15th Triennial World Congress, Barcelona, Spain

ROBUST OFFSET-FREE MODEL PREDICTIVE CONTROL

Gabriele Pannocchia *

* *Department of Chemical Engineering – University of Pisa*
Via Diotisalvi 2, 56100 Pisa (Italy)
E-mail: pannocchia@ing.unipi.it

Abstract: In this paper a method for designing robust offset-free MPC controllers for (possibly) nonzero targets is presented. The proposed controller is guaranteed to track the controlled variable to its target for any plant that lies in a polytopic region. First, an off-line design of a robust unconstrained offset-free controller is accomplished, and a corresponding invariant region is computed in which this controller is well defined and does not violate the constraints. Next, the online implementation requires the use of this unconstrained controller if the system state is in the invariant region or, if not, the solution of a min-max finite horizon optimization problem. An illustrative example is presented. *Copyright © 2002 IFAC*

Keywords: robust control, model based control, uncertain linear systems, minimax techniques, invariance

1. INTRODUCTION

Several formulations of robust model predictive control (RMPC) have been proposed during the last decade, starting from algorithms for systems described by finite impulse response (FIR) (Genceli and Nikolaou, 1993; Zheng and Morari, 1993; De Nicolao *et al.*, 1996*b*; Ralhan and Badgwell, 2000). Kassmann *et al.* (2000) propose a robust target calculation method to enhance the stability of QDMC. In the more general framework of state-space models, De Nicolao *et al.* (1996*a*) propose a robust controller with nominal tracking properties. Kothare *et al.* (1996) use a polytopic description of uncertain systems and propose an LMI based control strategy. Kouvarithakis *et al.* (2000) generate, off line, an ellipsoid invariant region and then compute on-line the free control moves to reach this invariant set ("dual-mode" paradigm). Also, Lee and Kouvaritakis (2000) reduce the computational complexity through the use of a linear programming approach.

One disadvantage of these current state-space formulations of RMPC is that they are able to control uncertain systems without steady-state offset only if the state target is the origin. Consider, for instance, a pure gain scalar system described by $x_{k+1} = bu_k$ in which b is an unknown constant scalar $\in [1, 2]$. If the state target is the origin, the corresponding input target is the origin, as well, regardless of the actual value of b in the uncertain region. Thus, if one uses a stabilizing gain, the state is driven to zero regardless of b. Conversely, if the state target is 1, the input target varies from 1 to 0.5, and the application of a stabilizing control law would lead to steady-state offset. Kothare *et al.* (1996, Section 4.2) propose to address this problem by expressing the state and the input as deviation from their corresponding target. However, these targets can be computed only if the system is known, thus making unnecessary the use of a robust control algorithm.

In this paper, a method for controlling unknown systems with zero steady-state offset is proposed. The method is based on a dual-mode paradigm in which the inner unconstrained controller is designed in a way that it leads to zero steady-state offset. An appropriate invariant region is defined, in which the controller does not violate the process constraints. The basic idea and many theoretical details arise from the feedback min-max approach proposed by Scokaert and Mayne (1998), in which the problem of controlling perfectly known systems in the presence of unknown bounded disturbances was addressed.

2. CONTROL ALGORITHM

We consider linear systems described by

$$\begin{aligned} x_{k+1} &= A(k)x_k + B(k)u_k, \\ x &\in \mathbb{R}^n, u \in \mathbb{R}^m, [A(k), B(k)] \in \Omega, \\ \Omega &= \mathrm{Co}\left\{[A_1, B_1], \ldots, [A_M, B_M]\right\}, \end{aligned} \tag{1}$$

in which $\mathrm{Co}\{\cdot\}$ denotes the convex hull (see *e.g.* (Kothare *et al.*, 1996)). This description includes linear time-varying systems (LTV) and uncertain linear time-invariant systems (ULTI). It is assumed that, at each sampling time, the state of the system x_k is measured. The controlled variable is defined as a known linear time-invariant combination of the state:

$$z = Hx, \qquad z \in \mathbb{R}^p, \tag{2}$$

in which $p \leq \min\{n, m\}$.

Assumption 1. For any $[A, B] \in \Omega$, we assume that (A, B) is stabilizable and (A, H) is detectable.

Let $\bar{z} \in \mathbb{R}^p$ be the desired target for the controlled variable. The objectives of the controller are:

(1) to stabilize any plant in Ω,
(2) to track without offset the controlled variable when the system is ULTI,

while respecting the process constraints. For ease, only input constraints are considered:

$$Du_k \leq e, \qquad e \in \mathbb{R}^q,\ e_i > 0\ . \tag{3}$$

We propose a robust dual-mode control law composed by an "outer" and an "inner" controller. The inner controller is associated with a robust invariant set and it is used whenever the system state enters this set. This controller is designed in a way that it drives the controlled variable to its target if the system is ULTI. The outer controller is found by solving, on line, a min-max optimization problem.

2.1 *Unconstrained controller and invariant set*

Let $[A_m, B_m] \in \Omega$ denote the state-space matrices of a particular model referred to as "nominal model". The "inner" controller is defined as an unconstrained MPC regulator based on the nominal model, designed for offset-free control. In MPC offset-free control is obtained by adding integrating disturbances to the process model. Rawlings *et al.* (1994) show that, for square non-integrating systems, an output disturbance guarantees offset-free performance. Recently, Pannocchia and Rawlings (2001*a*) presented results for generic linear systems. In particular, it is shown that a number of additional disturbances equal to the number of measurements is sufficient to guarantee absence of offset, provided that the closed-loop system is stable. Here, we use the same procedure and we choose the nominal model and the disturbance model in a way that the closed-loop system is stable for any plant in Ω, thus obtaining offset-free performance.

Let $\hat{x} \in \mathbb{R}^n$ and $\hat{d} \in \mathbb{R}^n$ be the state and the disturbance of the following augmented system:

$$\begin{aligned} \hat{x}_{k+1} &= A_m\hat{x}_k + B_m u_k + B_d\hat{d}_k \\ \hat{d}_{k+1} &= \hat{d}_k \\ x_k &= \hat{x}_k + C_d\hat{d}_k, \end{aligned} \tag{4}$$

in which B_d and C_d are matrices of appropriate dimension that satisfy

$$\mathrm{rank}\begin{bmatrix} I - A_m & -B_d \\ I & C_d \end{bmatrix} = 2n\ . \tag{5}$$

The state and the disturbance are computed, at each sampling time, from the plant state x by using a deterministic steady-state Kalman filter designed for (4), that is

$$\begin{aligned} \hat{x}_{k|k} &= \hat{x}_{k|k-1} + L_x(x_k - \hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \\ \hat{d}_{k|k} &= \hat{d}_{k|k-1} + L_d(x_k - \hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \end{aligned} \tag{6}$$

in which the common double-index filtering notation is used. Condition (5) implies that the augmented system (4) is detectable (Pannocchia and Rawlings, 2001*a*) and, therefore, such filter matrices L_x and L_d exist. The Kalman filter in (6) can also be written in the predictor form

$$\begin{aligned} \hat{x}_{k+1|k} &= A_m\hat{x}_{k|k-1} + B_m u_k + B_d\hat{d}_{k|k-1} \\ &\quad + L_1(x_k - \hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}) \\ \hat{d}_{k+1|k} &= \hat{d}_{k|k-1} \\ &\quad + L_2(x_k - \hat{x}_{k|k-1} - C_d\hat{d}_{k|k-1}), \end{aligned} \tag{7}$$

in which straightforward relations between (L_x, L_d) and (L_1, L_2) can be obtained.

Next, the controller is chosen as the solution of the following infinite horizon optimization problem:

$$\begin{aligned} \min_{u_k, u_{k+1}, \ldots} \Phi &= \sum_{j=k}^{\infty} (z_j - \bar{z})^T Q (z_j - \bar{z}) + \Delta u_j^T R \Delta u_j \\ &\text{s.t. the model (4)}, \Delta u_j = u_j - u_{j-1}, \end{aligned} \tag{8}$$

in which Q and R are symmetric positive definite matrices. This problem can be written as a standard LQ problem (see *e.g.* (Rao and Rawlings, 1999)) and its solution is the well-known feedback control law:

$$u_k = u_k^s + K_x(\hat{x}_{k|k} - x_k^s) + K_u(u_{k-1} - u_k^s) \quad (9)$$

in which K_x and K_u are computed from the corresponding Riccati equation, while x_k^s and u_k^s are obtained, at each sampling time, as the solution of the following target calculation problem (Muske and Rawlings, 1993):

$$\begin{aligned} &\min_{x_k^s, u_k^s} (u_k^s - \bar{u})^T R_s (u_k^s - \bar{u}) \\ &\text{s. t.} \\ &\quad x_k^s = A_m x_k^s + B_m u_k^s + B_d \hat{d}_{k|k} \\ &\quad \bar{z} = H(x_k^s + C_d \hat{d}_{k|k}), \end{aligned} \quad (10)$$

in which $\bar{u}$ is the desired input target and R_s is a symmetric positive definite matrix.

It is interesting to notice that the term u_{k-1} in the feedback law (9) is related to the presence of Δu terms in the objective function (8). Also, the target calculation (10) has a closed-form solution that is affine in $\hat{d}_{k|k}$. In fact, (10) is an equality-constrained convex quadratic program whose solution can be computed from the corresponding KKT system.

When the feedback control law (9) is applied to the plant (1), the closed-loop system has the following linear form (after some tedious algebraic calculations):

$$\begin{aligned} w_{k+1} &= \Lambda(k) w_k + \Gamma(k) v, \\ w_k &= \begin{bmatrix} x_k \\ \hat{x}_{k|k-1} \\ \hat{d}_{k|k-1} \\ u_{k-1} \end{bmatrix}, \quad v = \begin{bmatrix} \bar{z} \\ \bar{u} \end{bmatrix}, \end{aligned} \quad (11)$$

$$[\Lambda(k), \Gamma(k)] \in \Omega^* = \mathrm{Co}\left\{[\Lambda_1, \Gamma_1], \ldots, [\Lambda_M, \Gamma_M]\right\},$$

in which the matrices $[\Lambda_i, \Gamma_i]$ are not reported for the sake of space. We also have that the input is equal to

$$u_k = \Theta w_k + \Psi v, \quad (12)$$

in which Θ and Ψ are appropriate matrices (not reported). Notice that, once the matrices of the nominal model (4) are fixed and the Kalman filter (6) is designed, the matrices $[\Lambda_i, \Gamma_i]$, Θ and Ψ are well defined and known. Also, notice that the closed-loop system is stable if and only if any matrix Λ in the corresponding subspace of Ω^* is stable. Thus, the nominal model and the disturbance model need to be found a way that closed-loop stability is guaranteed for any plant in the uncertainty region.

This goal can be achieved by solving a min-max problem whose objective is the same as in (8) but the true values of z_j and Δu_j are used instead of the predicted ones. In this step, we consider ULTI systems for which a simple way to compute the true objective function based on the solution of a Lyapunov equation is available (Pannocchia and Rawlings, 2001*b*). Thus, the optimization problem is

$$\min_{\{A_m, B_m, B_d, C_d\}} \max_{\{A,B\}} \Phi \, . \quad (13)$$

The meaning of this optimization problem is to find a nominal model and a disturbance model that guarantee the minimum closed-loop objective function for the worst case of plant in Ω. Clearly, if the global maximum is found and the corresponding Φ is finite, the closed-loop system is stable for any possible plant in Ω. In practice this optimization problem is solved by using SQP methods and, since in general the problem is not convex, these methods cannot guarantee that a global maximum is found. However, once a "feasible" solution is returned by the optimizer, *i.e.* a nominal model and a disturbance model for which Φ is finite at the local maximum, a global closed-loop stability argument is obtained by constructing a robustly invariant set, as discussed below.

We use the theory of positively invariant sets (see *e.g.* (Blanchini, 1999) for a recent and extensive review), in order to find a region in which the unconstrained controller (9) does not violate the process constraints. In particular, the method proposed by Blanchini (1994) is applied to the unconstrained closed-loop system (11)-(12). Let

$$\mathcal{S} = \{w | D(\Theta w + \Psi v) \le e\} = \{w | Fw \le g\} \, . \quad (14)$$

Set $\mathcal{X}_0 = \mathcal{S}$, and consider the following sequence of sets:

$$\begin{aligned} \mathcal{X}_k &= \left\{ w | F^{(k)} w \le g^{(k)} \right\} \\ \mathcal{N}_k &= \left\{ w | F^{(k)}(\Lambda_j w + \Gamma_j v) \le g^{(k)}, j = 1, \ldots, M \right\} \\ \mathcal{X}_{k+1} &= \mathcal{N}_k \cap \mathcal{S} = \left\{ w | F^{(k+1)} w \le g^{(k+1)} \right\} \, . \end{aligned} \quad (15)$$

Blanchini (1994) show that the maximal invariant set contained in the feasible region $\mathcal{S}$ is given by

$$\mathcal{W} = \bigcap_{k=0}^{\infty} \mathcal{X}_k \, . \quad (16)$$

For discussions about the practical computation of such invariant set, we refer to the cited paper. Here we assume that an invariant region of the following form exists and has been computed:

$$\mathcal{W} = \{w | Gw \le h\} \, . \quad (17)$$

A number of comments are appropriate.

(1) The off-line design technique requires the solution of a min-max problem in order to select a nominal model and a disturbance model. After this choice has been made, an unconstrained feedback control policy is well defined.
(2) The existence of an invariant region $\mathcal{W}$, implies robust stability of the unconstrained closed-loop system for any LTV system in Ω. That is, any characteristic closed-loop matrix

Λ in the corresponding subspace of Ω^* is stable.

(3) Assume that the system is ULTI. If the initial closed-loop state is in the invariant set, it will reach a steady value. Using the results in (Pannocchia and Rawlings, 2001*a*) we have that if input and output reach a steady state, there is zero offset in the controlled variable, that is

$$\lim_{k\to\infty} z_k = Hx_k = \bar{z}\,. \qquad (18)$$

2.2 *Min-max MPC controller*

The "outer" controller is based on the solution of a min-max problem similar to the one proposed by Scokaert and Mayne (1998). Two interesting features of the algorithm by Scokaert and Mayne need to be noticed.

(1) Linearity of the system and convexity of the disturbance region are exploited in order to show that only "extreme realizations" of the disturbance need to be considered in the maximization.
(2) For each disturbance realization, a different control profile is computed (with the addition of a "causality constraint") and this renders the control law less likely to fail due to infeasible constraints.

Both these features can be included in the proposed algorithm. Indeed, we show that only extreme plant realizations need to be considered when maximizing. However, for simplicity of presentation we optimize over a single input profile. All the results that we present can be directly extended to the more general case of different control profiles for each plant realization.

At time k, let $\{A^\ell(k+j), B^\ell(k+j)\}_{j=0}^{N-1}$ denote the possible realizations of the plant (*i.e.* any sequence of plant matrices in Ω). Let $\pi(k) = \{u_{k|k}, \ldots, u_{k+N-1|k}\}$ be an input sequence. Let w_k be the current closed-loop augmented state as defined in (11). Combining (1) and (7) we can write the evolution of w_k over the l-th plant realization as

$$\begin{aligned} w^\ell_{k|k} &= w_k, \\ w^\ell_{k+j+1|k} &= \tilde{\Lambda}^\ell(k+j)w^\ell_{k+j|k} + \tilde{\Gamma}^\ell(k+j)u_{k+j|k}, \end{aligned} \qquad (19)$$

in which

$$\tilde{\Lambda}^\ell(k+j) = \begin{bmatrix} A^\ell(k+j) & 0 & 0 & 0 \\ L_1 & A_m - L_1 & B_d - L_1C_d & 0 \\ L_d & -L_d & I - L_dC_d & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}^\ell(k+j) = \begin{bmatrix} B^\ell(k+j) \\ B_m \\ 0 \\ I \end{bmatrix}.$$

Further, let $\mathcal{L}_v$ be the set of indexes ℓ, such that $\{A^\ell(k+j), B^\ell(k+j)\}_{j=0}^{N-1}$ take values only on the vertices of Ω. The following min-max problem is considered:

$$\begin{aligned} &\min_{\pi(k)} \max_{\ell\in\mathcal{L}_v} \sum_{j=0}^{N-1} L(w^\ell_{k+j|k}, u_{k+j|k}) \\ &\text{s.t. } (19),\ (3), \\ &\quad w^\ell_{k+N|k} \in \mathcal{W}, \end{aligned} \qquad (20)$$

in which the cost function is

$$L(w,u) = \begin{cases} 0 & \text{if } w \in \mathcal{W}, \\ (z-\bar{z})^TQ(z-\bar{z}) & \\ \quad +\Delta u^T R\Delta u & \text{if } w \notin \mathcal{W}. \end{cases} \qquad (21)$$

Notice that, since $z = Hx = H[I,\,0,\,0,\,0]w$, $\Delta u = u - [0,\,0,\,0,\,I]w$, and (Q,R) are positive definite matrices, the cost function is convex in (w,u). Also notice that, by construction, the invariant set $\mathcal{W}$ includes the stationary points for all possible plants, *i.e.* (x_s, u_s) such that $x_s = Ax_s + Bu_s$, $\bar{z} = Hx_s$ for any $(A,B) \in \Omega$.

Given this foundations, the proposed control algorithm is the following.

Algorithm 1. At time k, given the closed-loop state w_k, if $w_k \in \mathcal{W}$ set $u_k = \Theta w_k + \Psi v$. Otherwise, solve (20) and set u_k to the first term of the computed optimal sequence $\bar{\pi}(k)$.

2.3 *Properties*

We have the following results.

Theorem 1. Assume that a robustly invariant set $\mathcal{W}$ as in (17) exists and that the optimization problem (20) is feasible at time 0. Then, the feedback control law defined by Algorithm 1 drives the closed-loop state w_k to the invariant set $\mathcal{W}$.

Proof. At time k, assume that $w_k \notin \mathcal{W}$ (otherwise the proof is complete). The optimal input $\bar{u}_k$, injected in the plant, drives the closed-loop state from w_k to w_{k+1}, *i.e.* $w_{k+1} = \tilde{\Lambda}(k)w_k + \tilde{\Gamma}(k)\bar{u}_k$. Since the process is linear we have that $w_{k+1} \in \mathrm{Co}\left\{w^\ell_{k+1|k} | l \in \mathcal{L}_v\right\}$. Thus, it can be written as

$$w_{k+1} = \sum_{l\in\mathcal{L}_v} \mu_\ell w^\ell_{k+1|k}, \qquad (22)$$

in which μ_ℓ are appropriate scalar weights. At time $k+1$, we consider as a candidate input sequence

$$\left\{\bar{u}_{k+1|k}, \ldots, \bar{u}_{k+N-1|k}, \sum_{l\in\mathcal{L}_v} \mu_\ell(\Theta w^\ell_{k+N|k} + \Psi v)\right\}.$$

Under this control sequence, the augmented state predictions at time $k+1$ evolve in the convex hulls of the predictions made at time k. Moreover, the proposed sequence is feasible with respect to input constraints and such that $w^{\ell}_{k+N+1|k+1} \in \mathcal{W}$ for all $\ell \in \mathcal{L}_v$. Under the assumption that the problem (20) is feasible at time 0 (essentially, this means that the horizon is long enough to reach the invariant set after N moves), we have by induction that (20) remains feasible at any time. Given that the cost function is convex in its arguments, we have that, at time $k+1$ the candidate control sequence satisfies the constraints and yields a cost, say Φ_{k+1}, no larger than the optimal one at time k, say $\bar{\Phi}_k$. It is easy to see that $\Phi_{k+1} \le \bar{\Phi}_k - L(w_k, \bar{u}_k)$. Since, this sequence may be suboptimal, we have that the optimal cost at time $k+1$, denoted with $\bar{\Phi}_{k+1}$ satisfies

$$\bar{\Phi}_{k+1} \le \Phi_{k+1} \le \bar{\Phi}_k - L(w_k, \bar{u}_k)\ . \qquad (23)$$

The cost is monotonically non increasing, bounded below by zero and, therefore, it converges. Thus, we have that $\bar{\Phi}_k - \bar{\Phi}_{k+1} \to 0$ which implies that $L(w_k, \bar{u}_k) \to 0$. This means that, either w_k asymptotically enters $\mathcal{W}$ or $z_k \to \bar{z}$, $\Delta u_k \to 0$, *i.e.* the plant state and input reach stationary values. But, as previously remarked, all the stationary points for the closed-loop system are included in $\mathcal{W}$, and the proof is complete. □

Theorem 2. Under the assumptions of Theorem 1, also assume that the plant is ULTI. Then, the feedback control law defined by Algorithm 1 drives the controlled variable $z = Hx$ to its desired target $\bar{z}$ without offset.

Proof. From Theorem 1 we have that the closed-loop state enters the invariant set $\mathcal{W}$ and, since the system is ULTI, it reaches a steady state. Thus, by construction of the unconstrained control law that is applied in the invariant region, we have that $\lim_{k\to\infty} z_k = \bar{z}$. □

3. ILLUSTRATIVE EXAMPLE

We consider a LTV system as in (1) in which $(A_1 = 0.9, B_1 = 1)$, $(A_2 = 2, B_2 = 2)$. Notice that the convex hull includes both stable and unstable plants. The controlled variable is $z = x$ and its target is $\bar{z} = 1$. We choose as controller tuning matrices $Q = 1, R = 1$ and for the target calculation $R_s = R$, $\bar{u} = 0$. The offline optimization returns the following values for the nominal model and disturbance models:

$$A_m = 1.39, \quad B_m = 1.45,$$
$$B_d = 1.48, \quad C_d = 0.07\ .$$

The input is constrained:

$$-1.5 \le u_k \le 1.5,$$

and, using the outlined method (Blanchini, 1994), an invariant region $\mathcal{W} \subseteq \mathbb{R}^4$ for the closed-loop state is computed. This invariant region consists of ten linear inequalities.

Time interval	$A(k)$	$B(k)$
$0 \le k \le 19$	A_1	B_1
$20 \le k \le 39$	$\frac{A_1+A_2}{2}$	$\frac{B_1+B_2}{2}$
$40 \le k \le 60$	A_2	B_2

Table 1. Plant matrices.

We present the simulation results for the case in which the initial state is $x_0 = -2.5$. The plant matrices vary during the simulation as described in Table 1. The input u and the controlled variable z are reported in Figure 1.

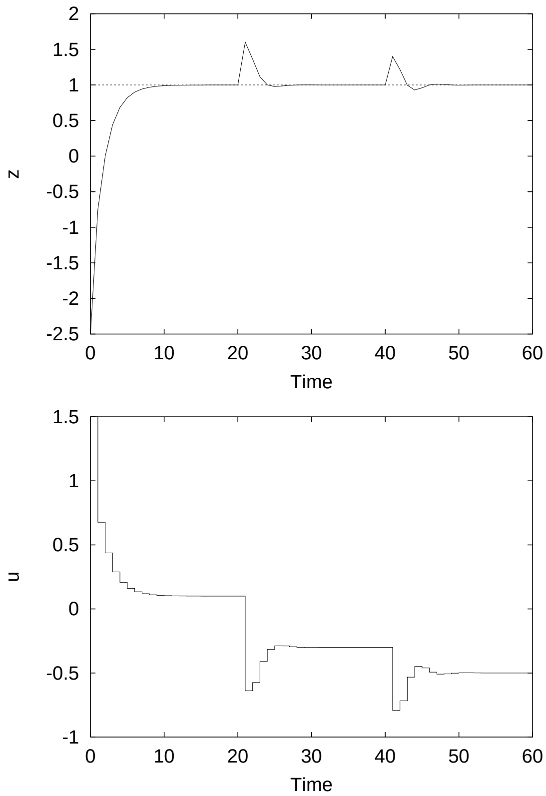

Fig. 1. Controlled variable and input.

4. COMMENTS AND REMARKS

As expected, the proposed control algorithm is able to track nonzero set points without offset, independently of which plant is running. It is interesting to notice that the input steady-state target changes when the actual plant changes. The strength of this algorithm is its ability to drive the input to its correct target without knowing the actual plant.

Another interesting simulation (not shown) regards the case in which the plant randomly varies in the uncertainty region. The closed-loop system is stable, since the closed-loop state reaches the invariant region (see Theorem 1), and the controlled variable remains within a neighborhood of the target. Obviously, it is impossible to achieve an asymptotic zero offset, since every time that the plant changes the input needs to be changed to its new target.

A straightforward extension of this method is to the case of time-varying set points (piece-wise constant). After having computed a nominal model and disturbance model, we need to compute (off line) for each set point a corresponding invariant region. The results presented in this work apply to this case provided that, every time that the set point changes, the corresponding initial optimization problem is feasible.

Finally, a further variation of the on-line optimization method is to consider the horizon as a decision variable. In this way, the horizon required to reach the invariant region can be minimized as discussed by Scokaert and Mayne (1998).

5. CONCLUSIONS

In this paper, a robust model predictive control formulation has been proposed. The controller has been formulated as a dual-mode regulator in which the inner unconstrained controller is designed (off line) in order to achieve offset-free performance for any plant in an uncertainty convex region (convex hull). By using the theory of robustly invariant sets, an appropriate region in which this unconstrained controller satisfies the constraints has been defined. Next, an on-line optimization min-max problem is solved to compute the input sequence that drives the terminal state to the invariant region, while satisfying the constraints. It has been shown that the proposed algorithm stabilizes any time-varying system in the convex hull. Moreover, when the system is unknown but time invariant it has been shown that the proposed controller drives the controlled variable to its corresponding (possibly nonzero) target. Finally, an application example has been presented, in which the offset-free properties of the controller have been shown.

6. REFERENCES

Blanchini, F. (1994). Ultimate boundedness control for uncertain discrete time systems via set-induced Lyapunov functions. *IEEE Trans. Auto. Cont.* **39**, 428–433.

Blanchini, F. (1999). Set invariance in control. *Automatica* **35**, 1747–1767.

De Nicolao, G., L. Magni and R. Scattolini (1996*a*). Robust predictive control of systems with parametric uncertainties. In: *UKACC International Conference on Control.* pp. 1362–1367.

De Nicolao, G., L. Magni and R. Scattolini (1996*b*). Robust predictive control of systems with uncertain impulse response. *Automatica* **32**, 1475–1479.

Genceli, H. and M. Nikolaou (1993). Robust stability analysis of constrained l_1-norm model predictive control. *AIChE J.* **39**, 1954–1965.

Kassmann, D. E., T. A. Badgwell and R. B. Hawkins (2000). Robust steady-state target calculation for model predictive control. *AIChE J.* **46**, 1007–1024.

Kothare, M. V., V. Balakrishnan and M. Morari (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica* **32**, 1361–1379.

Kouvarithakis, B., J. A. Rossiter and J. Schuurmans (2000). Efficient robust predictive control. *IEEE Trans. Auto. Cont.* **45**, 1545–1549.

Lee, Y. I. and B. Kouvaritakis (2000). A linear programming approach to constrained robust predictive control. *IEEE Trans. Auto. Cont.* **45**, 1765–1770.

Muske, K. R. and J. B. Rawlings (1993). Model predictive control with linear models. *AIChE J.* **39**, 262–287.

Pannocchia, G. and J. B. Rawlings (2001*a*). Disturbance models for offset-free MPC control. Submitted for publication in *AIChE J.*

Pannocchia, G. and J. B. Rawlings (2001*b*). Robustnees of MPC and disturbance models for multivariable ill-conditioned processes. Technical Report 2001–02. TWMCC, Department of Chemical Engineering, University of Wisconsin-Madison. URL: *http://www.che.wisc.edu/jbr-group/tech-reports/twmcc-2001-02.pdf*.

Ralhan, S. and T. A. Badgwell (2000). Robust disturbance rejection for FIR systems with bounded parameters. In: *Proceedings of IFAC AdChem.* Vol. II. Pisa, Italy. pp. 839–844.

Rao, C. V. and J. B. Rawlings (1999). Steady states and constraints in model predictive control. *AIChE J.* **45**, 1266–1278.

Rawlings, J. B., E. S. Meadows and K. R. Muske (1994). Nonlinear model predictive control: a tutorial and survey. In: *ADCHEM Conference.* Kyoto, Japan. pp. 203–214.

Scokaert, P. M. O. and D. Q. Mayne (1998). Min-max feedback model predictive control for constrained linear systems. *IEEE Trans. Auto. Cont.* **43**, 1136–1142.

Zheng, Z. Q. and M. Morari (1993). Robust stability of constrained model predictive control. In: *American Control Conference.* San Francisco, CA. pp. 379–383.