15th Triennial World Congress, Barcelona, Spain

OUTPUT FEEDBACK TRACKING CONTROL OF SURFACE SHIPS

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Abstract: A controller is developed to make surface ships track a reference trajectory using only position and heading measurements for feedback under the environmental disturbances. The controller is first developed for full state feedback. A nonlinear dynamic filter, which is fundamentally different from the linear and high gain filters used in literature, is then designed to construct surge, sway and yaw velocities. The solution utilizes several properties of the ocean surface ships. Numerical simulations on a monohull ship with the length of 32 illustrate the effectiveness of the proposed controller.

Keywords: nonlinear control, surface ship, output feedback, tracking.

1. INTRODUCTION

Measurements of the ship velocities for most ships are not available and often corrupted with noise. Therefore, for feedback ship control systems, ship velocities need to be computed from position and heading measurements. In conventional feedback ship control systems, the ship velocities are often estimated by using Kalman filter. The nonlinear ship model is linearized around the operating points. A typical example is the linearization of the kinematic equations of ship motion about a set of 36 constant yaw angles separated by 10 degrees in order to cover the whole operating area of 360 degrees. For each of these linearized models, Kalman filters and control gains have to be computed and then modified on line using gain scheduling techniques. Although some acceptable results have been achieved the aforementioned linear control systems have certain drawbacks. They are developed based on linear models while the ship motion dynamics are inherently nonlinear. The useful nonlinearities are not utilized in the control design. They require a considerable amount of tuning work. The ad hoc nature of the linearization approach does not guarantee the desired stability and convergence properties, which mean a poor performance of the closed loop system.

The problem of output feedback, i.e. only position measurements are available for feedback, control of surface ocean vessels has been a topic of considerable interest since velocity measurement sensors are often contaminated with noise and are expensive. However the previous work targeted at the output feedback control problem to achieved global results was only devoted to dynamic positioning of ships (see Fossen (2000), Fossen and Grovlen (1998), Aarset et al. (1998)). In these works, the square term of velocity due to the Coriolis matrix was ignored since the vessel operates at low speed in dynamic positioning. Then several types of observer were designed to estimate vessel velocities in surge, sway and yaw. A common feature of the proposed output feedback controller for dynamic positioning is that the observers are first designed such that the observer error is asymptotically stable then the control inputs are designed by using the information from the observers.

The output tracking control of surface ships in one degree of freedom has been addressed in literature. Paulsen et al. (1998) derived a passive control law without velocity measurements and achieved asymptotic stabilization result of the yaw angle. Vik and Fossen (1997) utilized the approach developed for robot control in Berghuis and Nijmeijer (1993) to design an observer based controller to semiglobally stabilize the heading angle. The approach in Vik and Fossen (1997) was extended to the ship tracking control in three degrees of freesom by Pettersen and Nijmeijer (1999). Semiglobal tracking result was derived. A different approach to deal with output feedback control of ships is to apply a state transformation to transform the ship dynamic system into the so-called output injection form. However as noted in Besancon (2000), the state transformation solution cannot be found in generally for the case of more than one degree of freedom.

In this paper, a controller is developed to force surface ships track a reference trajectory using only position and heading measurements for feedback under the environmental disturbances. The controller is first developed for full state feedback. A nonlinear dynamic filter, which is fundamentally different from the linear and high gain filters used in literature, is then designed to construct surge, sway and yaw velocities. The solution utilizes several properties of the ocean surface ships. Numerical simulations on a monohull ship with the length of 32 illustrate the effectiveness of the proposed controller.

2. PROBLEM FORMULATION

The mathematical model of the ship moving in surge, sway and yaw is obtained from the motion equation of the ship moving in six degrees of freedom by neglecting motion in heave, pitch and roll, disturbances induced by wave, wind and ocean current. Furthermore it is assumed that the inertia, added mass and damping matrices are diagonal. This assumption holds when the vessels have three planes of symmetry, for which the axes of the body-fixed reference frame are chosen to be parallel to the principal axis of the displaced fluid, which are equal to the principal axis of the vessel. Most ships have port/starboard symmetry, and moreover bottom/top symmetry is not required for horizontal motion. Nonsymmetry fore/aft of the ship implies that the off-diagonal terms of the inertia and damping matrices are nonzero. However these terms are small compared to the main diagonal terms. It is also assumed that the damping is linear. This assumption is valid for low speed application and for cruising at a constant speed. For detailed development of the mathematical model of a surface ship moving in six degrees of freedom, the reader is referred to Fossen [1994].

Under the aforementioned assumptions, the ship moving in surge, sway and yaw can be described as

$$\begin{aligned}\dot{\eta} &= J(\psi)v \\ M\dot{v} + C(v)v + Dv &= \tau + \tau_w(t)\end{aligned} \qquad (1)$$

where $\eta = [x, y, \psi]^T$ and $v = [u, \vartheta, r]^T$. The variables x, y, ψ, u, ϑ and r are surge, sway and yaw displacements, surge, sway and yaw velocities respectively. $\tau \in R^3$ is the control force vector while the environmental disturbance vector $\tau_w(t) \in R^3$ is assumed to be bounded, i.e. $\|\tau_w(t)\| \le \tau_{w\max} < \infty$. $J(\psi), M, C(v)$ and D are the kinematic, inertia, Coriolis and centripetal, and damping matrices including added mass effects respectively, and given by

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$$

$$C(v) = \begin{bmatrix} 0 & 0 & m_{22}\vartheta \\ 0 & 0 & -m_{11}u \\ -m_{22}\vartheta & m_{11}u & 0 \end{bmatrix}, D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}$$

The ship dynamics (1) are transformed into the earth fixed coordinate as

$$M_\eta(\psi)\ddot{\eta} + C_\eta(\psi, \dot{\eta})\dot{\eta} + D_\eta(\psi)\dot{\eta} = \tau_\eta + \tau_{\eta w}(t, \eta) \qquad (2)$$

where

$$\begin{aligned} M_\eta(\psi) &= J^{-T}(\psi) M J^{-1}(\psi) \\ C_\eta(\psi, \dot{\eta}) &= J^{-T}(\psi)\left(C(v) - MJ^{-1}(\psi)\dot{J}(\psi)\right)J^{-1}(\psi) \\ D_\eta(\psi) &= J^{-T}(\psi) D J^{-1}(\psi) \\ \tau_\eta &= J^{-T}(\psi)\tau \\ \tau_{\eta w}(t, \psi) &= J^{-T}(\psi)\tau_w \end{aligned} \qquad (3)$$

where we have kept $v = J^{-1}(\psi)\dot{\eta}$ for notation simplicity. From the aforementioned assumptions on $M, C(v)$ and D, it is direct to show that the following properties are satisfied.

Property 1.

$$M_\eta(\psi) = M_\eta^T(\psi) > 0, \ \forall \psi \in R \qquad (4)$$

Property 2.

$$0 < M_1 \le \|M_\eta(\psi)\| \le M_2 < \infty, \ \forall \psi \in R \qquad (5)$$

Property 3.

$$C_\eta(\psi, \alpha)\beta = C_\eta(\psi, \beta)\alpha, \ \forall \psi \in R, \alpha, \beta \in R^3 \qquad (6)$$

Property 4.

$$\|C_\eta(\psi, \alpha)\| \le C_M \|\alpha\|, C_M > 0, \ \forall \psi \in R, \alpha \in R^3 \qquad (7)$$

Property 5.

$$s^T\left(\dot{M}_\eta(\psi) - 2C_\eta(\psi, \dot{\eta})\right)s = 0, \ \forall s \in R^3 \qquad (8)$$

Property 6.

$$s^T D_\eta(\psi)s > 0,\ \forall\psi\in R, s\in R^3 \tag{9}$$

The control objective is formulated as: Design a control law τ such that the tracking error $e=\eta-\eta_d$, with $\eta_d\in R^3$ being the bounded twice differentiable reference trajectory vector, globally asymptotically converges to zero using only the position and heading angle vector η for feedback when there are no environmental disturbances. Furthermore the tracking error converges to an arbitrarily small ball centered at the origin in presence of bounded environmental disturbances.

3. CONTROL DESIGN

In this section, we first design a full state feedback controller to achieve the control objective. We will then construct a nonlinear dynamic filter to calculate velocity vector.We define a filtered regulation error as

$$s=\text{Atan}(e)+\text{Atan}(f)+\dot e \tag{10}$$

where

$$\begin{aligned}&\text{Atan}(e)=[\text{atan}(e_1)\quad \text{atan}(e_2)\quad \text{atan}(e_3)]^T,\\ &\text{Atan}(f)=[\text{atan}(f_1)\quad \text{atan}(f_2)\quad \text{atan}(f_3)]^T,\end{aligned} \tag{11}$$

with $e=[e_1\quad e_2\quad e_3]^T$, $f=[f_1\quad f_2\quad f_3]^T$, f is an auxiliary filter variable designed as

$$\begin{aligned}&\dot f=-\text{Atan}(f)+\text{Atan}(e)+ks,\\ &f(0)=0\end{aligned} \tag{12}$$

where k is chosen later.
We consider the following function

$$V=\frac{1}{2}s^T M_\eta(\psi)s+\sum_{i=1}^{3}\left(e_i\text{atan}(e_i)-\frac{1}{2}\log(1+e_i^2)+f_i\text{atan}(f_i)-\frac{1}{2}\log(1+f_i^2)\right). \tag{13}$$

From property 1, it can be seen that the first part of V is radially unbounded and globally positive definite in s. In appendix A, we also show that the second part of V is radially unbounded and globally positive definite in e_i and f_i. Therefore, the function V defined in (13) is a proper function. By taking the first time derivative of (13) along the solution of (2) and utilizing properties 1-6, we have

$$\begin{aligned}\dot V\le&-\text{Atan}(e)^T\text{Atan}(e)-\text{Atan}(f)^T\text{Atan}(f)+\\ &s^T\left(\text{C}_\text{e}+C_f k-C_\eta(\psi,(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d))\right)s+\\ &s^T\big(k\text{Atan}(f)+\text{Atan}(e)+\\ &M_\eta(\psi)\left(C_e(-\text{Atan}(f)-\text{Atan}(e))+C_f(-\text{Atan}(f)+\text{Atan}(e))\right)-\\ &C_\eta(\psi,(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d))(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d)-\\ &D_\eta(\psi)(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d)+\tau_\eta+\tau_{\eta w}(t,\psi)-M_\eta(\psi)\ddot\eta_d\big)\end{aligned} \tag{14}$$

where

$$C_e=\begin{bmatrix}\frac{1}{1+e_1^2}&0&0\\0&\frac{1}{1+e_2^2}&0\\0&0&\frac{1}{1+e_3^2}\end{bmatrix}\text{ and }C_f=\begin{bmatrix}\frac{1}{1+f_1^2}&0&0\\0&\frac{1}{1+f_2^2}&0\\0&0&\frac{1}{1+f_3^2}\end{bmatrix} \tag{15}$$

From (14) we propose the control law, when the ship parameters are known, as

$$\begin{aligned}\tau=&J^T(\psi)(-k\text{Atan}(f)-\text{Atan}(e)-\\ &M_\eta(\psi)\left(C_e(-\text{Atan}(f)-\text{Atan}(e))+C_f(-\text{Atan}(f)+\text{Atan}(e))-\ddot\eta_d\right)+\\ &C_\eta(\psi,(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d))(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d)+\\ &D_\eta(\psi)(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d))\end{aligned} \tag{16}$$

and k is chosen as
-when there are no environmental disturbances

$$k=C_f^{-1}\left(-1-k_1-C_M\left(\pi+\|\dot\eta_d\|_{\max}\right)\right):=C_f^{-1}k_2,\ k_1>0 \tag{17}$$

-when there are environmental disturbances

$$k=C_f^{-1}\left(-1-k_1-C_M\left(\pi+\|\dot\eta_d\|_{\max}\right)-\frac{\tau_{w\max}^2}{4\varepsilon}\right):=C_f^{-1}k_2,\quad k_1>0,\ \varepsilon>0 \tag{18}$$

There should not be confused in using the same constant k_2 in (17) and (18) since its meaning is clear.
By noting that

$$\left\|C_\eta(\psi,(-\text{Atan}(f)-\text{Atan}(e)+\dot\eta_d))\right\|\le C_M\left(\pi+\|\dot\eta_d\|_{\max}\right) \tag{19}$$

$$\|C_e\|\le 1$$
(20)

substituting (16) and (17) into (14) yields (noting that for this case $\tau_w(t)=0$)

$$\dot V\le-\text{Atan}(e)^T\text{Atan}(e)-\text{Atan}(f)^T\text{Atan}(f)-s^T k_1\text{s} \tag{21}$$

substituting (16) and (18) into (14) yields

$$\dot V\le-\text{Atan}(e)^T\text{Atan}(e)-\text{Atan}(f)^T\text{Atan}(f)-s^T k_1\text{s}+\varepsilon \tag{22}$$

From (16), (17) and (18), it can be seen that the control and update laws depend on only e and f. It is also seen that calculation of f in (12) requires the velocity vector. However we can construct a nonlinear filter to compute f based on e not on its derivative as follows.

Substituting (17) or (18) into (12) yields

$$C_f \dot{f} - k_2 \dot{e} = -C_f \operatorname{Atan}(f) + C_f \operatorname{Atan}(e) + k_2 (\operatorname{Atan}(e) + \operatorname{Atan}(f)) \tag{23}$$

From (29), f can be computed as

$$f = \operatorname{Tan}(\xi + k_2 e) \tag{24}$$

$$\begin{aligned} \dot{\xi} &= -C_{\operatorname{Tan}(\xi+k_2 e)}(\xi + k_2 e) + C_{\operatorname{Tan}(\xi+k_2)} \operatorname{Atan}(e) + \\ &\quad k_2 (\operatorname{Atan}(e) + (\xi + k_2 e)) \\ \xi(0) &= -k_2 e(0) \end{aligned} \tag{25}$$

where $C_{\operatorname{Tan}(\xi+k_2 e)}$ is calculated as C_f in (15) but f is replaced by $\operatorname{Tan}(\xi + k_2 e)$, and Tan(.) is calculated as in (11) but atan(.) is replace by tan(.).
We now ready to present out main result.

Theorem 1. The control objective posed in section 2 is solvable by the control law (16) with the control gain k chosen as in (17) for the case without environmental disturbances and as in (18) in the presence of the environmental disturbances.

Proof. We first proof for the case without environmental disturbances. From (13) we know that V is a radially unbounded, globally positive definite function for all $e(t), f(t)$ and $s(t)$. Since $\dot{V}$ in (21) is negative, we know that $V(e(t), f(t), s(t)) \in L_\infty$. Therefore we have $(e(t), f(t), s(t)) \in L_\infty$. From (2), (12) and (16) we have $(\dot{e}(t), \dot{f}(t), \dot{s}(t)) \in L_\infty$ as well. Since η_d and its first two derivatives are bounded, we conclude that $(\ddot{\eta}(t), \tau(t)) \in L_\infty$. Since the function $\operatorname{Atan}(e)$ is convex in e, we now can apply Barbalat's Lemma in (Krstic et al. (1995)) to state that $e(t)$ tends to zero.

For the case with the environmental disturbances, the proof is similar. Since $\dot{V}$ in (22) is negative when $\|\zeta\| > \sqrt{\varepsilon / \min(1, k_1)}$ with $\zeta = [\operatorname{Atan}(e) \quad \operatorname{Atan}(f) \quad s]^T$, using the similar arguments for the case without the environmental disturbances, we can state that $e(t)$ tends to $\tan\left(\sqrt{\varepsilon / \min(1, k_1)}\right)$ which can be made arbitrarily small by selecting k_1.

Furthermore, it is not difficult to solve that the tracking error $e(t)$ exponentially converges to zero for the case without the environmental disturbances and to $\tan\left(\sqrt{\varepsilon / \min(1, k_1)}\right)$ in the presence of the environmental disturbances.

4. NUMERICAL SIMULATION

This section validates the control law (16) by simulating them on a monohull ship with the length of 32 m, mass of 118×10^3 kg and other parameters calculated by using Marintek Ship Motion program version 3.18
as
$m_{11} = 120 \times 10^3$ kg, $m_{22} = 177.9 \times 10^3$ kg, $m_{33} = 636 \times 10^5$ kgm^2,
$d_{11} = 215 \times 10^2$ kgs^{-1}, $d_{22} = 497 \times 10^2$ kgs^{-1}, $d_{33} = 802 \times 10^4$ kgm$^2 s^{-1}$.

For the simulation, we take: $\eta_d = [10\sin(0.3t) \; 10\cos(0.3t) \; 0.5\sin(0.3t)]^T$, $[x(0) \; y(0), \psi(0)]^T = [5 \; 5 \; 0.5]^T$. Based on (17) and (18), for the case without the environmental disturbances, we take $k_2 = 3 \times 10^6$, and $\varepsilon = 0.1$, $k_2 = 5 \times 10^6$ for the case with the environmental disturbances. Simulation results for the case without the environmental disturbances are plotted in Figure 1. In presence of the environmental disturbances of random zero mean with magnitude of 2×10^6, the results are drawn in Figure 2. From these figures, it can be seen that the tracking error converges to zero or a ball centered at the origin as proven in Theorem 1.

5. CONCLUSIONS

A global output feedback controller has been developed to force surface ships to track a reference trajectory under random but bounded environmental disturbances using only position and heading measurements for feedback. The requirement of velocity measurement is removed by constructing nonlinear dynamic filter to calculate roll, pitch and yaw velocity. Simulation on a monohull validated our proposed controller. The effect of actuator dynamics was assumed to be neglectable. The ship parameters were assumed to be known. We are working on relaxing this assumption.

APPENDIX A.

In this appendix, we show that the function V defined in (13) by proving the following lemma.

Lemma 1. The following function is proper

$$V_x(x) = x\operatorname{atan}(x) - \frac{1}{2}\log(1+x^2), \qquad \forall x \in R. \text{ (A.1)}$$

That is the followings must be held.

1) $V_x(0)=0$; 2) $V_x(x)>0, \forall x \in R$;

3) $\lim_{x\to\infty} V_x(x) \to \infty$.

Proof. In order to prove that V_x is a proper function for all $x \in R$, we need to show that 1), 2) and 3) are satisfied.

1) It is clear that $V_x(0)=0$ since $\operatorname{atan}(0)=0$ and $\log(1)=0$.

2) It is straightforward that $\frac{\partial V_x}{\partial x} = \operatorname{atan}(x)$ and $\frac{\partial^2 V_x}{\partial x^2} = \frac{1}{1+x^2}$. Therefore $x=0$ is the minimum point of V_x and $V_x > 0, \forall x \neq 0$.

3) It is necessary to show that $x\operatorname{atan}(x)$ tends to infinity faster than $\frac{1}{2}\log(1+x^2)$ does. Obviously we have

$$\lim_{x\to\infty}\left(\frac{x\operatorname{atan}(x)}{0.5\log(1+x^2)}\right) = \lim_{x\to\infty}\left(\frac{\operatorname{atan}(x)+\frac{x}{1+x^2}}{\frac{x}{1+x^2}}\right) =$$

$$\lim_{x\to\infty}\left(\frac{\operatorname{atan}(x)(1+x^2)}{x}+1\right) = \infty$$

as desired.

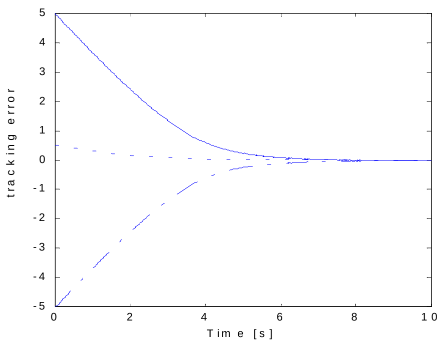

Figure 1. Without disturbances: $x-x_d$: solid, $y-y_d$: dash-dot, $\psi-\psi_d$: dot.

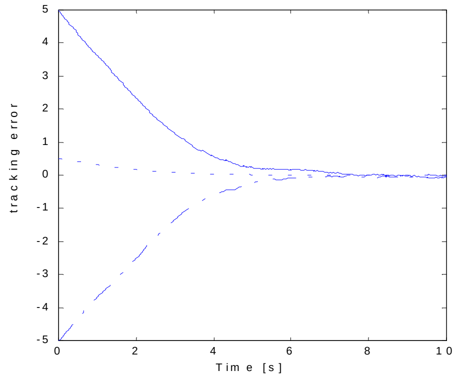

Figure 2. With disturbances: $x-x_d$: solid, $y-y_d$: dash-dot, $\psi-\psi_d$: dot.

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