15th Triennial World Congress, Barcelona, Spain

ON THE ROBUST FAULT ISOLATION OBSERVER BY ASSIGNING LEFT AND RIGHT EIGENVECTORS

Alessandro Casavola* Domenico Famularo ** Giuseppe Franzè*

* *DEIS - Università degli studi della Calabria, Rende(CS), 87036 ITALY,* {casavola, franze}@deis.unical.it
** *ISI - Consiglio Nazionale delle Ricerche, Rende (CS) 87036, ITALY,* famularo@isi.cs.cnr.it

Abstract: This note deals with the exact Fault Detection and Isolation problem. In the case of observer-based residual generation, the problem amounts to find two gain matrices such that the two problems are simultaneously solved. It is known that the freedom in eigenstructure assignment leads to well-conditioned design which is robust to unstructured uncertainties. The approach here considered aims to design a residual generator trough a full-order observer to achieve fault detection and isolation by means of right and left eigenstructure assignment. Moreover when diagonal isolation cannot be obtained, an almost isolation method, based on a functional minimization, is proposed. A numerical example shows the effectiveness of the contribution here presented.

Keywords: Fault Fault Detection, Fault Isolation, Geometric Approaches, Eigenstructure Assignment, Genetic Algorithms.

1. INTRODUCTION

Fault detection and isolation (FDI) techniques involve the generation and evaluation of fault accentuated signals on the basis of available measurements and a mathematical model of the system. FDI problem, which plays a central role in system diagnosis, has been extensively studied in recent past (Frank, 1996; Patton and Chen, 1999; Shen and Hsu, 1998). The problem has been studied under various hypotheses and using various resolution techniques. Among these approaches, the geometric one appeared to be very appealing (Massoumnia *et al.*, 1989; Patton and Chen, 2000).

In this paper, we are concerned with the FDI problem, where our aim is to design a filter which exactly detects and isolates fault signals. To do this, we build a residual generator through a full-order observer. More precisely, we look for the unknown gain matrix values which ensure that:

a) the transfer function from the disturbance to the residual is zero,
b) the transfer function from the faults to the residual has a diagonal structure.

The contribution of this paper aims to the design of a diagnostic observer to achieve exact fault detection and isolation by means of right and left eigenstructure assignment. We propose a procedure to eigenstructure assignment for FDI based on the assignment of some right eigenvectors parallel to the disturbance distribution directions. It is straightforward to observe that the observer design is a dual problem of the disturbance decoupling design one, and there are only a few studies about left eigenvectors assignment in control design problems (Choi *et al.*, 1995).

Next, the almost isolation problem is treated. The notion of almost fault identification is motivated by a vast control literature that exists on exact and almost disturbance decoupling. In almost fault isolation, we seek in a natural way an almost decoupling disturbance based on the residual signal rather than the exact disturbance decoupling (Saberi *et al.*, 2000).

2. FORMULATION PROBLEM AND PRELIMINARIES

We deal with linear continuous and discrete time-invariant systems. In order to show our approach, in the sequel we shall consider the following class of LTI continuous systems which are disturbed by an additive unknown disturbance

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + Rf(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the control input vector, $d(t) \in \mathcal{R}^d$ is the unknown disturbance vector, $f(t) \in \mathcal{R}^q$ is the fault vector and $y(t) \in \mathcal{R}^p$ is the output vector. A, B, C, E, are known matrices with appropriate dimensions. R denotes the fault distribution matrix.

The residual generator is based on the following full-order observer

$$\begin{cases} \dot{\hat{x}}(t) = (A + KC)\hat{x}(t) + Bu(t) + Ky(t) \\ \hat{y}(t) = C\hat{x} \\ r(t) = Q\left(y(t) - \hat{y}(t)\right). \end{cases} \quad (2)$$

When the residual generator (2) is applied to the system described by equations (1), the state estimation error and the residual are governed by

$$\begin{cases} \dot{e}(t) = (A + KC)\hat{e}(t) + Ed(t) + Ry(t) \\ r(t) = He(t) \end{cases} \quad (3)$$

where $H = QC$. Since the number of fault signals that can be isolated is bounded from the above by the number of measurements, the condition

$$p > q + d \quad (4)$$

is required to provide sufficient information for q multiple fault signals and d multiple unknown inputs.

The problem is then to determine the unknown matrices Q and K such that the residual $r(t)$ is not affected by the disturbances $d(t)$ and the transfer function from the fault signals $f(t)$ to the residuals has to be diagonal, that is

$$G_{rd}(s) = H(sI - A - KC)^{-1}E = 0 \quad (5)$$

$$\begin{aligned} G_{rf}(s) &= H(sI - A - KC)^{-1}R \quad (6) \\ &= \operatorname{diag}(g_{r_1 f_1}(s), \dots, g_{r_q f_q}(s)) \end{aligned}$$

Recall that an FDI (Fault Detection and Isolation) scheme, which makes use of the residuals with the disturbances de-coupling property, is robust.

Theorem 1. (Patton and Chen, 2000) The sufficient conditions for satisfying the disturbance decoupling requirement (6) are

1) $QCE = 0$
2) All columns of the matrix E are right eigenvectors of $(A + KC)$ corresponding to any eigenvalues.

Disturbance de-coupling does not impose any restriction on the choice of the remaining $n - d$ right eigenvectors and corresponding eigenvalues of $A + KC$. Under disturbance de-coupling by assigning right eigenvectors the transfer function between residuals and faults becomes

$$\begin{aligned} G_{rf}(s) &= \sum_{i=1}^{n} H \frac{v_i l_i^T}{s - \lambda_i} R \\ &= \sum_{i=1}^{n-d} H \frac{v_i l_i^T}{s - \lambda_i} R, \end{aligned} \quad (7)$$

where $V = [v_1, \dots, v_n]$, $L = [l_1, \dots, l_n]^T$ and $\Lambda = [\lambda_1, \dots, \lambda_n]$ are right eigenvectors, left eigenvectors and eigenvalues of $A + KC$ respectively.

After having de-coupled the disturbance by assigning right eigenvectors, in order to obtain a structure of the transfer function (7), the following equations have to be satisfied:

$$H v_i = 0,\ i = n - q + 1, \dots, n - d \quad (8)$$

$$\operatorname{row}(H)_i\, v_j = 0,\ i, j = 1, \dots, q;\ i \neq j \quad (9)$$

$$\operatorname{row}(H)_i\, v_i l_i^T \operatorname{col}(R)_j = 0,\ i, j = 1, \dots, q;\ i \neq j \quad (10)$$

In the sequel, we propose a factorization of the right (left) eigenvectors of a matrix, which depends on a parameter vector belonging to $\mathcal{R}$. Let $p, q \in \mathcal{N}$; we will denote $\chi(p,q)$ the q-th *fundamental symmetric function* of points $x_1, \ldots, x_n$; it is defined as

$$\chi(p,q) = \begin{cases} 1, & p = 0, 1, \ldots; q = 0; \\ 0, & q < 0 \,||\, p < 0 \,||\, q > p; \\ \chi(p-1,q) + x_p \chi(p-1,q-1), & \\ \textit{otherwise}. & \end{cases} \quad (11)$$

Proposition 2. Let $H \in \mathcal{R}^{n \times n}$ with all distinct eigenvalues α_i, $i = 1, \ldots, n$. The right eigenvectors w_i, $i = 1, \ldots, n$, of H are given by

$$w_i = \sum_{k=0}^{n-1} \sum_{s=0}^{n-1-k} (-1)^{s+k} \chi(n, n-1-k-s) \alpha_i^k H^s h \quad (12)$$
$$h \in \mathcal{R}^n, \;\; i = 1, \ldots, n,$$

where $h^T w_i \neq 0$, $i = 1, \ldots, n$ and $\chi(n,k)$ denotes the kth fundamental symmetric function of the eigenvalues of H, $\alpha_1, \ldots, \alpha_n$.

Note that the left eigenvectors can be computed by simply substituting in (12) H^T to H.

3. OBSERVER POLE PLACEMENT PROBLEM

The right eigenvector assignment problem amounts to determine a matrix K which assigns the eigenvalues of the observer dynamic matrix $A + KC$ in the left-hand side of the complex plane. This is only possible when (C, A) is a detectable pair. In the following, we propose a pole placement procedure that allows to place the eigenvalues of $A + KC$ in prescribed locations by assigning the right eigenvectors of $A + KC$. Let the eigenvalue problem be given

$$(A + KC)\, v_i = \lambda_i\, v_i, \quad i = 1, \ldots, n, \quad (13)$$

and let $\pi_1, \ldots, \pi_n$ the eigenvalues of A, $U = [u_1, \ldots, u_n]$ and $W = [w_1, \ldots, w_n]$ the matrices of right and left eigenvectors of A. Let Γ represent the right eigenvector of $A + KC$ v_i in the basis $U : U\Gamma = V$. The n vectors equations (13) can be rewritten as

$$\sum_{i=1}^{n} \gamma_k^i w_s^T KC u_k = (\lambda_i - \pi_s)\gamma_s^i, \quad i, s = 1, \ldots, n. \quad (14)$$

For this pole placement problem, the first step is to construct a gain matrix $K \in \mathcal{R}^{n \times p}$ that assigns p eigenvalues. To this purpose let $\Lambda_p = \{\lambda_1, \ldots, \lambda_p\}$ be a symmetric set of complex numbers. It can be showed that if there exist p independent eigenvectors

$$\begin{aligned} v_k &\in C(\lambda_k I - A)^{-1}, \quad k = 1, \ldots, p, \\ v_k &= [\theta_1(k), \ldots, \theta_{p-1}(k),\ 1] C(\lambda_k I - A)^{-1}, \\ &\quad k = 1, \ldots, p, \end{aligned} \quad (15)$$

then there exists K such that $\lambda_1, \ldots, \lambda_p$ are eigenvalues of $A + KC$.

Proposition 3. Let $\{\lambda_1, \ldots, \lambda_p\}$ be a subset of Λ, the $n \cdot p$ equations

$$\begin{cases} w_1^T KC u_j = \gamma_j^1 (\lambda_j - \pi_1) \\ w_2^T KC u_j = \gamma_j^2 (\lambda_j - \pi_2) \\ \quad \vdots \quad = \quad \vdots \\ w_n^T KC u_j = \gamma_j^n (\lambda_j - \pi_n),\ j = 1, \ldots, p, \end{cases} \quad (16)$$

extracted from (14) have a solution K for almost all choices of $u_k \in C(\lambda_k I - A)^{-1}$, $k = 1, \ldots, p$.

The remaining $n - p$ eigenvalues of $A + KC$ are computed using the following lemma.

Lemma 4. Let K be a gain matrix which satisfies Proposition 3. The remaining eigenvalues of the matrix $A + KC$ are the eigenvalues of $(n-p) \times (n-p)$ matrix

$$A_T = A_{22} - LA_{12} \quad (17)$$

where $A_{22} = \bar{C}AT_2$, $A_{12} = CAT_2$, $L = \bar{C}V_p[CV_p]^{-1}$, $U_p = [u_1, \ldots, u_p]$, and $\bar{C}$ is any $(n-p) \times (n-p)$ matrix such that the matrix $\begin{bmatrix} C \\ \bar{C} \end{bmatrix} = [T_1, T_2]$ is nonsingular.

4. SOLUTION OF THE FDI PROBLEM BY ASSIGNING RIGHT AND LEFT EIGENVECTORS

In this section, we propose a parametric procedure that allows to solve the disturbed fault detection and isolation problem by imposing the right and left eigenvectors of the full-order observer.

The procedure is based on the use of the free parameters which characterize the first p right eigenvectors u_i of $A + KC$, as showed in the previous section, and the left eigenvectors l_i^T given in the parameterized form (12). The number of free parameters is obtained by the union of the following sets:

- Number of free parameters Θ_r of the right eigenvectors:

$$\begin{cases} \Theta_r &= \{\theta_1(1), \dots, \theta_{p-1}(1), \dots, \\ & \quad \theta_1(p-d), \dots, \theta_{p-1}(p-d)\} \\ N_{\Theta_r} &= n\,(p-d-1) + d, \end{cases} \quad (18)$$

- Number of free parameters Θ_l of the left eigenvectors:

$$\begin{cases} \Theta_l &= h \\ N_{\Theta_l} &= n-1, \end{cases} \quad (19)$$

Remark Note that N_{Θ_r} is obtained by taking into account $n-p$ free parameters that must be used to assign the remaining $n-p$ eigenvalues of $A+KC$, and d right closed-loop eigenvector are imposed equal to the disturbance directions E.

Numerical Procedure

Step 1 Compute the residual weighting matrix Q such that $QCE = 0$

Step 2 Choose the desired observer eigenvalues Λ and partition the spectrum into $\Lambda_p = \{\lambda_1, \dots, \lambda_p\}$ and $\Lambda_{n-p} = \{\lambda_{p+1}, \dots, \lambda_n\}$

Step 3 Using equations (15), calculate explicit parametric expressions of the p right eigenvectors associated to Λ_p

Step 4 Impose the d columns of E as right eigenvectors of $A+KC$

Step 5 Solve the sets of equations (8) and (9) with respect to Θ_r

Step 6 Compute the gain matrix K applying Proposition 3 and Lemma 4

Step 7 Construct parametric expressions of the left eigenvectors of $A+KC$ using formula (12)

Step 8 Numerical solve equations (10) with respect to Θ_l

Step 9 Compute

$$G_{rf}(s) = \sum_{i=1}^{n-d} H \frac{v_i l_i^T}{s-\lambda_i} R$$

5. ALMOST ISOLATION

Fault detection problems makes sense only when there is a possibility of the occurrence of multiple fault signals. In that case, in addition to detecting that a single fault or multiple faults occurred, one has to identify as to what individual fault or faults have occurred. It is easy to recognize that the task of exactly isolating or identifying (as proposed in the previous section) every individual fault could require too restrictive conditions. Therefore we can weaken this requirement by requiring that the conditions are satisfied arbitrarily well but not perfectly.

As it is well known, disturbance de-coupling does not place any restriction on the choice of right eigenvectors v_i, $i = n-d+1, \dots, n$ and the corresponding eigenvalues λ_i, $i = n-d+1, \dots, n$. Therefore, the free parameters explicited in the previous section can be used to maximize the fault effect as

$$G_{rf}(s) = \sum_{1}^{n-d} H \frac{v_i l_i^T}{s-\lambda_i} R\,.$$

As pointed out in (Patton and Chen, 2000) the most factor in fault detectability is the steady-state gain matrix $G_{rf}(0)$, hence a performance index, to be maximized for increasing fault detectability, is defined as:

$$J(\Lambda) = \left\| \sum_{1}^{n-d} H \frac{v_i l_i^T}{-\lambda_i} R \right\|_F, \quad (20)$$

where $\|\cdot\|_F$ denotes the Frobenius norm.

To maximize the fault effect and, subsequently fault detectability, the performance index $J(\Lambda)$ should be maximized. Consequently, the problem can be formulated as an optimization one.

Let $\Sigma = \Theta_r \cup \Theta_l$ be the free parameter vector and $\nu = \dim(\Sigma)$:

Problem 1

$$\max_{\Sigma \in S_\nu} J(\Lambda) \quad (21)$$

where S_ν is a compact set of $\mathcal{R}^\nu$.

Note that the maximization of $J(\Lambda)$ is a constrained optimization problem because all the elements of Λ must be on the left hand-side of the complex plane.

The optimization problem can be solved by any suitable numerical search method. In this note, we have faced **Problem 1** by a genetic-like algorithm (Goldberg, 1989), which has a minimum degree of problem dependence.

Consequently, the new posed problem (**Problem 1**) imposes a variation on the numerical procedure of the previous section. In fact, *Step 8* must be modified in the sense that it solves an optimization problem: the maximization of the detectability index (20) which depends on the same parameter vector by using any genetic scheme.

6. ILLUSTRATIVE EXAMPLE

In this section, we present a numerical example that allows to verify the reliability of the proposed method for robust detection and isolation problem. As clearly comes out from the nature of the approach, we can also apply our technique to discrete time-invariant systems, therefore we shall consider an example taken from (Shen and Hsu, 1998), which represents the mathematical model of an automotive engine. The nominal system matrices are obtained by using standard least-squares error technique:

$$A = \begin{bmatrix} -0.0960 & -0.1306 & -0.1910 & -4.6833 & 0.7491 \\ 1 & 0 & 0 & 0 & 0 \\ 0.0885 & 0.2358 & 0.9911 & 0 & -29328 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 26.3821 & -0.1055 \\ 0 & 0 \\ 0 & -0.4518 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The unknown input distribution matrix E is defined as

$$E = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Finally, the fault entry matrix for actuator fault diagnosis in this case is simply defined as

$$R = B.$$

The weighting matrix Q to satisfy $QCE = 0$ can be easily found as

$$Q = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Let $\Lambda = \{0.1, 0.2, 0.5, -0.25, -0.15\}$ be the closed-loop spectrum, which is divided into two subsets $\Lambda_p = \{0.1, 0.2, 0.5, -0.25\}$ and $\Lambda_{n-p} = \{-0.15\}$. The vectors which parameterize the bases of the characteristic subspaces, are defined as $\theta_i(1) = [\theta_{i1}(1)\ 1]^T,\ i = 1, \ldots, 4$.

Now, let us impose that the column of the disturbance matrix E is a right eigenvector of $A+KC$: in particular, we choose the eigenvector corresponding to the eigenvalue $\lambda_4 \in \Lambda_p$ is chosen. Therefore, the parameterized closed-loop eigenvectors corresponding to the elements of the subset Λ_p are represented as

$$v_1 = C(0.1I - A)^{-1}\theta_1(1)$$
$$v_2 = C(0.2I - A)^{-1}\theta_2(1)$$
$$v_3 = C(0.5I - A)^{-1}\theta_3(1)$$

Hence, the free parameter Θ_r coming from the right closed-loop eigenvectors is

$$\Theta_r = \{\theta_{11}, \ldots, \theta_{14}, \ldots, \theta_{31}, \ldots, \theta_{34}\}.$$

In order to compute the gain matrix K, which assigns the desired closed-loop spectrum, and to achieve diagonal fault isolation, we simultaneously have solved some equations of sets (8) and (9) inside the assignment procedure (*Step 5* and *Step 6*) , where the parameter $\theta_{11}(1)$ is used to impose the remaining closed-loop eigenvalues of Λ_{n-p}:

$$Hv_3 = 0,$$
$$\mathrm{row}(H)_1 v_2 = 0,$$
$$\mathrm{row}(H)_2 v_1 = 0.$$

Thse equations are parameterized by the following subset of Θ_r: $\{\theta_{12}(1), \theta_{21}(1), \theta_{31}(1), \theta_{32}(1)\}$, so that, the gain matrix K is calculated as

$$K = \begin{bmatrix} -0.154 & -2.21111 & 4.65164 & -2.03748 \\ -1 & 0.714172 & 0.00831456 & 0.266679 \\ -0.0885 & -0.327467 & -0.00211946 & 3.30927 \\ 0 & 0.7136327 & 0.2037032 & 0.417336 \\ 0 & -0.713633 & -0.00370321 & -0.217336 \end{bmatrix}. \quad (22)$$

Now, we can construct parametric expressions of left and right closed-loop eigenvectors by formula (12). Let $\psi_i,\ i = 1, \ldots, n$ the left closed-loop eigenvectors of $A + KC$:

$$\psi_1 = \sum_{k=0}^{n-1} \sum_{s=0}^{n-1-k} (-1)^{s+k} \chi(n, n-1-k-s) \lambda_1^k ((A+KC)^T)^s\, h^T,$$
$$\psi_2 = \sum_{k=0}^{n-1} \sum_{s=0}^{n-1-k} (-1)^{s+k} \chi(n, n-1-k-s) \lambda_2^k ((A+KC)^T)^s\, h^T,$$
$$\psi_5 = \sum_{k=0}^{n-1} \sum_{s=0}^{n-1-k} (-1)^{s+k} \chi(n, n-1-k-s) \lambda_5^k ((A+KC)^T)^s\, h^T,$$

$$v_5 = \sum_{k=0}^{n-1} \sum_{s=0}^{n-1-k} (-1)^{s+k} \chi(n, n-1-k-s) \lambda_5^k (A+KC)^s \, h^T,$$

where $h = [h_{11}, h_{21}, h_{31}, h_{41},\ 1]$ and $\Theta_l = \{h_{11}, h_{21}, h_{31}, h_{41}\}$.

Hence, the following equations of set (10) and the further equation $Hv_5 = 0$,

$$\begin{aligned} \text{row}(H)_1 \;\; v_1 \;\; \psi_1^T \;\; \text{col}(R)_2 &= 0, \\ \text{row}(H)_2 \;\; v_2 \;\; \psi_2^T \;\; \text{col}(R)_1 &= 0, \\ Hv_5 &= 0, \end{aligned}$$

are solved with respect to the elements of Θ_l. Consequently the closed-loop right and left eigenvectors matrices are

$$V = \begin{bmatrix} 0.403659 & 0.608021 & 0.481425 & 1.09542 & -1.09542 \\ 26.4605 & -8.29576 & 7.61914 & -609.613 & -7.61914 \\ -0.455945 & 0.246734 & 0.270656 & 0.270656 & -0.270656 \\ 1 & 0 & 0 & 0 & 0 \\ 0.581668 & -0.129742 & -0.160659 & -0.160659 & 0.160659 \end{bmatrix},$$

$$L^T = \begin{bmatrix} -0.003476 & -0.013213 & -0.002292 & 0.031565 & -0.001402 \\ -0.001810 & -0.026421 & -0.008845 & 0.047777 & -0.012029 \\ 0.004804 & -0.086071 & -0.062707 & 0.117166 & -0.078080 \\ -0.015997 & 0.007474 & 0.016703 & -0.015121 & 0.025581 \\ -0.009992 & 0.001658 & 0.003700 & 0.003581 & 0.011239 \end{bmatrix}.$$

Hence, the transfer matrix from the faults to residuals is

$$G_{rf}(s) = \begin{bmatrix} \dfrac{0.036938}{s-0.1} & 0 \\ 0 & \dfrac{4.8404}{s-0.2} \end{bmatrix}. \qquad (23)$$

Finally, it is worth to note that the above numerical results depend on the fact that some free parameters of Θ_r ($\Theta_r - \{\theta_{12}(1), \theta_{12}(1), \theta_{21}(1),$ $\theta_{31}(1), \theta_{32}(1)\}$) has been fixed to particular values. As a consequence by picking other values in the real parameter space, we can obtain different realization for the gain matrix K, the closed-loop right eigenvector (V) and the closed-loop left eigenvector (L^T) respectively.

7. CONCLUSIONS

A geometric approach to the FDI problem by means of right/left eigenstructure assignment has been here proposed. The method gives a way to explicitly represent the structure of the observer matrices and a simple computational procedure to achieve such result has been listed using a right eigenstructure assignment procedure. The results from a numerical experiment taken from literature are encouraging shown the strenghtness of the geometric paradigm. A non secondary observation to the experiments is that exactly the same results are obtainable by duality, if a left-eigenvector assignment procedure is used.

ACKNOWLEDGEMENTS

This work has been supported by MURST Project *Fault Detection and Diagnosis, Supervision and Control Reconfiguration in Industrial Process Automation*

8. REFERENCES

Patton, R. J. and Chen, J. (2000). On eigenstructure for robust fault diagnosis. *International Journal of Robust and Nonlinear Control* **10**, 1193–1208.

Frank, P. M. (1996). Analytical and quantitive model based fault diagnosis - a survey and some new results. *European Journal of Control* **2**, 6–28.

Commault, C. (1999). On the disturbed fault detection and isolation problem. *Systems & Control Letters* **38**, 73–78.

Saberi, A., Stoorvogel, A. A., Sannuti, P. and Niemann, H. (2000). Fundamental problems in fault detection and identification. *International Journal of Robust and Nonlinear Control* **10**, 1209–1236.

Massoumnia, M. A., Verghese, G. C. and Willsky, A. S. (1989). Failure Detection and Identification. *IEEE Trans Auto. Control* **34**, 316–321.

Shen, L. C. and Hsu, P. L. (1998). Robust Design of Fault Isolation Observers. *Automatica* **34**, 1421–1429.

Choi, J. W., Lee, J. G., Kim, Y. and Kang, T. (1995). Design of an effective controller via disturbance accomodating left eigenstructure assignment. *AIAA Journal of Guidance, Control & Dynamics* **18**, 347–354.

Carotenuto, L., Franzé, G. and Muraca, P. (2001). A new computational procedure to the pole placement problem by static output feedback. *IEE Proccedings Control Theory and Applications* **148**, 466–471.

Kailath, T. (1980). *Linear Systems.* Prentice-Hall Inc.

Chen, J. and Patton, R. J. (1999). *Robust Model-based fault diagnosis for dynamic systems* . Kluwer Academic Publisher: Dordrecht, The Nederlands.

Goldberg, D.E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning.* Addison-Wesley.