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NON-LINEAR KALMAN FILTER ESTIMATION FOR ACTIVE STEERING OF PROFILED RAIL-WHEELS

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Abstract: This paper presents a study of state estimation for active steering of profiled wheels. Fundamental characteristics of a solid-axle wheelset are explained and the potential benefits of active steering are discussed. The paper studies the non-linearity of the wheelset caused by profiled wheel-rail contact surfaces and develops a recursive non-linear Kalman filter to provide reliable and accurate estimations of the wheelset state. Real wheel and rail profiles are used in the study and a non-linear model is developed to represent the wheelset. Computer simulations are used to verify the design and assess its performance. *Copyright © 2002 IFAC*

Keyword: Active Control, State estimation, Railway wheelset, Wheel-rail profiles, Kalman filter, Non-linear system

1. INTRODUCTION

Active controls for railway vehicles have been studied for many years and recently attracted increased interest (Goodall, 1997). It has been demonstrated that, by using advanced control technology and mechatronic concepts to replace the traditionally mechanical components of the suspensions, the performance of railway vehicles can be significantly improved. Active tilt control for trains is now becoming a standard technology and tilting trains are in commercial operation throughout European. However an important emerging area of interest is to consider active control for the basic guidance function of a railway vehicle. At the moment this is provided mechanically by means of wheelsets in which coned/profiled wheels are connected by a solid axle, but there are some difficult trade-offs between ensuring stability and making sure the vehicle goes round curves without creating noise and wheel/rail wear. Active control provides the possibility both to provide the essential stability control and to improve the vehicle performance on curves (Mei and Goodall, 2000). For practical applications different controls can be developed to achieve different steering/stability strategies, e.g. minimising the wear or balancing the track shifting forces. A number of papers have studied various control strategies as well as actuation configurations (Leo, 1985; Aknin et al 1991; Wickens, 1991; Shen and Goodall, 1997; Mei and Goodall 2000; Gretzschel and Bose, 2000).

However one of the well known difficulties is that active controls often require feedback signals that are not readily available and cannot be easily and economically measured in practice. To enable practical implementations of the active solutions, it is therefore essential to find effective methods that will provide accurate and reliable information of those signals. A study of model based estimation techniques has been carried out, where a re-formulated Kalman filter is used to estimate not only the state variables of a railway wheelset but also to calculate parameters such as curve radius and cant of the railway track on which the wheelset is travelling (Li and Goodall, 1999). The technique is also extended to a two-axle vehicle and the filter output is used for an optimal controller developed to actively steer the two wheelsets. In both studies, coned wheels with constant conicity (i.e. linear profiles) are used. For wheels with non-linear profiles, which are almost always the case in practice, studies have found that the assumption of constant conicity would result in an inaccurate estimation for some of the state variables although it does not appear to affect the stability of the control system. Clearly the Kalman filters produced will need to be developed further to tackle the issue of the variation in conicity in order to improve the estimation accuracy.

This paper explores the possibility of state estimation for real profiled wheels, and the study is based on the fact that the profile of a wheel tread can be made known, and accessible to the estimator. In section 2, key properties of a solid-axle wheelset are explained and non-linear profiled wheels are introduced. Section 2 also introduces a mathematical model of the wheelset. A linerisation process of the wheel-rail contact geometry is carried out and the development of a recursive Kalman filter is presented in section 3. Simulation results and performance assessments are given in section 4.

This study concentrates on a single wheelset only in order to develop the basic principle of such an approach, but the outcome can be easily extended to vehicles with multiple axles.

2. SOLID-AXLE WHEELSET

As shown in Figure 1, a solid-axle wheelset consists of two coned/profiled wheels connected rigidly to a common axle so that they must rotate at the same angular velocity. The advantage of the arrangement is that the wheelset has the ability of natural curving and centring. This can only happen when the wheels are coned or profiled such that a difference in contact radius between the two wheels can be formed to cover the different travel distances of the outer and inner rails when the wheelset is off the centre line or on a curved track.

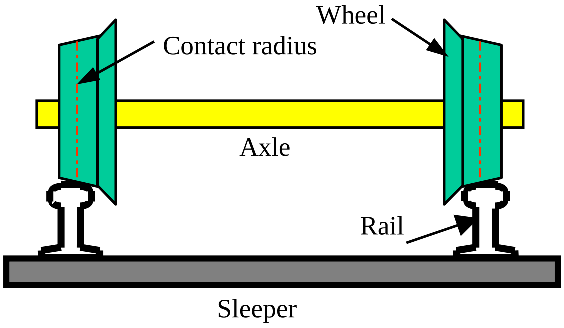


Figure 1. Solid-Axle Wheelset

However the wheelset alone also exhibits a sustained kinematic oscillation in the lateral plane commonly referred as the "*wheelset hunting*". This is overcome on conventional railway vehicles with the use of a yaw stiffness, which on the other hand degrades the ability of the wheelset to curve and results in severe wear of the wheels and rails. To solve this difficult design conflict between the curving, the dynamic performance and the stability, active steering controls have been proposed (Mei and Goodall, 2000) - the motivation of this study to provide necessary feedback signals.

It is common that the coned wheels with constant conicty are used in the study of wheel-rail contact mechanics, a useful simplification of a very complex problem. However in practice real wheel rim and rail-head are of non-linear profiles and the rolling radius of the two wheels at the contact point varies with relative wheel-rail lateral position. Figure 2 shows a typical non-linear profile for the wheel and rail which will be used in the study.

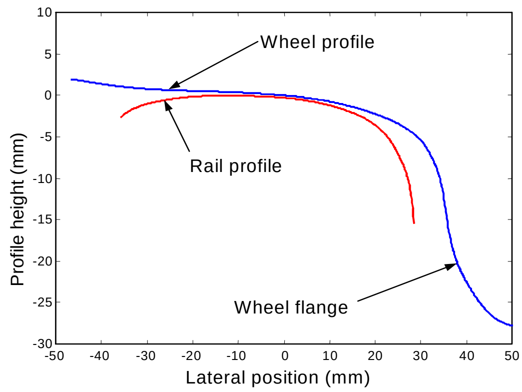


Figure 2. Non-linear Wheel and Rail Profiles

In this case the conicity is no longer a constant value and therefore the model for constant conicity, which can be found in most published literature, is no longer adequate to represent the wheel rolling radius accurately. Figure 3 demonstrates how the wheel radius changes with the position. A general form for the representation of the rolling radii can be expressed in equation 1, in which (y-y_t) is the relative lateral movement between the wheel and rail, and r_{L0}(.) and r_{R0}(.) are functions that represent the non-linearity.

$$\begin{cases} r_L = r_0 + r_{L0}(y - y_t) \\ r_R = r_0 + r_{R0}(y - y_t) \end{cases} \quad (1)$$

For the purpose of this study, the wheelset is stabilised by the use of a spring-damper in series placed in the lateral direction. The equation of motions for the wheelset with the stabilisation can be derived as given in equations 2-4.

$$\ddot{y} = -\frac{2f_{22}}{mv}\dot{y} + \frac{2f_{22}}{m}\psi + \frac{2k_y}{m}y_1 - \frac{2k_y}{m}y + \frac{v^2}{R} - g\theta \tag{2}$$

$$\ddot{\psi} = -\frac{2f_{11}l^2}{Iv}\dot{\psi} + \frac{f_{11}l}{Ir_0}r_{L0} - \frac{f_{11}l}{Ir_0}r_{R0} + \frac{2f_{11}l^2}{IR} \tag{3}$$

$$\dot{y}_1 = -\frac{k_y}{c_y}(y_1 - y) \tag{4}$$

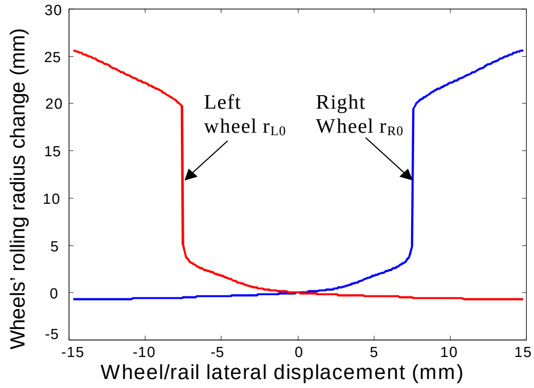


Figure 3. Rolling radius changes with profiled wheel-rail

3. KALMAN FILTER DESIGN

In the development of the Kalman filter, the changes of rolling radius in the two wheels of a wheelset shown in Figure 3 are linearised and expressed in equations 5 and 6, where $y_d = y\text{-}y_t$ is the wheel-rail lateral displacement.

$$\begin{cases} r_{L0} = -0.067 \cdot y_d & y_d \geq -0.0021 \\ r_{L0} = -0.6 \cdot (y_d + 0.00195) & -0.0075 \leq y_d < -0.0021 \\ r_{L0} = -120 \cdot (y_d + 0.00745) & y_d < -0.0075 \end{cases} \tag{5}$$

$$\begin{cases} r_{R0} = 0.067 \cdot y_d & y_d \leq 0.0021 \\ r_{R0} = 0.6 \cdot (y_d - 0.00195) & 0.0021 < y_d \leq 0.0075 \\ r_{R0} = 120 \cdot (y_d - 0.00745) & y_d > 0.0075 \end{cases} \tag{6}$$

From equations 5 and 6, a general form of the radius changes can be represented as follows:

$$r_{L0} = a_1 \cdot ((y - y_t) + b_1) \tag{7}$$

$$r_{R0} = a_2 \cdot ((y - y_t) + b_2) \tag{8}$$

And hence

$$\dot{r}_{L0} = a_1 \cdot (\dot{y} - \dot{y}_t) \tag{9}$$

$$\dot{r}_{R0} = a_2 \cdot (\dot{y} - \dot{y}_t) \tag{10}$$

From equations 2, 3, 4, 9 and 10, a discrete time state-space form model can be formulated as given in equation 11. In the model, the random track irregularity (y_t) and deterministic track features (R, θ) are included as the part of the states - an approach which has shown to be very effective for the estimation of all wheelset state variables on both straight and curved tracks (Li and Goodall, 1999; Mei etc, 1999). In addition, the radius changes of the right and left wheels are also formulated as two extra state variables to deal with the variation in conicity between the two wheels. Three inertial sensors (lateral accelerometer, yaw and roll gyros) are used to provide the measurements for the Kalman filter and equation 12 gives the output equation.

$$\begin{bmatrix} \dot{y} \\ y_d \\ \dot{\psi} \\ \psi \\ y_1 \\ y_t \\ r_{L0} \\ r_{R0} \\ 1/R \\ \theta \end{bmatrix}_{n+1} = \begin{bmatrix} 1-T\frac{2f_{22}}{mv} & -T\frac{2k_y}{m} & 0 & T\frac{2f_{22}}{m} & T\frac{2k_y}{m} & -T\frac{2k_y}{m} & 0 & 0 & Tv^2 & -Tg \\ T & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-T\frac{2f_{11}l^2}{Iv} & 0 & 0 & 0 & T\frac{f_{11}l}{Ir_0} & -T\frac{f_{11}l}{Ir_0} & T\frac{2f_{11}l^2}{I} & 0 \\ 0 & 0 & T & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & T\frac{k_y}{c_y} & 0 & 0 & 1-T\frac{k_y}{c_y} & T\frac{k_y}{c_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ Ta_1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ Ta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{y} \\ y_d \\ \dot{\psi} \\ \psi \\ y_1 \\ y_t \\ r_{L0} \\ r_{R0} \\ 1/R \\ \theta \end{bmatrix}_n + \begin{bmatrix} 0 & 0 & 0 \\ -T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ T & 0 & 0 \\ -Ta_1 & 0 & 0 \\ -Ta_2 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} \dot{y}_t \\ (\dot{1/R}) \\ \dot{\theta} \end{bmatrix}_n \tag{11}$$

$$Y_m = \begin{bmatrix} \ddot{y}_m \\ \dot{\psi}_m \\ \dot{\theta}_m \end{bmatrix}_n = \begin{bmatrix} -\frac{2f_{22}}{mv} & -\frac{2k_y}{m} & 0 & \frac{2f_{22}}{m} & \frac{2k_y}{m} & -\frac{2k_y}{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -v & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ y_d \\ \dot{\psi} \\ \psi \\ y_1 \\ y_t \\ r_{L0} \\ r_{R0} \\ 1/R \\ \theta \end{bmatrix}_n + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_n + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_n \qquad (12)$$

Equations 11 and 12 can be re-expressed as shown in equations 11a and 12a

$$x_{n+1} = Ax_n + Gu \qquad (11a)$$

$$Y_n = Cx_n + \underline{v} \qquad (12a)$$

Equations 11a and 12a form the basis for the design of the recursive Kalman filter. The flow chart in Figure 4 shows the execution sequence of the re-formulated Kalman filter, and the design detail of the Kalman filter can be found in (Li, 2001).

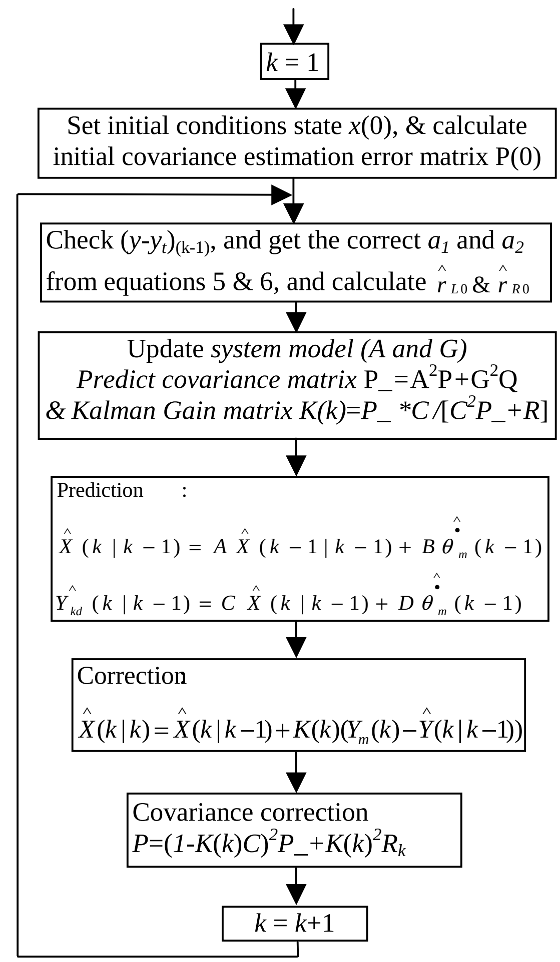


Figure 4. Flow chart diagram of Kalman filter

As the contact geometry of each wheel is partitioned into three linearised sections, the Kalman filter is required to determine for each wheel which section is in contact with the rail and hence to decide its operating point. In each sampling period all the state variables are estimated including the radius changes r_{L0} and r_{R0}. The outcome is then used to update the operating point by re-computing the state space matrices in equation 11 and to re-calculate the Kalman filter gain matrix $K(k)$. Consequently, a linearised model is calculated at each iteration based upon the current estimated state vector. This is often referred to as an Extended Kalman Filter or EKF in short (Maybeck, 1979). The Extended Kalman Filter algorithm is well documented in the literature (Maybeck 1979), and although global convergence cannot be guaranteed, experience has shown the algorithm to be robust.

4. SIMULATION STUDY

To study the performance of the Kalman filter, both deterministic and random track inputs are used in the simulation. The deterministic track used is a typical low speed curve, including a constant curve with the curve radius of 200m and a transition lasting around two seconds on either side to the straight track; the random track is derived from a filtered white noise generator to give an appropriate power spectrum for the lateral deviations. Note also that the track is

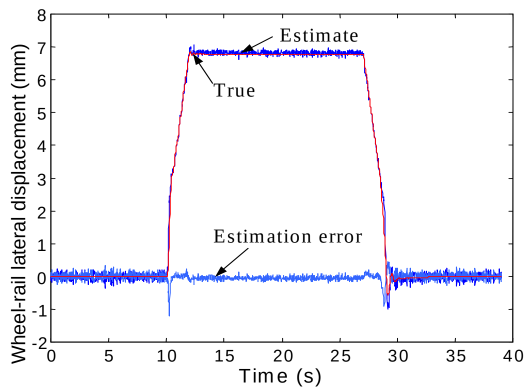


Figure 5. Wheel-rail lateral displacement

canted during the curve to reduce the lateral acceleration experienced by the passengers.

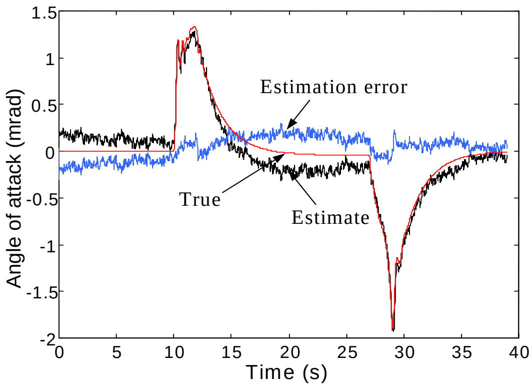


Figure 6. Wheelset angle of attack

Figures 5 shows the true and estimated wheel-rail lateral displacement, and the estimation error of the profiled wheelset on the curved track. The results for the angle of attack, i.e. the relative angle between the wheelset and the track in the yaw direction, and its estimation error are given in Figure 6. Clearly good estimates for the wheel-rail lateral displacement and angle of attack are obtained and its performance is very similar to the results obtained for the three-sensor Kalman-Bucy filter estimation for the coned wheelset where the physical model and estimator use the same conicity value (Li and Goodall, 1999). The small error is mainly due to the measurement noise. In the simulation all measurement noises are set to 2% of their maximum values, which is determined by taking 3 times their true (rms) value on the straight track with irregularities, plus the peak value of their responses on the pure curved track.

As the changes of the left and right wheel radius at the contact point with rail are included as two extra state variables, the Kalman filte also estimates those two parameters well, as demonstrated in Figure 7. The estimated radius changes are very close to the actual changes, although the estimation accuracy obviously depends on the closeness of the linearisised curve to the real one.

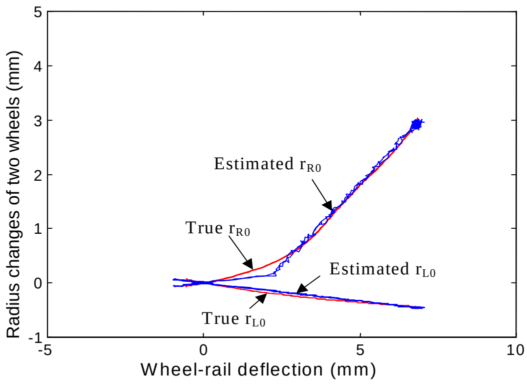
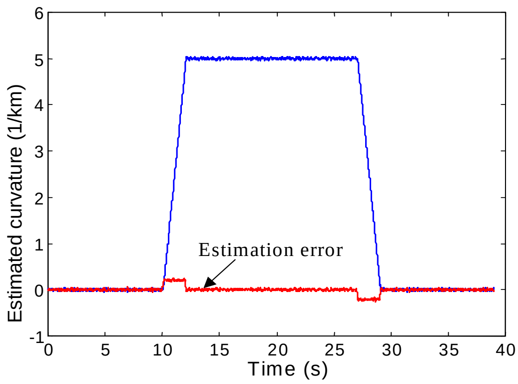


Figure 7. Radius changes of two wheels

The curvature and cant angle of the track on which the wheelset is travelling can also be estimated as the two are also formed as part of the state vector, even though the track deterministic features are strictly not part of the vehicle dynamics. Figure 8 shows the estimated track curvature and estimation error, and a similar result is obtained for the cant angle.



·Figure 8. Estimation of track curvature

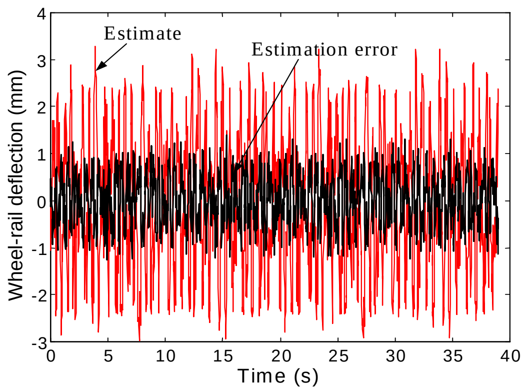


Figure 9. Wheel-rail deflection (random track)

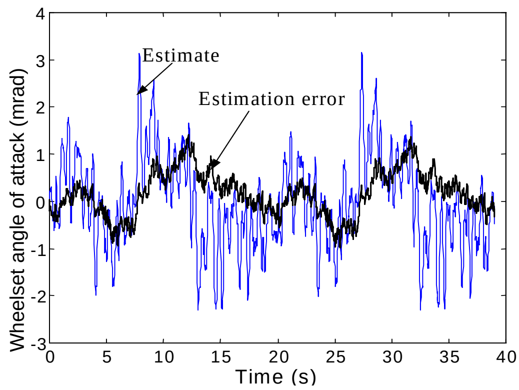


Figure 10. Wheelset angle of attack (random track)

On random track similar estimation performance is obtained. Figure 9 gives the estimated wheel-rail lateral displacement and its estimation error. It can be seen that the estimation error is relatively large, but mostly within ±1mm. Figure 10 shows the estimated angle of attack and its estimation error. There is a low frequency component of the estimation error on the angle of attack, which is very similar to what has been observed before (Li and Goodall, 1999; and Mei etc, 1999). The cause of the relatively large estimation error is not entirely clear, but the sensor noises and the low level of the attack angle in the study are contributing factors

5. CONCLUSIONS

This paper has presented a detailed study of state estimation for profiled wheels. The study has firstly discussed the need of a radical solution via active means to solve the problems associated with the design conflict between stability and curving of a solid-axle wheelset. It has then concentrated on the development of a recursive non-linear Kalman filter to provide reliable and accurate state estimations of a wheelset with non-linear wheel and rail profiles.

The performance of the Kalman filter has been evaluated by using computer simulations. It has been demonstrated that the Kalman filter deals with the non-linearity well and gives adequate estimations of the state variables of the wheelset on both random and curved tracks. This performance has been made possible by augmenting the estimator to include track and wheel-rail contact parameters as extra states.

For further work, it will be necessary to study the performance and robustness of the estimator in particular for the profile variation due to wear and tear of the wheels and track. The study could also be extended to investigate overlapping of the local models in the sense of fuzzy or probabilistic local model networks. In addition the authors are involved in practical implementations of active steering and it is hoped that the data from relevant tests will be used for a more comprehensive assessment.

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APPENDIX. SYMBOLS AND PARAMETERS

c_y	Lateral damping per wheel (6400 *kN s/m*)
f_{11}	Longitudinal creep coefficient (10 *MN*)
f_{22}	Lateral creep coefficient (10 *MN*)
g	Gravity constant (9.81 m/s^2)
I	Wheelset yaw inertia (700 *kg* m^2)
k_y	Lateral stiffness per wheel (3200 *kN/m*)
l	Half gauge (0.75 *m*)
m	Vehicle mass (1250 *kg*)
R	Track curve radius (200 *m*)
r_0	Nominal wheel radius (0.45 *m*)
v	Vehicle travel speed (18.4 *m/s*)
y	Wheelset lateral displacement
y_1	Displacement of the connection point between the lateral stiffness and damper
y_t	Lateral random track input
θ	Track cant angles (6^0)
ψ	Wheelset angle of attack
A	State space model system matrix
G	State space model input matrix
C	State space model measurement matrix
P	Kalman Filter state estimation error covariance matrix (corrected)
P_	Kalman Filter state estimation error covariance matrix (predicted)
K	Kalman filter gain matrix
k	iteration step of Kalman filter
Q	Process noise matrix
R	Measurement noise matrix
x	State space model state vector
Y	State space model output vector
^	denotes estimated vectors within the Kalman filter