the FIFO.
- $T'_{Wj}$ : a labeled sample for which the best feeding strategy is the FM.

By convention, the signs to identify each class of samples were assigned such that:

$$\begin{aligned} \text{LDF}(T'_{Wi}) &> 0 \\ \text{LDF}(T'_{Wj}) &< 0 \end{aligned}$$

The dead zone between the two classes is defined by the linear functions given by

$$\text{LDF}(T_W) = W \cdot T_W = \pm d$$

The lost function computed when a sample  $T'_{Wi}$  (i.e. with FIFO as best strategy) is classified as FM is (see for instance  $d_1$  and  $d_2$  in Figure 6):

$$L_I(T'_{Wi}) = \begin{cases} 0 & \text{if } \text{LDF}(T'_{Wi}) > d \\ d - (W \cdot T_W) & \text{if } \text{LDF}(T'_{Wi}) < d \end{cases}$$

and the lost function computed when a sample  $T'_{Wj}$  (i.e. with FM as best strategy) is classified as FIFO is (see for instance  $d_3$  and  $d_4$  in Figure 6):

$$L_J(T'_{Wj}) = \begin{cases} 0 & \text{if } \text{LDF}(T'_{Wj}) < -d \\ d + (W \cdot T_W) & \text{if } \text{LDF}(T'_{Wj}) > -d \end{cases}$$

Then, the function to be minimized in order to look for the optimum LDF is

$$R = \sum_i L_I(T'_{Wi}) + \sum_j L_J(T'_{Wj})$$

The minimization of  $R$  was done using MATLAB Optimization Toolbox (The Math Works Inc., 1996). Considering  $d = 25$  (determined empirically according to the distance between the samples) the result of the minimization process was a lost  $R = 0.017$  for the discriminant parameters

$$W = \begin{pmatrix} w_a \\ w_b \\ w_c \end{pmatrix} = \begin{pmatrix} 0.27083 \\ 0.21527 \\ -100 \end{pmatrix}$$

so the optimum Linear Discriminant Function can be written as

$$\text{LDF}(t_{wa}, t_{wb}) = 0.27083t_{wa} + 0.21527t_{wb} - 100 = 0$$

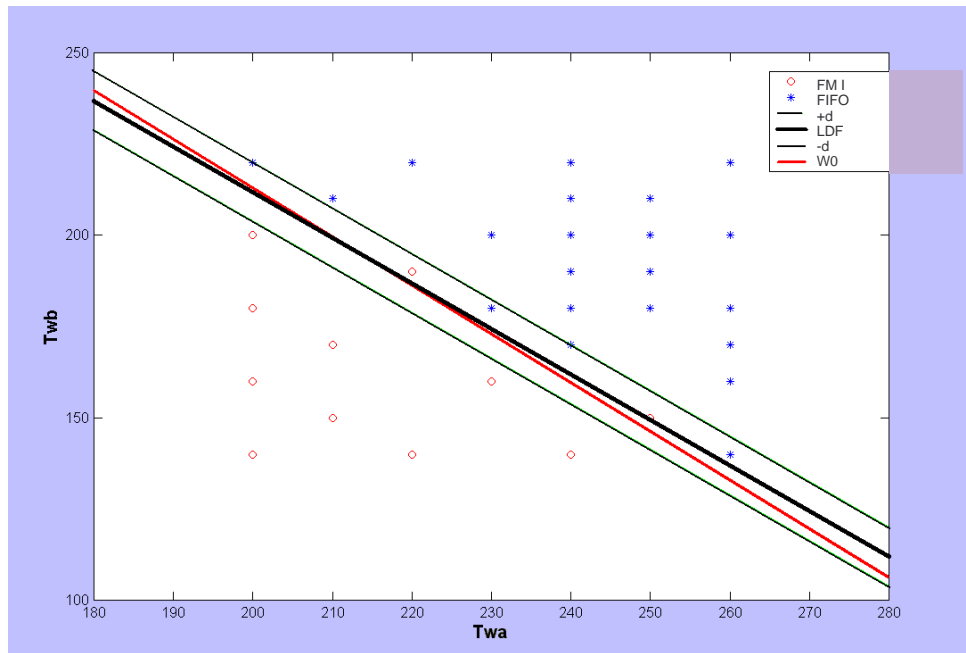


Fig. 7. Labeled samples from simulations and the obtained LDF: the three parallel lines represent  $LDF=0$  and  $LDF=\pm d$ , the other line is the input to the algorithm that search the optimum LDF.

Figure 7 shows the obtained LDF with the corresponding samples obtained by simulation.

As an example of the use of the discriminant consider a given part with processing times  $t_{wa}=222$  and  $t_{wb}=163$ ; then  $LDF(222, 163) = 0.27083 \cdot 222 + 0.21527 \cdot 163 - 100 = -4.786730 < 0$  and therefore the best feeding sequence is FM.

### 3. CONCLUSIONS

The problem of feeding in an optimal way a manufacturing cell composed by four parallel machines was addressed using the theory of linear discriminants. The cell is located in a car-parts manufacturing company and has some features that do not make the problem to be a standard one, and therefore there is no general solution available in the literature. For some particular conditions, differences in the unproductive time of the machines can be quite significant so choosing the best feeding sequence is really important. The cell was modelled and a discrete event simulator of the cell was implemented. Simulations results for different feeding sequences and working times were then used to determine a linear discriminant function. The application of linear discriminant functions, frequently used in pattern recognition, allows the determination of the best feeding strategy in a very simply way just by solving a linear equation. Special preparation or knowledge is not necessary at the time of the sequence selection in the plant and the cell operators can easily do it. Special theoretical knowledge is necessary only for the work presented here, i.e. for the obtention of the linear discriminant.

The proposed approach is a simple solution to an open problem without a precise deterministic solution (new theoretical approaches has still to be developed), and therefore it is a first step in the search of a practical solution for a real industrial application.

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