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GUARANTEED COST CONTROL FOR MULTI–INVENTORY SYSTEMS WITH UNCERTAIN DEMAND ⋆

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Abstract: In this paper we consider the problem of controlling a multi–inventory system in the presence of uncertain demand. The demand is unknown but bounded in an assigned compact set. The control input is assumed to be also constrained in a compact set. We consider an integral cost function of the buffer level and we face the problem of minimizing the worst case cost. We show that the optimal cost of a suitably constructed auxiliary problem with no uncertainties is always an upper bound for the original problem. In the special case of minimum–time control, this upper bound is tight, namely its optimal cost is equal to the worst case cost for the original system. Furthermore the result is constructive, since the optimal control law can be explicitly computed.

1. INTRODUCTION

Multi–inventory dynamical systems are constituted by processes that produce and/or transfer goods, possibly storing them temporarily in warehouses. Such systems form a class of fundamental importance in practice since they are met in several different contexts, e.g., manufacturing (Forrester, 1961; Boukas *et al.*, 1995; Bertsekas, 2000; Kimemia and Gershwin, 1983; Narahari *et al.*, 1999), communications (Ephremides and Verdú, 1989), water distribution (Larson and Keckler, 1999), logistics and traffic control (Moreno and Papageorgiou, 1995).

A multi–inventory system aims at satisfying the demand of final goods. All the storing and processing operations are consequently decided to pursue this objective by possibly minimizing some operational costs. In this context, static optimization methods can be applied when the system operating conditions such as demand, external inputs and link structure are known.

Unfortunately, many real systems work in uncertain and varying conditions. Under such circumstances a feedback approach is sometime preferable (Iftar and Davison, 1990; Moss and Segall, 1982; Kimemia and Gershwin, 1983). Actually, a feedback control can make a system robust against uncertain events such as failures or unknown demand rate.

The authors of this work have recently pursued a deterministic approach in dealing with uncertain events unknown but bounded inside given constraint sets. In particular, the problems of keeping the buffer levels within assigned constraints while driving a system to the “least storage level" are faced in (Blanchini *et al.*, 1997). In (Blanchini *et al.*, 2000) it is shown that for continuous–time models there exists a strategy assuring convergence to any target buffer level if a certain control dominance necessary and sufficient condition is satisfied.

The previous contributions deal mainly with stability instead of control optimality. Although stability is fundamental, because it assures that the system is kept in a desired working point (i.e. at given buffer levels), it is also fundamental to investigate the transient cost necessary to reach such a point. Several previous

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references have dealt with the problem of transient optimality (see for instance (Bertsekas, 2000; Shu C. and R., 2000; Moss and Segall, 1982)). However, with the exception of (Boukas *et al.*, 1995), very few of them explicitly consider unknown–but–bounded disturbances.

In this paper, we consider an integral cost of the buffer levels and we deal with the problem of estimating the worst–case cost. We introduce an auxiliary optimal control problem with no uncertainty and, provided that the necessary and sufficient condition of (Blanchini *et al.*, 2000) are satisfied, we prove the following results:

- the auxiliary optimal cost is an upper bound for the worst case cost of the original problem;
- in the minimum–time case the provided upper bound is tight, namely it is equal to the worst case cost.

The mentioned results are constructive since we will provide the guaranteed cost control. We also show how to compute a lower bound, namely the optimal best–case cost.

2. PROBLEM STATEMENT

We consider the following dynamic model of a multi–inventory system

$$\dot{x}(t) = v(t) - w(t), \tag{1}$$

where $v(t) \in \mathbb{R}^n$ is the control and $w(t) \in \mathbb{R}^n$ is an unknown external input, whose components model respectively the controlled resource flows and the demands, or more in general, the non-controllable flows. We assume that the control v and the input w are bounded:

$$v(t) \in \mathscr{V} \tag{2}$$

$$w(t) \in \mathscr{W} \tag{3}$$

where $\mathscr{V}$ and $\mathscr{W}$ are assigned convex and compact sets. Note that in (Blanchini *et al.*, 1997; Blanchini *et al.*, 2000) the notation $v = Bu$ and $w = -Ed$ is used. In our case there is no reason to consider matrices E and B. In (Blanchini *et al.*, 1997; Blanchini *et al.*, 2000) $\mathscr{W} \subset int\{\mathscr{V}\}$ is shown to be a necessary and sufficient condition for stabilizability. Provided that such a condition is satisfied, a bang–bang stabilizing (discontinuous) strategy exists. This strategy drives the buffer level to the origin (or any target level) in finite time. For any given initial state we consider the following cost

$$J = \int_0^T g(x(t))dt \tag{4}$$

being $g : \mathbb{R}^n \to \mathbb{R}^+$ a given positive–definite and convex function. Let $\mathscr{F}_{\mathscr{V}}$ be the set of all feedback control functions of the state

$$\Phi : \mathbb{R}^n \to \mathscr{V}$$

and $\mathscr{P}_{\mathscr{W}}$ the set of all piecewise continuous functions

$$w : \mathbb{R}^+ \to \mathscr{W}$$

Consider the following problem with free terminal time T

$$\begin{aligned} &\Psi(x_0) = \inf_{\Phi \in \mathscr{F}_{\mathscr{V}}} \sup_{w \in \mathscr{P}_{\mathscr{W}}} J, \\ &\text{s.t.,} \\ &x(0) = x_0, \ \ x(T) = 0 \end{aligned} \tag{5}$$

This is a typical min–max problem in which the control goal is that of minimizing the worst–case cost over all admissible (i.e. compatible with the constraints) disturbances w.

Let us now consider the next modified set $\hat{\mathscr{V}}$

$$\hat{\mathscr{V}} = \{\hat{v} \in \mathbb{R}^n : \hat{v} + w \in \mathscr{V}, \ \forall \ w \in \mathscr{W}\} \tag{6}$$

Note that $\mathscr{W} \subset int\{\mathscr{V}\}$ implies that $\hat{\mathscr{V}}$ is non–empty and includes 0 as an interior point. Consider the following auxiliary system

$$\dot{x}(t) = \hat{v}(t), \tag{7}$$

with

$$\hat{v}(t) \in \hat{\mathscr{V}} \tag{8}$$

Let $\hat{\mathscr{F}}_{\hat{\mathscr{V}}}$ be the set of all feedback control functions

$$\Phi : \mathbb{R}^n \to \hat{\mathscr{V}}$$

and consider the following optimal control problem for (7)

$$\begin{aligned} &\hat{\Psi}(x_0) = \min_{\Phi \in \hat{\mathscr{F}}_{\hat{\mathscr{V}}}} J \\ &\text{s.t.,} \\ &x(0) = x_0, \ \ x(T) = 0 \end{aligned} \tag{9}$$

Problem (9) for (7) is quite simpler than (5) because no uncertainties are present. Conversely (5) is very hard, since the control is not aware, at each time, of the future actions of w. In the next section we state how the two problems are related to each other.

Remark 2.1. The set $\hat{\mathscr{V}}$ can be easily determined if the support functional of $\mathscr{V}$ is known. If $\mathscr{V}$ is a polytope, $\hat{\mathscr{V}}$ is also a polytope computable via linear programming.

3. MAIN RESULTS

As it is well known, an optimal control problem can be faced by means of dynamic programming. The HJB equation for problem (9) is

$$\min_{\hat{v} \in \hat{\mathscr{V}}} \left[\nabla \hat{\Psi}(x)\hat{v}\right] + g(x) = 0 \tag{10}$$

where $\hat{\Psi}(x)$ is the cost–to–go function. If such equation is satisfied, then $\hat{\Psi}(x_0)$ represents the optimal cost with initial condition x_0(Bertsekas, 2000). The following theorem holds.

Theorem 3.1. Assume that the cost–to–go function $\hat{\Psi}(x)$ is smooth everywhere with $x \neq 0$, and satisfies the HJB equation (10). Then the control

$$\hat{\Phi}(x) = \arg\min_{v \in \mathscr{V}} \left[\nabla \hat{\Psi}(x) v\right] \qquad (11)$$

applied to (1) guarantees a cost

$$J \leq \hat{\Psi}(x_0),$$

for each $x(0) = x_0$ and for all $w(\cdot) \in \mathscr{P}_{\mathscr{W}}$.

The following corollary is an immediate consequence of Theorem 3.1.

Corollary 3.1. Under the assumptions of Theorem 3.1

$$\Psi(x) \leq \hat{\Psi}(x)$$

for all x.

The previous theorem and corollary provide a guaranteed cost control, but say nothing concerning the tightness of the upper bound. However, in the special case of minimum time we can show that this upper bound is tight. The minimum–time HJB equation for problem (5) is

$$\min_{\hat{v} \in \hat{\mathscr{V}}} \left[\nabla \hat{\Psi}(x) \hat{v}\right] + 1 = 0 \qquad (12)$$

where now $\hat{\Psi}(x_0)$ is the minimum time necessary for the auxiliary system state to reach the origin starting from x_0. Correspondingly let $\Psi(x_0)$ be the minimum time which is necessary to drive the actual system state to zero in system (1). We have the following theorem

Theorem 3.2. Let $g(x) \equiv 1$. Assume that the cost–to–go function $\hat{\Psi}(x)$ is smooth everywhere, with $x \neq 0$, and satisfies the HJB equation (12). Then

$$\Psi(x) = \hat{\Psi}(x)$$

Solving the HJB equation is usually a hard task. Fortunately, in the minimum–time case we have an explicit solution according to the next theorem.

Theorem 3.3. Assume that $\hat{\mathscr{V}}$ is smooth (i.e. its boundary is smooth). Then $\hat{\Psi}(x)$ is the Minkowski function of $-\hat{\mathscr{V}}$, namely

$$\hat{\Psi}(x) = min\left\{\lambda > 0 : \quad -x \in \lambda \hat{\mathscr{V}}\right\}$$

Remark 3.1. The requirement of $\hat{\mathscr{V}}$ being smooth is not restrictive. Indeed, if, for instance, $\hat{\mathscr{V}}$ is a polytope, we can always approximate it by means of a smooth convex set with arbitrary precision. In this case the Minkowski function admits a nice analytic expression (Blanchini and Miani, 1999).

We show now that the bound $J \leq \hat{\Psi}(x_0)$ is non–tight in general.

Example Consider system (1) with $n = 2$ and

$$\mathscr{V} = \{v : \ \|v\|_1 \leq 2\}$$
$$\mathscr{W} = \{w : \ w_1 = 0, |w_2| \leq 1\}$$
$$g(x) = \max\{|x_1 + \gamma x_2|, |x_1 - \gamma x_2|\}$$

where $\gamma > 0$ is a parameter. It is easy to show that

$$\hat{\mathscr{V}} = \{v : \ \|v\|_1 \leq 1\}.$$

Note also that g is linear in any quadrant and it is linear along the positive and negative axes. The level surfaces of function g are as depicted in Fig. 1 (the dotted lines). Consider the initial condition $A = [1\ 0]^T$. The optimal strategy for the auxiliary system is clearly

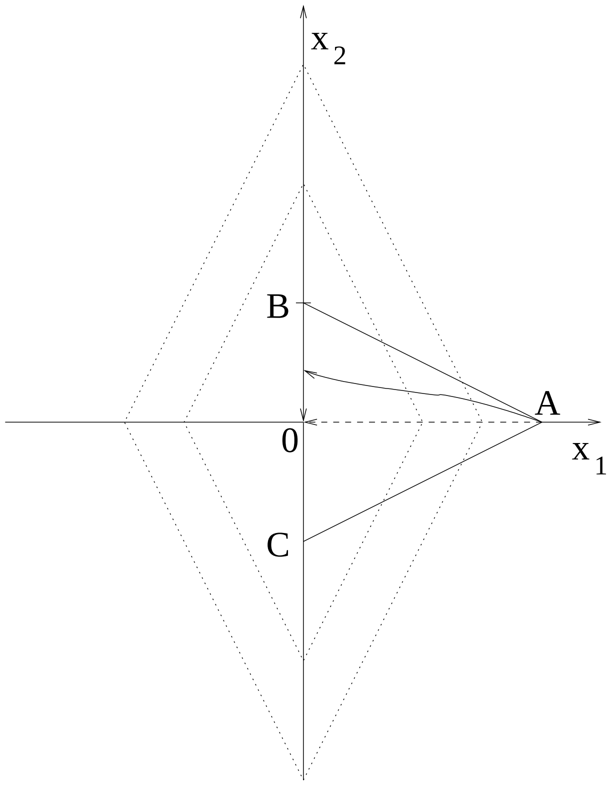

Fig. 1. The system trajectories

$$\hat{v}(t) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad 0 \leq t \leq 1$$

The corresponding transient is $x(t) = (1-t)A$ (the dashed line from A to the origin in Figure 1) which implies the (optimal) cost $\hat{\Psi}(A) = 1/2$. Now consider the system with disturbances. Among the admissible strategies we have the following. Take

$$v(t) = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \quad 0 \leq t \leq 1/2$$

Since $w_1 = 0$, the disturbance has no influence on the horizontal motion, precisely, $x_1(t) = \int_0^t v_1(\sigma) d\sigma =$

$-2t$. Then at $t = 1/2$ the state reaches the vertical axis (see the curved trajectory in Fig 1) say $x_1(1/2) = 0$. Note that since $|w_2| \leq 1$ we have $|x_2(t)| = |\int_0^t w_2(\sigma)d\sigma| \leq t$. Then, necessarily, at $t = 1/2$ the second component of x, $x_2(1/2)$, is such that $|x_2(1/2)| \leq 1/2$. Therefore $x(1/2)$, is on the vertical axis between points B and C (which correspond to the values $1/2$ and $-1/2$, respectively). Henceforth by means of a control

$$v_2 = -2sgn[x_2]$$

the state is driven to 0. It is quite easy to see that the worst case cost is achieved if w_2 pushes at full force the state away from the origin. So let us assume $w_2 = 1$ (the opposite case $w_2 = -1$ leads to the same conclusions). Now note that when x is on the positive vertical axis with the considered control action we have $\dot{x}_2 = -2+1 = -1$. Since $x_2(1/2) \leq 1/2$ it takes at most further $1/2$ time units to reach the origin. Significantly the total amount of time is $T = 1$, the same time of the auxiliary system, as expected, notwithstanding the fact that the perturbed trajectory is completely different. Now the cost of the "extremal trajectory" is the cost of the path from A to B plus the cost of the path from B to 0. Now being these paths segments and being $x(t)$ linear in t on these paths, and since the function g is linear on the first close quadrant, the total cost is easy computed by means of the trapezoidal formula:

$$J = [\frac{1}{2}g(A) + \frac{1}{2}g(B)]\frac{T}{2} + [\frac{1}{2}g(B)]\frac{T}{2} = \\ = \frac{T}{4}g(A) + \frac{T}{2}g(B)$$

Now $g(A) = 1$, $g(B) = \gamma/2$ and $T = 1$. Therefore if we take $\gamma < 1$

$$\Psi(A) \leq \frac{1}{4}(1+\gamma) < \hat{\Psi}(A).$$

4. PROOFS OF THE MAIN RESULTS

In this section we prove Theorems 3.1-3.3. To this aim we introduce the following preliminary results. A convex and compact set $\mathscr{S} \subset \mathbb{R}^n$ can be always represented as follows

$$\mathscr{S} = \{s : \; z^T s \geq \mu_{\mathscr{S}}(z), \; \text{ for all } \; z \in \mathbb{R}^n\}$$

where the function

$$\mu_{\mathscr{S}}(z) \doteq \min_{s \in \mathscr{S}} z^T s$$

is positively homogeneous of order one. Such a representation holds for $\mathscr{V}$, $\mathscr{W}$ and $\hat{\mathscr{V}}$. The set $\hat{\mathscr{V}}$ can be represented as follows

$$\hat{\mathscr{V}} = \{\hat{v} : \; z^T \hat{v} \geq \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z), \; \text{ for all } \; z \in \mathbb{R}^n\}$$

thus, in general, for any $z^T \in \mathbb{R}^n$, we have

$$\mu_{\hat{\mathscr{V}}}(z) \geq \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z). \tag{13}$$

Proof of Theorem 3.1. First note that for all $z \in \mathbb{R}^n$ the following inequality holds for all $w \in \mathscr{W}$

$$\min_{v \in \mathscr{V}} z^T(v-w) \leq \min_{v \in \mathscr{V}} z^T v - \min_{w \in \mathscr{W}} z^T w = \\ = \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z) \leq \min_{\hat{v} \in \hat{\mathscr{V}}} z^T \hat{v}.$$

Consider the cost–to–go function $\hat{\Psi}(x)$ for problem (9). If we apply the control

$$\Phi(x) = \arg\min_{v \in \mathscr{V}} \nabla\hat{\Psi}(x)v, \tag{14}$$

denoting by $x(t)$ the trajectory of system (1) with control (14), the corresponding Lyapunov derivative is

$$\frac{d}{dt}\hat{\Psi}(x(t)) = \dot{\hat{\Psi}}(x) = \nabla\hat{\Psi}(x)[\Phi(x) - w] = \\ = \min_{v \in \mathscr{V}} \nabla\hat{\Psi}(x)[v-w] \\ \leq \min_{\hat{v} \in \hat{\mathscr{V}}} \nabla\hat{\Psi}(x)\hat{v} = -g(x)$$

where the last equality comes from the assumption that (10) holds. By integrating both sizes we have

$$J = \int_0^T g(x(t))dt \leq \hat{\Psi}(x(0)) - \hat{\Psi}(x(T)) = \hat{\Psi}(x(0))$$

The last inequality obviously implies that

$$\Psi(x(0)) \leq \hat{\Psi}(x(0))$$

□

The next step is to show that in the minimum–time case, the latter inequality is indeed an equality.

Proof of Theorem 3.2 The proof is only sketched for brevity.

We have seen that $\Psi(x) \leq \hat{\Psi}(x)$. Now we prove that $\Psi(x) \geq \hat{\Psi}(x)$ by showing that, given x_0, there exists a trajectory $\hat{x}(t)$ of the auxiliary system (7) which reaches the origin before a "worst case" trajectory of the original system (1).

Consider the set $\hat{\mathscr{V}}$ and an initial state x_0 and let $\hat{v}_0$ be the unique vector on the boundary of $\hat{\mathscr{V}}$ which is aligned with $-x_0$, i.e. $\hat{v}_0 = -\lambda x_0$ for some $\lambda \geq 0$ (note that such a λ exists because $0 \in int\{\hat{\mathscr{V}}\}$). It is possible to show that there exists a direction z^T such that the plane

$$\Pi = \{v : z^T(v - \hat{v}_0) = 0\}$$

is tangent to $\hat{\mathscr{V}}$ in $\hat{v}_0$,

$$\mu_{\hat{\mathscr{V}}}(z) = \min_{\hat{v} \in \hat{\mathscr{V}}} z^T \hat{v} = z^T \hat{v}_0$$

(roughly z points from v_0 toward the interior of $\hat{\mathscr{V}}$) and

$$\mu_{\hat{\mathscr{V}}}(z) = \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z),$$

namely inequality (13) is actually an equality. It can also be proved that, since $\hat{v}_0$ is on the boundary of $\hat{\mathscr{V}}$, z is such that $z^T x_0 > 0$. Consider the Lyapunov–like function $V(x) = z^T x$. If for initial condition x_0 we apply the constant control input $\hat{v}(t) \equiv \hat{v}_0$ to the auxiliary system (7) we have, denoting by $\hat{x}(t)$ the corresponding solution,

$$\dot{V}(\hat{x}(t)) = z^T\hat{v}_0 = \mu_{\hat{\mathscr{V}}}(z) = \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z)$$

Now, let $x(t)$ be the solution of system (1) when $w(t)$ is constant and such that $z^T w = \mu_{\mathscr{W}}(z)$. We get

$$\begin{aligned}\dot{V}(x(t)) &= z^T(v-w)\\ &\geq \mu_{\mathscr{V}}(z) - \mu_{\mathscr{W}}(z) =\\ &= \mu_{\hat{\mathscr{V}}}(z) = \dot{V}(\hat{x}(t))\end{aligned}$$

For initial state $\hat{x}(0) = x(0) = x_0$, condition $\dot{V}(x(t)) \geq \dot{V}(\hat{x}(t))$ implies that

$$V(\hat{x}(t)) \leq V(x(t)), \quad t \geq 0.$$

The proof is completed by noticing that the trajectory $\hat{x}(t)$ reaches the origin, let us say at time T. Conversely, $x(t) = 0$ at time t only if $V(x(t)) = 0$ is satisfied, and, in view of the inequality above, this condition cannot hold before T, when $V(\hat{x}(T)) = 0$ is achieved. □

Remark 4.1. Since we had previously shown that $\Psi(x) \leq \hat{\Psi}(x)$, namely that the minimum time for the perturbed system (1) does not exceed the minimum time for the auxiliary system (7), the previous proof shows, in passing, that $\hat{x}(t)$ is indeed the optimal trajectory for the auxiliary system. Namely, the optimal minimum–time trajectory for the system without disturbances is achieved open-loop by taking the control action pointing to the origin. This fact is already known (see (Moss and Segall, 1982)). Clearly the open–loop solution is useless as long as we have to cope with disturbances.

Proof of theorem 3.3 The time necessary to reach the origin from x_0 for the auxiliary system (7) is the same time necessary for the system $\dot{x} = -\hat{v}$ to reach x_0 from the origin. The reachable set at time t is $-t\hat{\mathscr{V}}$. Therefore the reachable set hits x_0 at time

$$T_{min} = \min\{t \geq 0 : \ x_0 \in -t\hat{\mathscr{V}}\}.$$

Clearly $T_{min} = \hat{\Psi}(x_0)$. □

5. CONCLUSIONS AND DISCUSSION

In this paper we have shown that an optimal control problem for multi–inventory systems with unknown–but–bounded demand admits a guaranteed cost solution achieved by considering a suitable auxiliary problem. Such guaranteed cost is an upper bound for the worst–case cost which is shown to be tight in the minimum–time case. The results are constructive as long as we can solve the optimal auxiliary problem. In the minimum time case we provide a solution for the HJB equations being the Minkowski function of the opposite of the constraint set of the auxiliary problem.

Often, the sets $\mathscr{V}$ and $\mathscr{W}$ are polytopes, having a finite representation. If this is the case, $\hat{\mathscr{V}}$ is also a polytope (Bertsekas and Rhodes, 1977). Then, the Minkowski function of the opposite of $\hat{\mathscr{V}}$ is non–smooth. However, we can always approximate it by means of a smooth function as it is shown in (Blanchini and Miani, 1999).

If we consider a generic function $g(x)$, then finding an exact solution is a hard task. It can be shown that approximate solutions can be found, providing a guaranteed cost.

As a final conclusion, notice that the best–case cost, namely when v and w cooperate to reach the origin, can be computed by considering (7) with

$$\hat{v} \in \check{\mathscr{V}} = \mathscr{V} - \mathscr{W},$$

the Minkowski sum of $\mathscr{V}$ and $-\mathscr{W}$. Therefore, denoting by $\check{\Psi}(x_0)$ the cost–to–go function with initial condition x_0 and control constrained as $v \in \check{\mathscr{V}}$, we can derive the upper and lower bounds

$$\check{\Psi}(x_0) \leq J \leq \hat{\Psi}(x_0).$$

Clearly the lower bound is tight for any $g(x)$. Namely the cost is actually reached for some favorable realization of $w(\cdot)$.

We have shown that the upper bound obtained by solving the optimal control problem for the auxiliary system is not tight in general. Determining if there are special cases of cost functions $g(x)$, beside $g(x) \equiv 1$, for which the bound is non conservative is still under investigation.

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