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REFERENCE POINT AND FUZZY APPROACHES FOR DECISION SUPPORT IN MULTIOBJECTIVE PROGRAMMING

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Abstract: The study of the interactions between the energy sector, its effects on the environment and the corresponding impacts on a national economy must explicitly address multiple, conflicting and incommensurate aspects of evaluation. Multiple objective programming models enable the decision makers to rationalize the comparisons among distinct alternative solutions, providing them with a better perception of the conflicting aspects under evaluation and the ability to grasp the tradeoffs to be made. Reference point approaches provide a framework to aid decision makers to search for "satisfactory" efficient solutions. Moreover, it is possible to interpret the degree of satisfaction with the values of the objective functions by means of fuzzy membership functions. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The energy sector is of outstanding importance to the analysis of an economy on a national level, because of direct and indirect consequences on several well-being indicators ranging from economical aspects to social and environmental ones. In countries where the primary energy resources are scarce, it is even more important to provide decision makers (DMs) with well-founded information concerning the study of the interactions and tradeoffs between the energy sector, its effects on the environment and the corresponding impacts on the economic system.

However, models aimed at this purpose must explicitly address multiple, conflicting and incommensurate aspects of evaluation. These are generally operationalized in mathematical models by means of objective functions expressing aspects of distinct nature such as economical, social, environmental, technical, etc. In single objective models these evaluation aspects are generally encompassed by an aggregate economic indicator. Therefore, it is not possible to identify the tradeoffs between them which are a crucial information to evaluate the merit of alternative solutions.

Multiobjective mathematical models become more adequate to provide decision support in actual decision situations by enabling the DMs to rationalize the comparisons among distinct alternative courses of action, providing them with a better perception of the conflicting aspects under evaluation and the ability to grasp the nature of tradeoffs to be made. Since the objective functions are generally in conflict, there is not a prominent solution that optimizes all the objective functions simultaneously. The concept of optimal solution to a single objective problem gives thus place, in a multiple objective context, to the concept of efficient solutions: feasible solutions for which no improvement in any objective function is possible without sacrificing on at least one of the other objective functions.

Several approaches exist to compute efficient solutions. These processes are generally called scalarizing processes because they involve the resolution of a scalar optimization problem in a way that the optimal solution to this problem is an efficient solution to the multiobjective problem. Reference point-based approaches provide an appealing framework, both from theoretical and cognitive perspectives, to aid the decision maker to strive for "satisfactory" efficient solutions. Furthermore, it enables to interpret the degree of satisfaction with the values of the objective functions by means of fuzzy membership functions.

The multiobjective model based on input-output analysis to study economy-energy-environment interactions is briefly described in section 2. In section 3 the main ideas regarding the reference point approach are presented. The use of aspiration and reservation levels in the framework of fuzzy analysis is discussed in section 4. Some illustrative results obtained by the application of this methodological approach to the multiobjective linear programming (MOLP) model are presented in section 5.

2. A MULTIOBJECTIVE MODEL

A MOLP model is developed by using input-output analysis to express the flows of good and services within an economy. This model has been supplied with data from the Portuguese case.

The input-output approach considers 21 economic sectors and 23 artificial sectors (used for distributing the output of the oil refining sector and the by-products through the consuming sectors). It consists of: a (44x44) matrix which represents the inter- and intra-sector flows, 6 column vectors with the components of final demand (private consumption, collective consumption, gross fixed capital formation, positive and negative stock changes and exports), 1 column vector for the competitive imports (imports which have endogenous equivalent) and 3 row vectors for the primary inputs (wages, net indirect taxes, operating surplus). The sectors are classified as follows: energy sectors (coal, oil, electricity, city gas and self-production of electricity), hydrocarbons (crude oil, shale oil, propylene, LPG, gasoline, petroleum, jets, diesel oil, fuel oil, naphtha, lubricants, bitumen, paraffin, solvents and petroleum coke), by-products used in self-production of electricity (incondensable gases, hydrogen, black liquors, other by-products, pitch, coke oven gas, coke gas and biogas) and industrial sectors (encompassing industry and services). The total output of each sector is represented by a decision variable. The technical coefficients matrix is obtained from the transactions table of a given year taken as the basis of the study. This matrix shows the relationships among the different sectors of the economy that are used to define coherence constraints of the MOLP model (namely imposing that the use of a specific good or service, for intermediate consumption and final demand, cannot exceed the resources available, resulting from national production and competitive imports).

The input-output approach (Leontieff, 1951) has been used to develop a table which structures the national economy in activity sectors and displays the economic flows between them, thus providing a systemic overview of all activities in the country and enabling to take into account their interactions. An input-output table disaggregates an economic system into a number of sectors, each of which producing a particular type of output, with the output structure assumed to be fixed, and no substitution between the outputs of the different sectors. The aim of this study is to model the interactions between the economy and the energy sector on a national level. The amount of primary energy to produce a good or service (either as an input for other sectors or for final demand) is computed. The use of fossil fuels can be then associated with the activity level of each sector to compute the resulting amount of emissions of atmospheric pollutants. The top-down methodology of IPCC (IPCC, 1996), which is based on the principles of combustion and composition of fuels, has been used to model CO_2 emissions.

The objective functions are: - the maximization of employment (as a surrogate for social well-being); - the minimization of energy imports (taking into account the energy dependence of the country); - the maximization of Gross Domestic Product (GDP) (as a measure of the performance of the national economy); - the minimization of carbon dioxide (CO_2) emissions (due to the impact of energy resources on the environment, namely regarding air pollution). Several sets of constraints are considered related to production capacity, bounds on imports and exports, public deficit (according to European Union requirements), balance of payments (imposing a given level of external equilibrium), gross added value, self-production of electricity (upper and lower bounds on the use of alternative forms of energy, aimed at encouraging the recycle of wastes, energy economies and the minimization of waste disposal), storage capacity and security stocks for hydrocarbons.

Further details on the mathematical model can be seen in (Oliveira and Antunes, 2000) and (Antunes *et al.*, 2002).

3. REFERENCE POINT APPROACHES

Reference point approaches can be considered as generalized goal programming. Goal programming is a well established optimization model based on the concept of setting a goal in the objective space and compute the feasible solution closest (in a certain sense) to it. The goal can be attainable or not. A distance minimization is underlying. However, as Wierzbicki (1983; 2000) points out this is mathematically inconsistent with the concept of vector-optimality or efficiency, for a function to produce a vector-optimal outcome a requirement is its monotonicity. That is, the distance function is not monotone when its arguments cross zero.

The reference point can be formed by asking the DM to specify his/her aspiration levels regarding each objective function, that is the (preferably ambitious) levels he/she would like to attain in each aspect of evaluation. Setting these aspiration levels in the objective space and proceeding to come as close as possible to them is quite appealing from the perspective of the cognitive effort required from the DM. However, whenever the aspiration levels are all simultaneously attainable the DM can be confronted with non-efficient decisions (because the reference point is itself non-efficient). In this case an improved solution compared to it must be obtained. In the framework of reference point methodologies the aspiration levels must be improved, not just reached, whenever they are attainable (that is, better in all evaluation aspects can be achieved). Using the terminology of Wierzbicki (1983; 2000) this leads to a "quasi-satisficing" decision which is obtained by optimizing a so-called achievement function, departing from Simon's "satisficing" decision. Those order-consistent achievement functions are similar but not equivalent to distance functions.

Reservation levels can also be specified as the worst values the decision maker is willing to accept (in a final solution) for each objective function. Both aspiration levels and reservation levels are used as "soft constraints", in the sense that they are not definitively incorporated in the problem formulation and they are not rigid since they can be revised in subsequent interactions with the decision maker as more knowledge about the problem as well as his/her own preferences is gathered.

4. FUZZY ANALYSIS IN MOLP

The MOLP problem with p objective functions and m constraints is generally stated as

$$\text{“max” } \mathbf{z} = C\,\mathbf{x} \qquad (1)$$
$$\text{s. t.}$$
$$\mathbf{x} \in X = \{\mathbf{x} \in R^n\text{: } A\mathbf{x} \le \mathbf{b},\ \mathbf{x} \ge \mathbf{0}\}$$

where A is a mxn matrix, **b** is the m right-hand side (RHS) column vector and C is a pxn matrix of objective functions coefficients. "max" denotes the operation of computing efficient solutions.

In a fuzzy environment a great diversity of possible modifications to (1) has been proposed.

The mathematical relations involved may be fuzzy (fuzzy objectives and/or constraints). The DM may not be interested in optimizing some of the objective functions; rather he/she might want to "improve" as much as possible their values in order to reach some "aspiration levels" which may not be crisply defined. The constraints may also be fuzzy, that is the '$\le$' sign might not be met in the strictly mathematical sense but the DM may accept small violations on it.

The coefficients of the vector **b** or the matrices C or A can also have a fuzzy character either because they are fuzzy in nature or their perception is fuzzy.

Moreover, the solution of a fuzzy linear programming problem may be crisp or fuzzy. In the latter case a solution set (of all fuzzy efficient solutions) is presented to the DM and he/she must select his/her preferred one according to his/her preferences (which can be explicitly or implicitly modelled by an analytical construct such as a value or utility function).

Different ways to deal with these types of fuzziness on LP models are widely reported in the literature. The Zimmermann's symmetrical approach (Zimmermann, 1983; 1992) is generally used when the objectives and/or some constraints are fuzzy.

4.1 The Zimmermann's symmetrical approach.

We shall assume that, regarding model (1), the DM is able to establish a p-vector of aspiration levels, $\mathbf{Z_0}$, for the objective functions and that '$\lesssim$' means something as "essentially smaller than or equal" (that is, fuzzy relations are at stake).

The obtained model is fully symmetrical with respect to objective functions and constraints. This can be made clearer by using the following substitutions $A' = [-C\ A]^T$ and $\mathbf{b}' = [-\mathbf{Z_0}\ \mathbf{b}]^T$. Model (1) becomes:

$$\text{Find } \mathbf{x} \qquad (2)$$
$$\text{s. t.}$$
$$\mathbf{x} \in X' = \{\mathbf{x} \in R^n\text{: } A'\mathbf{x} \lesssim \mathbf{b}',\ \mathbf{x} \ge \mathbf{0}\}$$

Each of the (m+p) rows of (2) shall now be represented by fuzzy sets, each one defined by the membership function $\mu_i(\mathbf{x})$. Considering the Bellman and Zadeh (1970) decision model the membership function of the fuzzy set decision to (2) is

$$\mu_{\tilde{D}}(\mathbf{x}) = \min\ \{\mu_i(\mathbf{x})\} \qquad (3)$$

If the DM is only interested in a crisp optimal solution then

$$\max\ [\min\ \{\mu_i(\mathbf{x})\}] = \max\{\mu_{\tilde{D}}(\mathbf{x})\} \qquad (4)$$

$\mu_i(\mathbf{x})$ should be '0' if the constraints (including objectives) are strongly violated, '1' if they are very well satisfied (i. e., satisfied in the crisp sense) and $\mu_i(\mathbf{x})$ should increase monotonously from '0' to '1'.

Problem (4) is transformed into

$$\max\ \lambda \qquad (5)$$
$$\text{s. t.}$$
$$\lambda \le \mu_i(\mathbf{x})$$
$$\mathbf{x} \ge \mathbf{0},\ \lambda \in [0, 1],\ i=1, 2, \dots m+p.$$

$\lambda = \max[\min\ \{\mu_i(\mathbf{x})\}]$ and can be interpreted as the degree to which **x** fulfils (satisfies) the fuzzy inequality $(A'\mathbf{x})_i \le b'_i$ (associated with the ith row).

The resolution of this problem leads to an efficient solution to the original MOLP (1), and even in cases where multiple optimal solutions to (5) exist at least one is strictly efficient.

The complexity of problem (5) is associated with the considered membership function $\mu_i(\mathbf{x})$. The simplest type of membership function is linearly increasing over the tolerance interval p_i (fig. 1), and the following linear programming is obtained

$$\max \lambda \quad (6)$$
$$\text{s. t.}$$
$$\lambda p_i - (A'\mathbf{x})_i \leq b'_i - p_i$$
$$\mathbf{x} \geq \mathbf{0},\ \lambda \in [0, 1],\ i=1, 2, \ldots m+p.$$

If the ith constraint is of type $\tilde{\geq}$ then $\lambda \leq \mu_i(\mathbf{x})$ is converted in $\lambda p_i + (A'\mathbf{x})_i \leq b'_i + p_i$. If it is of type $\tilde{=}$ then both relations must be considered. If for some objectives the DM can specify precisely the aspiration levels those could be grouped with the constraints that remain crisp in X.

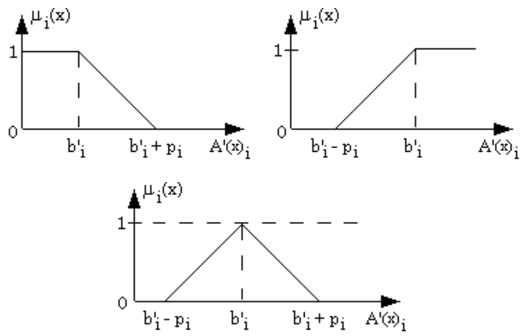

Fig. 1. Linear membership functions.

4.2 Reference point versus symmetrical approaches.

Werners (1987) drew attention to the fact that problem (5) mostly resembles goal programming with a special norm. In (5), the nearness of the objective values to the aspiration levels, defined by the DM, is maximized. This is done by considering a new decision variable to be maximized with an upper bound ($\lambda \leq 1$).

For the sake of illustration, let us consider the example in fig. 2 (Zimmermann, 1983) whose solution P_F is characterized in table 1. If the DM considers the values that maximize each objective (point P_I) as aspiration levels and the differences from these to the worst objective values over the efficient region as tolerance values (that is, the worst objective values over the efficient region be seen as reservation levels), then the solution to (6) (point P_F) is the one that minimizes the distance between the objective values and P_I. Moreover, if it is considered that the distance from P' to P_I is 1, then the distance from P' to P_F is λ =0.742 and from P_F to P_I is (1-λ) =(1-0.742)=0.258.

For problems with two membership functions the slacks of the corresponding constraints in (5) are always zero, that is, they are strictly satisfied. Point P_F is associated with the same value for both $\mu_i(\mathbf{x})$. If the number of membership functions is greater, then some of the slacks associated with the membership functions' constraints in (5) may be non-zero for a given solution, meaning that the corresponding satisfaction degree value is higher than the overall satisfaction degree.

Table 1 Illustrative example solutions

	P_F (Fig. 2)	P_G (Fig. 3)
$f_1(\mathbf{x})_{Asp.}$	14.000	7.000
$f_2(\mathbf{x})_{Asp.}$	21.000	13.000
$f_1(\mathbf{x})_{Res.}$	-3.000	-3.000
$f_2(\mathbf{x})_{Res.}$	7.000	7.000
λ	0.742	1.000
$f_1(\mathbf{x})$	9.613	11.375
$f_2(\mathbf{x})$	17.387	15.625
slack $\mu_1(\mathbf{x})$	0.000	0.000
slack $\mu_2(\mathbf{x})$	0.000	0.000

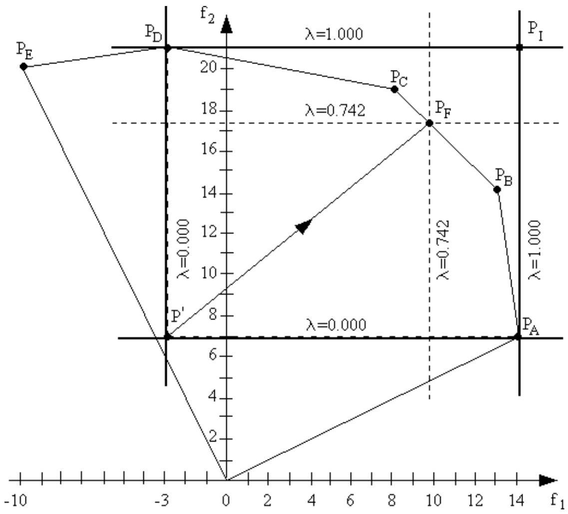

Fig. 2. Illustrative example (non-attainable aspiration levels).

Fig. 3 displays the previous example but considering attainable aspiration levels.

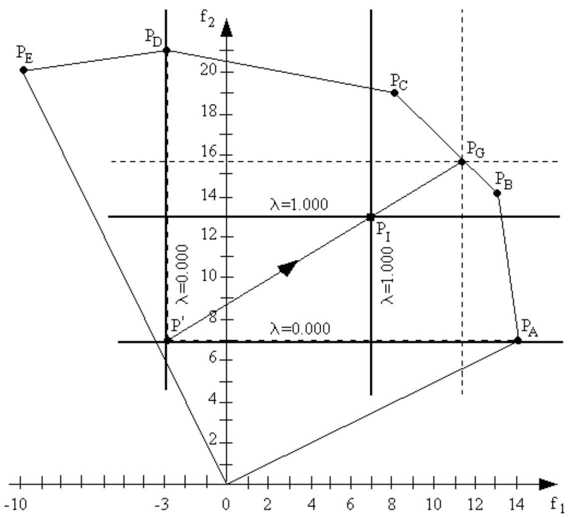

Fig. 3. Illustrative example (attainable aspiration levels).

Using Zimmermann's approach the obtained solution is one of the alternative optima to problem (6) with the overall satisfaction degree λ=1.0 but that is situated on the efficient region to the original MOLP problem (1).

Based on the membership functions that the DM has specified, an increasing direction for λ (removing its upper bound) can be defined, using a similar idea as in the reference point approach (see figs. 2 and 3). If the obtained solution is associated with a λ>1 (as in the case of fig. 3), then all the fuzzy constraints are completely satisfied and λ=1. In this case, the decision aid system shall propose an improved solution to the DM that is efficient to problem (1), such as point P_G.

This procedure considering the modified fuzzy membership functions (associated to each objective function/constraint) can be interpreted similarly to the one presented by Wierzbicki (2000), in the framework of reference point methodologies considering piece-wise linear partial achievement functions σ_i (monotone and concave), where the slope is always the same (fig. 4).

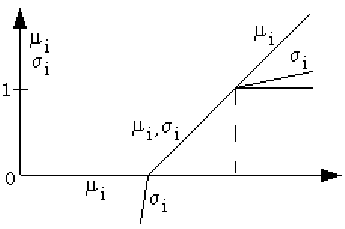

Fig. 4. Similarities between the modified fuzzy membership function and the partial achievement function (for a maximizing objective function).

Considering the decision process as a learning process about the problem as well as about the DM's preference structure, an interactive analysis supported by the approach described above can be carried out. The preferential model is specified by means of aspiration and reservation levels, which initially can be the optimal values for each objective function and the worst values over the efficient region, respectively. Thereafter, the approach is flexible in order to interactively incorporate the DM's preferences changes. The DM might modify his/her preferences by specifying which objective value(s) should be improved/worsened in order to analyze different courses of action. Moreover, it is not required that the DM's preferences should be consistent through out the study.

5. AN ECONOMY- ENERGY-ENVIRONMENT MODEL - SOME ILLUSTRATIVE RESULTS

In this section some illustrative results are presented by using the methodological approach described above to provide decision support in the study of economy- energy-environment interactions. The data supplied to the model have been collected from several sources such as INE (Portuguese National Statistics Institute), DGE (General Directorate for Energy) and IPCC.

Let us suppose that the efficient solutions that individually optimize each objective function are computed. This enables to have a first overview of the characteristics of well dispersed solutions and the range of the objective function values over the efficient solution set (table 2).

Let us consider that, without having further information, the DM accepts the optimal values for each objective function as the initial aspiration levels. The worst values in each column of table 2 are taken as reservation levels, which are not necessarily the worst values over the efficient region. Therefore, the initial tolerance values are p_i=$best_i$-$worst_i$.

The Zimmermann's solution presented in the second column of table 3 is computed. Owing to the fact that the slacks associated with all $\mu_i(\mathbf{x})$ are zero, the corresponding membership function values are the overall satisfaction degree, 0.768.

From herein the DM can change the aspiration/reservation levels in order to improve and/or worsen some of the objective function values.

For instance, if the DM is willing to relax the value of Employment (=5 629 579) then he/she could decrease the corresponding reservation level or aspiration level (and improve the overall satisfaction degree). The second computed solution (third column of table 3) has been obtained considering that the aspiration level associated with Employment is decreased by 1% of (the initial) p_i. All the membership function values equal the overall satisfaction degree of 0.770, meaning that the aspiration level associated with Employment can be decreased and different efficient solutions with lower Employment values can be determined. The solution in the fourth column has been computed considering the initial aspiration level associated with Employment decreased by 5% of (the initial) p_i. As the slack associated with $\mu_{Employment}$ for this solution is not zero the corresponding membership function has higher value than the overall satisfaction degree of 0.771. If a lower aspiration level for Employment is considered no further changes of the obtained solution occur.

If the DM also accepts a higher value for CO_2 emissions then he/she could increase the corresponding reservation level or the corresponding aspiration level. This leads to the solution in the fifth column of table 3, where the CO_2 emissions aspiration level has been increased by 10% of (the initial) p_i. The slacks associated with $\mu_{Employment}$ and slack $\mu_{CO_2\ emissions}$ are not zero, meaning that the corresponding membership functions have higher values than the overall satisfaction degree.

If the DM wants to improve GDP he/she can, for example, increase the corresponding aspiration level as in the fifth computed solution where it is improved by 20% of (the initial) p_i. Note that, in this solution (column six of table 3), the Employment value has also improved.

Table 2 Efficient solutions that individually optimize each objective function.

	Employment (#)	Energy imports (toes)	GDP (PTE)	CO_2 emissions (Gg)
Max Employment	**5 735 829**	22 150 792	19 996 334	***63 260***
Min Energy imports	5 285 951	**19 315 123**	18 476 526	58 433
Max GDP	5 727 488	***22 241 143***	**20 023 324**	63 220
Min CO_2 emissions	***5 277 375***	21 039 177	***18 470 058***	**58 402**
Initial p_i = $best_i$-$worst_i$	458 454	2 926 020	1 553 266	4 858

Table 3 Interactive analysis of the Economy- Energy-Environment planning model.

	Symmetrical Solution	Loss in Employment (1)	Loss in Employment (2)	Loss in CO_2 emissions	Improvement in GDP
$Employment_{Asp.}$	5 735 829	5 731 244	5 712 906	5 712 906	5 712 906
$Energy\ imports_{Asp.}$	19 315 123	19 315 123	19 315 123	19 315 123	19 315 123
$GDP_{Asp.}$	20 023 324	20 023 324	20 023 324	20 023 324	20 178 651
$CO_2\ emissions_{Asp.}$	58 402	58 402	58 402	59 374	59 374
$Employment_{Res.}$	5 277 375	5 277 375	5 277 375	5 277 375	5 277 375
$Energy\ imports_{Res.}$	22 150 792	22 150 792	22 150 792	22 150 792	22 150 792
$GDP_{Res.}$	18 470 058	18 470 058	18 470 058	18 470 058	18 470 058
$CO_2\ emissions_{Res.}$	63 260	63 260	63 260	63 260	63 260
λ	0.768	0.770	0.771	0.851	0.790
Employment	5 629 579	5 627 061	5 626 674	5 662 485	5 670 759
Energy imports	19 993 245	19 986 775	19 985 781	19 752 654	19 928 485
GDP	19 663 346	19 666 780	19 667 308	19 791 063	19 820 490
CO_2 emissions	59 528	59 518	59 516	59 723	60 189
slack $\mu_{Employment}$	0	0	135 938	147 044	491 509
slack $\mu_{Energy\ imports}$	0	0	0	0	0
slack μ_{GDP}	0	0	0	0	0
slack $\mu_{CO_2\ emissions}$	0	0	0	232 447	0

The study could proceed in the same manner until the DM considers to have gathered sufficient information to make a final decision. Although in this example only the aspiration levels of $\mu_i(\mathbf{x})$ have been changed, a similar study can be performed considering different reservation levels.

6. CONCLUSION

It has been shown how reference point approaches provide a decision aid framework to support the computation of efficient solutions adapted to the evolutionary DM's preferences. The degree of satisfaction with the objective function values can be interpreted as fuzzy membership functions.

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