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DETECTION OF PARAMETRIC FAULTS

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Abstract: Fault detection and fault isolation for systems including parametric faults is considered in this paper. By using a linear fractional transformation (LFT) description of parametric fault systems, the systems are transformed into equivalent systems including additive fault signals instead. The theory for systems including additive faults is extended to handle the parametric fault case in the same setting. A number of fundamental problems for parametric fault detection in linear systems is formulated and the corresponding solvability conditions are given.

Keywords: Fault detection, fault isolation, disturbance rejection, parameteric variation.

1. INTRODUCTION

The main part of papers in the area of fault detection and isolation (FDI) deals with fault diagnosis for systems including additive faults, see e.g. (Basseville and Nikiforov, 1993; Chen and Patton, 1998; Frank, 1990; Gertler, 1998; Niemann *et al.*, 1999; Willsky, 1976) and the references herein. The fault diagnosis problem for systems including parametric faults has not achieved the same attention as systems with additive faults. Fault diagnosis for systems with parametric faults has been considered in e.g. (Basseville and Nikiforov, 1993; Frank, 1996; Gertler, 1998; Isermann, 1984; Isermann, 1997; Isermann, 1993; Patton, 1994; Stoustrup and Niemann, 1999) to mention a few references.

In a number of fault diagnosis cases, it is natural to assume that the faults are reflected in the physical system parameters, as e.g. mass, friction, viscosity etc. This fact that the parametric faults are associated with system parameters, make it obvious that various system identification methods can be applied for fault diagnosis, see e.g. (Frank, 1996). Thus, detection of faults (parameter changes) can be done by an on-line estimation of the system parameters followed by a validation of the discrepancy between the nominal parameters and the estimated parameters.

In connection with fault diagnosis, Willsky, (Willsky, 1976), has pointed out that it consist of three tasks, fault detection as the lower task, fault isolation as the middle task and then fault estimation/identification as the highest task. In the lower task, which is the simpler task, it is detected whether any fault has occurred in the system. In the isolation task, the faults which have occurred in the system are identified. In the last task, the quantitative extent of the occurring faults is determined.

Based on this, it might not be an optimal method to detect and/or isolate parametric faults based on a parameter estimation. Detection and/or isolation of parameter faults might instead be based on detection and/or isolation effects in the system from parametric changes/faults. This requires that it is possible to measure these effects from the measurement signals y. If this is possible, fault detection/isolation can then be done in a similar way as detection/isolation of additive faults.

One issue to be considered in this paper is to show how it is possible to describe systems with parametric faults in a form such that the fault detection and isolation can be based on the effects in the measurement outputs from the system.

The approach applied in this paper can be considered as an algebraic approach to handle fault detection and fault isolation of system including parameteric faults. However, as an alternative to this approach, it is also possible to apply a geometric approach. It has been shown in (Edelmayer *et al.*, 1997*a*; Edelmayer *et al.*, 1997*b*) that it is possible to transform a system with parametric faults (perturbations) into an equivalent system including only additive faults based on (C, A)-invariant subspaces, i.e. the two system representations have equivalent (C, A)-invariant subspaces. Further, note that geometrical methods has been applied in connection with fault diagnosis in a number of papers, see e.g. (Chen and Speyer, 2000; Edelmayer *et al.*, 1997*a*; Edelmayer *et al.*, 1997*b*; Massoumnia, 1986; Niemann *et al.*, 1999; Zad and Massoumnia, 1999). An introduction to geometric control theory can be found in (Trentelman *et al.*, 2001).

Another issue that will be considered in this paper is the fundamental diagnosis problem for systems with parametric faults. The fundamental diagnosis problem, (Massoumnia *et al.*, 1989), is the problem of detection, isolation and/or estimation faults with zero threshold or almost zero threshold. This requires that it is possible to decouple the disturbances exactly or almost from the residual signals. The fundamental detection and isolation problem for systems with additive faults has been investigated in details in (Saberi *et al.*, 2000). The estimation problem has been investigated in (Niemann *et al.*, 1999; Niemann *et al.*, 2000).

The rest of this paper is organized as follows. In Section 2, systems including parametric faults are described. Further, it is shown how it is possible to transform systems with parametric faults into systems with additive faults. Section 3 includes definitions of parametric fault detection and parametric fault isolation together with a number of fundamental parametric fault detection problems. The solvability conditions to these problems are given in Section 4. The paper is closed with a conclusion in Section 5.

2. SYSTEM SETUP

Consider the following system

$$\begin{aligned} y(s) &= G_{yd}(\theta,s)d(s) + G_{yu}(\theta,s)u(s) \\ &= \begin{pmatrix} G_{yd}(\theta,s) & G_{yu}(\theta,s) \end{pmatrix} \begin{pmatrix} d(s) \\ u(s) \end{pmatrix} \end{aligned} \quad (1)$$

where $u \in \mathcal{R}^m$ is the control input vector, $d \in \mathcal{R}^q$ is the disturbance input vector and $y \in \mathcal{R}^p$ is the measurement vector. The parameter vector $\theta \in \mathcal{R}^k$ is the parametric faults in the system. Without loss of generality, $\theta = 0$ can be assumed to represent the nominal value of the fault parameter vector θ.

Below, in connection with a more detailed description of the system in (1), θ will be a $k \times k$ diagonal matrix with the k fault parameters in the diagonal.

The system given in (1) is quite general and cannot be applied directly in connection with parametric fault detection and isolation. A more detailed system description is needed in order to give explicit design equations. The interconnection between the nominal system and the parameter fault vector is described by using an LFT description, see (Zhou *et al.*, 1996) for a description of LFT. Let the system given in (1) be extended with an additional input vector w and an additional output vector z. The extended system is given by:

$$\begin{aligned} &\begin{pmatrix} z(s) \\ y(s) \end{pmatrix} = \\ &\begin{pmatrix} G_{zw}(s) & G_{zd}(s) & G_{zu}(s) \\ G_{yw}(s) & G_{yd}(s) & G_{yu}(s) \end{pmatrix} \begin{pmatrix} w(s) \\ d(s) \\ u(s) \end{pmatrix} \end{aligned} \quad (2)$$

where the connection between w and z is given by

$$w = \theta z$$

with θ given by a $k \times k$ diagonal matrix. The above LFT description of the system in (2) including a parameter fault vector θ is very general. The two transfer matrices in (1) are then given by

$$\begin{aligned} G_{yd}(\theta,s) &= G_{yd}(s) \\ &\quad + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zd} \\ &= G_{yd}(s) + G_{yw}\theta\Xi_d(\theta) \\ G_{yu}(\theta,s) &= G_{yu}(s) \\ &\quad + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu} \\ &= G_{yu}(s) + G_{yw}\theta\Xi_u(\theta) \end{aligned}$$

where $\Xi_d(\theta)$ and $\Xi_u(\theta)$ are given by

$$\begin{aligned} \Xi_d(\theta) &= (I - G_{zw}\theta)^{-1}G_{zd} \\ \Xi_u(\theta) &= (I - G_{zw}\theta)^{-1}G_{zu} \end{aligned}$$

Using the above description of the two transfer matrices in (1), the system can now be written as

$$\begin{aligned} y(s) &= (G_{yd}(s) + G_{yw}\theta\Xi_d)d(s) \\ &\quad + (G_{yu}(s) + G_{yw}\theta\Xi_u)u(s) \end{aligned} \quad (3)$$

Let the residual signal/vector r be given by

$$r = H(y - G_{yu}u) = \Psi(\theta, d, u) \quad (4)$$

where r is a time function that takes values in $\mathcal{R}^q$. In general, we might have to take H to be a nonlinear bounded-input, bounded-output stable operator, which makes Ψ also a nonlinear operator. In the case that

H is a linear operator, there exist transfer matrices $G_{rd}(\theta, s)$ and $G_{ru}(\theta, s)$ such that

$$r = G_{rd}(\theta, s)d + G_{ru}(\theta, s)u$$

or using the system matrices $G_{yd}(\theta, s)$ and $G_{yu}(\theta, s)$ given above, in the equation for the residual vector r gives the following equation for r

$$\begin{aligned} r &= H(G_{yd}(s) + G_{yw}(s)\theta\Xi_d(\theta, s))d \\ &\quad + HG_{yw}(s)\theta\Xi_u(\theta, s)u \end{aligned} \tag{5}$$

Note that it is not possible to remove the effect from the control signal completely in the residual vector due to the change of the system caused by the parameter fault vector θ.

It is possible to rewrite (5) into

$$\begin{aligned} r &= HG_{yd}(s)d \\ &\quad + HG_{yw}(s)\theta(\Xi_d(\theta)d + \Xi_u(\theta)u) \\ &= HG_{yd}(s)d + HG_{yw}(s)\theta\xi \end{aligned} \tag{6}$$

where the vector ξ is given by

$$\xi = \Xi_d(\theta, s)d + \Xi_u(\theta)u$$

Note that ξ depends on parameter fault vector θ, the disturbance input vector d as well as the control input vector u.

Now, let f be the fault vector defined by

$$f = \theta\xi \tag{7}$$

where the individual fault signals f_i are given by

$$f_i = \theta_i\xi_i$$

Using the fault vector f introduced in (6) gives the following system:

$$\begin{aligned} y &= G_{yd}(s)d \\ &\quad + \left(G_{yw,1}(s) \ \cdots \ G_{yw,k}(s)\right)\begin{pmatrix} f_1 \\ \vdots \\ f_k \end{pmatrix} \\ &= G_{yd}(s)d + G_{yw}(s)f \end{aligned} \tag{8}$$

and the following residual vector r

$$r = G_{rd}(s)d + G_{rf}(s)f$$

It is assumed that the matrix G_{yw} does not include columns with zeros only. This is without loss of generality. If G_{yw} includes a zero column, say $G_{yw,i}$, then it will be impossible to detect the fault parameter θ_i from the measurement output y. In this case, the fault θ_i will be an internal fault that does not have any effect on the system output.

The parametric fault system has then been transformed into a additive fault system.

The fault vector f will depend on both the disturbance d, the control input u as well as the parameter fault vector θ, i.e. $f = f(\theta, d, u)$. An important fact with the fault vector f defined in (7) is that there is a direct connection between θ_i and f_i. From the definition of f in (7) we have directly

- $f_i = 0$ if $\theta_i = 0$
- $f_i \neq 0$ if $\theta_i \neq 0$ and ξ (ξ_i) is non-zero

It is important to note that when both the input signal as well as the disturbance signal is zero, the vector ξ will also be zero. As a result of this, the fault vector f in (7) will be zero as well. It will therefore be impossible to detect and/isolate parametric faults in this case. This is not particular to the method - a fault in a component which is not active can never be detected. In spite of the fact that the parametric fault system has been transformed into a additive fault system in (8), the parametric fault detection problem is not completely equivalent with the additive fault detection problem. The problem that a non-zero control and/or disturbance input is required for fault detection/isolation in the parametric fault case is not required in the additive fault case. This condition is equivalent with the requirement of persistent excitation in system identification. The derived model in (7) cannot be applied in connection with fault estimation, due to the fact that the fault vector f does not represent the real fault vector.

A transformation of parametric/multiplicative faults into a similar additive fault setup has also been considered in (Gertler, 1998) and in a state space setting in (Stoustrup and Niemann, 1999). The model description applied in (Gertler, 1998) is included in the above parametric fault model.

Based on the reformulation of the parametric fault detection as an additive fault detection problem in (8), the residual vector in (4) can be rewritten as

$$\begin{aligned} r &= \Psi(\theta, d, u) \\ &= \Psi(f(\theta, d, u), d) \end{aligned} \tag{9}$$

One issue in connection with fault detection and isolation is whether one can achieve this when the system is affected by disturbances. A direct consequence of this is that the residual vector needs to be insensitive to the disturbances. That is, we need to have that

$$\Psi(f(\theta, d, u), d) = \Psi(f(\theta, d, u), 0)$$

for all disturbances d and parametric faults θ. However, this is an ideal case, that cannot always be achieved. This will be considered in the next section. If it is not possible to achieve an exact or an almost disturbance decoupling in the residual generator, then one has to look for design of the residual generator, such that the effect from the disturbances on the residual vector is reduced as much as possible.

3. DEFINITIONS AND PROBLEM FORMULATION

First, let us give the definition for parametric fault detection (PFD) and parametric fault isolation (PFI). The standard definitions for fault detection and isolation for additive fault systems, considered in e.g. (Niemann *et al.*, 1999; Saberi *et al.*, 2000), cannot be applied directly as pointed out in Section 2. The condition that the system needs to be excited to detect/isolate parametric faults needs to be included in the definitions. To define what we mean by excitation of a system, we have the following definition.

Definition 1. Let a system $G(s)$ including parametric faults θ be given. Let disturbances d and control inputs u be the external inputs to the system. The system is said to be **parametric fault excited** if a parametric fault θ_i will give a nonzero fault signal f_i,

$$\theta_i \neq 0 \;\Rightarrow\; f_i \neq 0$$

With this definition of parametric fault excitation, we have the following definitions for PFD and PFI

Definition 2. Let the residual generator $H \in \mathcal{RH}_\infty$ be given and let the system be parametric fault excited by the control input u. The residual r is said to achieve parametric fault detection (PFD) without disturbances if a non-zero parametric fault vector θ and $d = 0$ results in a non-zero residual r.

Definition 3. Given the residual generator $H \in \mathcal{RH}_\infty$ and let the system be parametric fault excited by the control input u. The residual r is said to achieve parametric fault detection and isolation (PFDI) without disturbances if for any two different parametric fault vectors θ_i and θ_j and $d = 0$ the corresponding residuals r_i and r_j are different.

In connection with the problem formulation below, it is assumed that all k parametric faults θ_i can appear simultaneously if nothing else is indicated.

As pointed out in the introduction, the fundamental fault diagnosis problems where it is possible to decouple the effect from the disturbances on the residual vector exactly or almost are very important. Now, a number of different fault detection and isolation problems are given for systems including parametric faults.

First, let us consider the case of detection a single parametric fault.

Problem 1. Suppose there exists a single fault, say the θ_i-th fault. Further, assume that the system is parametric fault excited. Then, for the system given in (8), the problem of (exact) **parametric fault detection** of a single fault is finding, if existent, a bounded-input bounded-output stable residual generator H_i whose output is a scalar residual signal $r_i = \Psi_i(\theta_i, d)$ such that

(1) $\Psi_i(0, d) = 0$ for all disturbances d,
(2) $\Psi_i(\theta_i, d) \neq 0$ for all $f_i \neq 0$ and all disturbances d.

We say that the i-th parametric fault θ_i is exactly detectable if the above problem is solvable.

When the residual generator is linear, then we impose the condition that $H_i \in \mathcal{RH}_\infty$ such that $G_{rd} = 0$ and G_{rf_i} is non-zero.

If it is not possible to achieve exact parametric fault detection, one can consider the problem of obtaining almost parametric fault detection. However, it has been shown in (Saberi *et al.*, 2000) in connection with fault detection and isolation of additive faults, that if almost fault detection/isolation can be obtained, exact fault detection/isolation can also be obtained. Therefore, formulations of almost parametric fault detection/isolation have been omitted in this paper.

Let us consider the case with no condition on the number of parametric faults that can appear simultaneously. Then, we have the following problem.

Problem 2. Consider the system given in (8) and assume that the system is parametric fault excited. The problem of (exact) **parametric fault detection** of a set of multiple parametric faults θ is defined as the problem of finding, if existent, a bounded-input bounded-output stable residual generator H whose output is a scalar residual signal $r = \Psi(\theta, d)$ such that

(1) $\Psi(0, d) = 0$ for all disturbances d.
(2) $\Psi(\theta, d) \neq 0$ for all faults $\theta \neq 0$ and all disturbances d.

We say that the set of multiple parametric faults is exactly detectable if the above problem is solvable for it.

It turns out that the above problem for parametric fault detection of a set of multiple parametric faults is quite restrictive. The problem formulation as it is given in Problem 2 also take care of the case where there is a relationship between the faults such that the effect from the faults vanished. This is a quite rare situation in practice. To modify the above fault detection problem, the notation of generic fault detection has been considered first in (Massoumnia *et al.*, 1989) and then extended in (Saberi *et al.*, 2000). Due to space limitations, the generic parametric fault detection as well as the generic parametric fault isolation will not be considered in this paper. See instead (Massoumnia *et al.*, 1989; Saberi *et al.*, 2000) for further details about generic fault detection and isolation.

Let us continue with the parametric fault isolation problem. Following the line from above, we have the following precise formulation of the problem for exact parametric fault isolation.

Problem 3. Consider the system given in (8). Assume that the system is parametric fault excited. Then, the problem of (exact) **individual parameter fault isolation** for a set of faults θ is defined as a problem of finding, if existent, a bounded-input bounded-output stable residual generator H which generates a residual vector $r = \Psi(\theta, d)$ such that for any fault θ_i, $i = 1, 2, \ldots, k$, there exists a dedicated component r_i of r and the operator Ψ_i from d and θ to r_i has the following properties:

- $\Psi_i(\theta, d) = 0$ for any disturbance d and any fault θ such that θ_i is identical to zero.
- $\Psi_i(\theta, d) \neq 0$ for any disturbance d and any fault θ such that θ_i is not identical to zero.

The set of parametric faults θ is said to be individually isolable if the problem of individual fault isolation is solvable.

In the case where it is not possible to individually isolate all parametric faults θ in the system, then one can ask for the largest subset of parametric faults that are isolable. We then have the following problem.

Problem 4. Consider the system given in (8) under the assumption that the system is parametric fault excited. Assume that the given set of parametric faults θ is not individually isolable. Then, the problem is to obtain the largest subset of parametric faults θ_s such that it is (exactly) individually isolable.

4. SOLVABILITY CONDITIONS

The solvability conditions for the parametric fault detection and isolation problems given in Section 3 are given without proofs. Proofs for the results can be derived by using the results in (Saberi *et al.*, 2000).

The solvability conditions given below are all stated in terms of the normal rank of certain transfer matrices. Thus, $\operatorname{normrank} G$ denotes the normal rank of the transfer matrix G, i.e. the rank of $G(s)$ for all $s \in \mathcal{C}$ but finitely many points.

First let us give the solvability conditions for the parametric fault detection problems.

Theorem 4.1. Consider the system given (8) with a single parametric fault θ_i. Further, let the system be parametric fault excited. Then, the problem of (exact) **parametric fault detection** of a single fault is solvable if and only if

$$\operatorname{normrank}\begin{pmatrix} G_{yd} & G_{yw,i} \end{pmatrix} > \operatorname{normrank}(G_{yd}).$$

Moreover, whenever the normal rank condition given above is satisfied, one can construct a linear residual generator that solves the exact parametric fault detection of a single fault.

Theorem 4.2 presents the results of our study. In presenting these results and elsewhere, we denote by $\#\Omega_\alpha$ the number of elements in the set Ω_α and by f_{Ω_α} the subset of all faults in Ω_α.

Theorem 4.2. Consider the system given in (8) under the assumption that all parametric faults can occur simultaneously and that the system is parametric fault excited.

Then, the problem of (exact) **parametric fault detection** is solvable if and only if

$$\operatorname{normrank}\begin{pmatrix} G_{yd} & G_{yw,\Omega_\alpha} \end{pmatrix} \geq \operatorname{normrank}(G_{yd}) + \#\Omega_\alpha$$

for $\alpha = 1, \ldots, \ell$, where $\#\Omega_\alpha$ denotes the number of elements in the set Ω_α and θ_{Ω_α} the subset of all faults in Ω_α. ℓ is the number of subsets of faults.

The solvability conditions for the two parametric fault isolation problems are now given.

Theorem 4.3. Consider the system given (8) under the assumption that all parametric faults can occur simultaneously and that the system is parametric fault excited. Then, the problem of (exact) **individual parametric fault isolation** for a set of parametric faults θ is solvable if and only if the following condition is satisfied:

$$\operatorname{normrank}\begin{pmatrix} G_{yd} & G_{yw,\Omega_\alpha} \end{pmatrix} \geq \operatorname{normrank}(G_{yd}) + \#\Omega_\alpha,$$

for all $\alpha \in \{1, \ldots, \ell\}$.

As a direct consequence of the above theorem, we have the following corollary regarding the total number of parametric faults that can be isolated exactly.

Corollary 4.4. Consider the system (8) under the assumption that all parametric faults can occur simultaneously and that the system is parametric fault excited. The total number of parametric faults that can be isolated while solving the exact parametric fault isolation problem is equal to

$$\operatorname{normrank}\begin{pmatrix} G_{yd} & G_{yw} \end{pmatrix} - \operatorname{normrank}(G_{yd}).$$

5. CONCLUSION

Four different fault detection and isolation problems for continuous-time systems including parametric faults has been considered in this paper. It has been shown how it is possible to transform systems with parametric faults into systems with additive faults.

The theory for systems with additive faults can then be applied with a minor modification. Detecting and isolating parametric faults requires that the system is sufficiently excited. This condition is equivalent with the requirement of persistent excitation of dynamical system in connection with system identification.

Only the continuous-time case has been considered in this paper. However, the discrete-time case is equivalent with the continuous-time case, and it can therefore be handled in the same way as the continuous-time case.

The parametric fault model that has been used in this paper is based on the so-called LFT setup. In the case where the parametric faults appear as nonlinear interconnections in the system, it is also possible to apply the derived results in this paper. In this case, a linearization of the system needs to be done before the diagnosis method is applied.

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