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# ADAPTIVE BACKSTEPPING CONTROL OF A POWER PLANT STATION MODEL

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**Abstract:** The backstepping technique was used in order to synthesise a non-linear control law for the non-linear model of the power plant station. The non-linearities in the model are related to steam flow through the super-heater and action of turbine valves. The time constant of the boiler is considered as an unknown parameter. Its value has to be adapted. The backstepping procedure presents a straightforward method to implement the law of adaptation, which ensures the stability. Symbolic computations were used to derive control law. The performed simulations have shown that the performance and quality of adaptive non-linear control system is the same across the entire range of operation and the adapted parameter converges to its nominal value. *Copyright © 2002 IFAC*

**Key words.** Power plant, non-linear control, backstepping, adaptive control

## 1. INTRODUCTION

The non-linear control law has an advantage that it works with the same performance and quality over the entire range of non-linear plant operation. A linear control law is able to perform well only around the nominal point of operation. Therefore, there is a need to design control system with better performance than the linear controller can offer. One of possibilities is to implement a non-linear power plant model in the control loop (Pitscheider *et al.* 2000), (Prasad *et al.* 2000), (Sindelar 1996), (Klefenz and Krieger 1992). This approach is heuristic in nature and it is difficult to prove its global properties, e.g. stability. The other method is to derive a non-linear control law in an analytical way. Basically, there are two ways, which enable to obtain such control law for smooth, continuous plants. The first of them is feedback linearization, but this technique is considered to be sensitive to model uncertainties. The second method is based on Lyapunov function approach. It is called backstepping (Krstic *et al.* 1995), (Ioannou and Sun 1996). This method provides a procedure for designing Lyapunov function for consecutive plant equations. It also gives a straightforward method to implement the law of adaptation.

In this paper a non-linear, adaptive control law for a power plant is synthesised using the backstepping procedure. At the first stage, a certainty equivalence control is derived (Bolek and Wisniewski 2000). The appropriate Lyapunov function and the control law are obtained. It can be done, provided that all parameters of the plant are known. Subsequently, the time constant of the boiler dynamics is considered as a parameter, which should be adapted. The term containing adaptation error is included in Lyapunov function. The adaptation law in form of differential equation is derived in order to make negative the derivative of this adaptive control Lyapunov function. The performance of the synthesised control system is illustrated by simulation examples.

## 2. POWER PLANT MODEL

The power plant station is a non-linear, dynamic, multi-variable system composed of boiler, turbine and generator. In order to synthesise a model that is adequate for control applications, the following simplifying assumptions were made:

- the control system of the water level in the boiler's drum works ideally,
- the control system of the combustion air does not affect power (steam) produced by boiler,
- the control system of the super-heated steam temperature does not affect power produced (steam).

The model (1) developed for the power plant shown in Fig. 1 is based on recommendations made by de Mello *et al.* (1991). The non-linearities come from the Flugl-Stodola equation and relation for steam flow through super-heater.

$$\begin{aligned}
T_h \dot{x}_1 &= -x_1 + p_T u_1 \\
C_{sh} \dot{p}_T &= k\sqrt{\gamma p_D - p_T} - p_T u_1 \\
C_d \dot{p}_D &= x_4 - k\sqrt{\gamma p_D - p_T} \\
T_w \dot{m}_w &= -m_w + u_2
\end{aligned} \qquad (1)$$

$T_w$ – boiler time constant – the parameter, which has to be adapted; its nominal value is 260,s.
$C_D$ =120,s – time constant due to drum capacity
$T_h$ =7,s – time constant due to volumes in turbine and re-heater
$C_{sh}$ =20,s – time constant due to volume in super-heater
$\gamma = p_{Do}/p_{To} = 1.2$ $\quad k = \frac{1}{\sqrt{\gamma - 1}}$ coefficients related to the flow of steam.

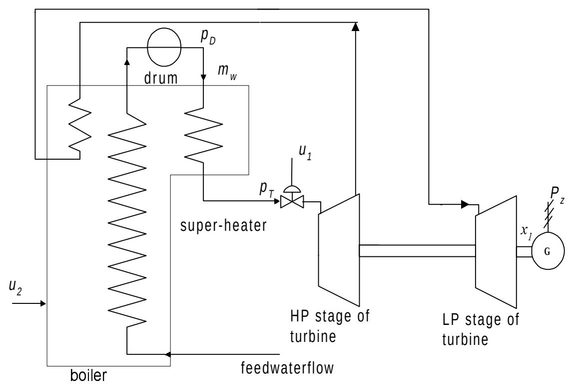


*Fig. 1. Technological diagram of power plant station.*

$x_1 = P_e$ – per unit electric power generated by power station
$x_2 = p_T$ – per unit pressure before the turbine
$x_3 = p_D$ – per unit pressure in the drum
$x_4 = m_w$ – per unit steam efficiency of the boiler (mass flow)
$u_1$- turbine valve opening
$u_2$ – flow of the fuel
$P_z$ – demanded power.

## 3. SYNTHESIS

The synthesis will be made in two steps. In first, a certainty equivalence method is considered. In the second the adaptation law is synthesised.

### *3.1 The control law synthesis for certainty equivalence case.*

The equilibrium point for the system (1)

$$u_2 = m_w = k\sqrt{\gamma p_D - p_T} = p_T u_1 = x_1 \qquad (2)$$

The system (1) can achieve equilibrium for any power $x_1$ and any pressure before turbine $p_T$. The control task of the system is to produce demanded power $P_z$ at constant pressure $p_T = 1$. Two integrations must be added in order to move equilibrium to these set points. The plant model with integrations is given in (3).

$$\begin{aligned}
T_h \dot{x}_1 &= -x_1 + p_T u_1 \\
T_{i1} \dot{e}_1 &= x_1 - P_z \\
T_{i2} \dot{e}_2 &= p_T - 1 \\
C_{sh} \dot{p}_T &= k\sqrt{\gamma p_D - p_T} - p_T u_1 \\
C_D \dot{p}_D &= m_w - k\sqrt{\gamma p_D - p_T} \\
T_w \dot{m}_w &= -m_w + u_2
\end{aligned} \qquad (3)$$

$T_{i1}$, $T_{i2}$ – integrator time constants

New co-ordinates (4) (denoted by ~) are introduced in order to move the set point to the origin. In this transformation the non-linearity in control $u_1$ can be also easily eliminated.

$$x_1 = \tilde{x}_1 + P \qquad p_T = \tilde{p}_T + 1 \qquad u_1 = \frac{\tilde{u}_1 + P_z}{\tilde{p}_T + 1}$$

$$p_D = \tilde{p}_D + \frac{k^2 + P_z^2}{\gamma k^2} \qquad (4)$$

$$m_w = \tilde{m}_w + P \qquad u_2 = \tilde{u}_2 + P_z$$

Plant model in new co-ordinates is given in (5).

$$\begin{aligned}
T_h \dot{\tilde{x}}_1 &= -\tilde{x}_1 + \tilde{u}_1 \\
T_{i1} \dot{e}_1 &= \tilde{x}_1 \\
T_{i2} \dot{e}_2 &= \tilde{p}_T
\end{aligned} \qquad (5a)$$

$$C_{sh}\dot{\tilde{p}}_T = k\sqrt{\gamma\tilde{p}_D - \tilde{p}_T + \left(\tfrac{P_z}{k}\right)^2} - \tilde{u}_1 - P_z$$

$$C_D\dot{\tilde{p}}_D = \tilde{m}_w + P_z - k\sqrt{\gamma\tilde{p}_D - \tilde{p}_T + \left(\tfrac{P_z}{k}\right)^2} \qquad (5b)$$

$$T_w\dot{\tilde{m}}_w = -\tilde{m}_w + \tilde{u}_2$$

Virtual controls $\alpha_1$, $\alpha_2$, $\alpha_3$ are computed successively in order to stabilise consecutive sets of equations. The results are summarised briefly below.

It occurs that sub-system (5a) is linear and it can be stabilised by linear state feedback. In this case the variable $\tilde{p}_T$ will be considered as first virtual control. The sub-system (5a) can be rewritten in a matrix form (6).

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\begin{bmatrix}\tilde{u}_1\\ \alpha_1\end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix}\tilde{x}_1\\ e_1\\ e_2\end{bmatrix} \qquad (6)$$

$$\mathbf{A} = \begin{bmatrix}-\frac{1}{T_h} & 0 & 0\\ \frac{1}{T_{i1}} & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \qquad \mathbf{B} = [\mathbf{b}_1 \quad \mathbf{b}_2] = \begin{bmatrix}\frac{1}{T_h} & 0\\ 0 & 0\\ 0 & \frac{1}{T_{i2}}\end{bmatrix}$$

For such system, it is possible to apply a linear-quadratic controller. The state feedback is computed accordingly to the positive definite, symmetric matrix **P**, which is the solution of Riccati equation.

The controls (actual and virtual) are given in (7).

$$\begin{bmatrix}\tilde{u}_1\\ \alpha_1\end{bmatrix} = \mathbf{g}\cdot\mathbf{x} = \begin{bmatrix}\mathbf{g}_1\\ \mathbf{g}_2\end{bmatrix}\cdot\begin{bmatrix}x_1\\ e_1\\ e_2\end{bmatrix} = \begin{bmatrix}g_{11} & g_{12} & g_{13}\\ g_{21} & g_{22} & g_{23}\end{bmatrix}\cdot\begin{bmatrix}x_1\\ e_1\\ e_2\end{bmatrix} \qquad (7)$$

The next virtual control is given in (8).

$$\alpha_2 = \frac{k^2\tilde{p}_T - P_z^2}{\gamma k^2} + \frac{\left(C_{sh}\left(-c_4(\tilde{p}_T - \alpha_1) - 2x^T P b_2 + \dot{\alpha}_1\right) + g_1 x + P_z\right)^2}{\gamma k^2} \qquad (8)$$

with time derivative $\alpha_1$ is evaluated analytically

$$\dot{\alpha}_1 = \frac{d\alpha_1}{dt} = \frac{\partial\alpha_1}{\partial\mathbf{x}}\dot{\mathbf{x}} = \frac{\partial(\mathbf{g}_2\mathbf{x})}{\partial\mathbf{x}}\left(\mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\begin{bmatrix}\tilde{u}_1\\ \tilde{p}_T\end{bmatrix}\right) = \mathbf{g}_2\left(\mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\begin{bmatrix}\mathbf{g}_1\mathbf{x}\\ \tilde{p}_T\end{bmatrix}\right). \qquad (9)$$

The third virtual control is given in (10).

$$\alpha_3 = \tilde{m}_w = -c_5C_D(\tilde{p}_D - \alpha_2) + k\sqrt{\gamma\tilde{p}_D - p_T + \left(\tfrac{P_z}{k}\right)^2} + {} - P_z + C_D\dot{\alpha}_2 + {} - \frac{C_Dk\gamma}{C_{sh}}\frac{(\tilde{p}_T - \alpha_1)}{\sqrt{\gamma\tilde{p}_D - \tilde{p}_T + \left(\tfrac{P_z}{k}\right)^2} + \sqrt{\gamma\alpha_2 - \tilde{p}_T + \left(\tfrac{P_z}{k}\right)^2}} \qquad (10)$$

The time derivative $\dot{\alpha}_2$ (11) is evaluated analytically. This evaluation is not given here explicitly, because it is quite involving.

$$\dot{\alpha}_2 = \frac{\partial\alpha_2}{\partial\mathbf{x}}\dot{\mathbf{x}} + \frac{\partial\alpha_2}{\partial\tilde{p}_T}\dot{\tilde{p}}_T \qquad (11)$$

Eventually the actual control $\tilde{u}_2$ is derived in (12).

$$\tilde{u}_2 = \tilde{m}_w - T_w\left(c_6(\tilde{m}_w - \alpha_3) + \tfrac{1}{C_D}(\tilde{p}_D - \alpha_2) - \dot{\alpha}_3\right) \qquad (12)$$

The time derivative $\dot{\alpha}_3$ is evaluated analytically. The control law for $\tilde{u}_1$ comes from (7).

$$\tilde{u}_1 = \mathbf{g}_1\mathbf{x} \qquad (13)$$

*3.2 The adaptation law synthesis*

Since $T_w$ is an unknown parameter, it can not be used to evaluate control (12). The $\tilde{u}_2$ can be evaluated using only an estimate $\hat{T}_w$. The adaptive version of (12) becomes (14).

$$\tilde{u}_2 = \tilde{m}_w - \hat{T}_w\left(c_6(\tilde{m}_w - \alpha_3) + \tfrac{1}{C_D}(\tilde{p}_D - \alpha_2) - \dot{\alpha}_3\right) \qquad (14)$$

It occurs that it is much easier to adapt the inverse of $T_w$. A new notation is introduced (15) below:

$$\rho = \frac{1}{T_w}\text{, and its estimate } \hat{\rho} = \frac{1}{\hat{T}_w} \qquad (15)$$

The estimation error is $\tilde{\rho} = \rho - \hat{\rho}$.

The adaptation law will be obtained in the form of differential equation. Its current solution will be used to evaluate the current value of control $\tilde{u}_2$. In the standard backstepping procedure, the term $\frac{1}{2}\tilde{\rho}^2$ is added to the Lyapunov function. Unfortunately, the derived on this basis adaptation law does not perform sufficiently well. The value of time constant does not change the equilibrium point. The discrepancy between estimated and actual values of time constant is visible only in transients. In the adaptation procedure, in order to benefit from this phenomenon, an estimation of state variable $\tilde{m}_w$ is introduced (Ioannou and Sun 1996). The estimate $\hat{\tilde{m}}_w$ is evaluated by differential equation (16).

$$\dot{\hat{\tilde{m}}}_w = \hat{\rho}\left(-\tilde{m}_w - \tilde{u}_2\right) + c_7\left(\tilde{m}_w - \hat{\tilde{m}}_w\right), \quad c_7>0 \qquad (16)$$

The error of this estimation is defined as

$$\varepsilon = \tilde{m}_w - \hat{\tilde{m}}_w$$

The adaptive Lyapunov function consists of the same term as in certainty equivalence case $V$ and the terms containing estimation error.

$$V_a = V + \tfrac{1}{2}\varepsilon^2 + \tfrac{1}{2c_8}\tilde{\rho}^2, \quad c_8>0 \qquad (17)$$

Its derivative:

$$\begin{aligned}\dot{V}_a &= -W - c_7\varepsilon^2 + \left(\tilde{m}_w - \alpha_3\right)\left(-\tilde{m}_w + \tilde{u}_2\right)\tilde{\rho} + \\ &+ \left(\tilde{m}_w - \hat{\tilde{m}}_w\right)\left(-\tilde{m}_w + \tilde{u}_2\right)\tilde{\rho} - \frac{1}{c_8}\tilde{\rho}\dot{\tilde{\rho}}\end{aligned} \qquad (18)$$

$W$ – is a positive definite function obtained in certainty equivalence case; the unknown parameter $\rho$ is constant, therefore $\dot{\tilde{\rho}} = -\dot{\hat{\rho}}$.

The terms in (18), which contain $\tilde{\rho}$ will vanish, if (19) holds.

$$\dot{\hat{\rho}} = c_8\left(-\tilde{m}_w - \tilde{u}_2\right)\left(2\tilde{m}_w - \alpha_3 - \hat{\tilde{m}}_w\right) \qquad (19)$$

When (14) is substituted into (16) and (19), then two differential equations, which are used in adaptation, are derived.

$$\dot{\hat{\tilde{m}}}_w = \left[c_6\left(\tilde{m}_w - \alpha_3\right) + \tfrac{1}{C_D}\left(\tilde{p}_D - \alpha_2\right) - \dot{\alpha}_3\right] + c_7\left(\tilde{m}_w - \hat{\tilde{m}}_w\right)$$

$$\dot{\hat{\rho}} = -c_8\tfrac{1}{\hat{\rho}}\left(2\tilde{m}_w - \alpha_3 - \hat{\tilde{m}}_w\right)\left[c_6\left(\tilde{m}_w - \alpha_3\right) + \tfrac{1}{C_D}\left(\tilde{p}_D - \alpha_2\right) - \dot{\alpha}_3\right]$$

(20)

## 4. SIMULATION EXAMPLES

Control laws (12) and (13) and adaptive law (20) were derived using symbolic computations. The control system (Fig. 1) was modelled in Simulink (MATLAB).

The non-linear control law was applied with following parameters. For linear part (5a) the state feedback was evaluated. The matrices in quadratic criterion are given below.

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 10^{-8} \end{bmatrix} \quad \mathbf{R} = 1000 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Eventually, the eigenvalues of the linear part are:

$$\begin{aligned} &-0.14244 \\ &-0.01182 \\ &-2.6352 \cdot 10^{-6} \end{aligned}$$

Time constants of integrators:

$T_{i1} = 0.6$ [s], $T_{i2} = 1.2$ [s].

Parameters in Lyapunov function

$c_4 = 0.23$; $c_5 = 0.001$; $c_6 = 0.00001$.

It was noticed that the control system was very sensitive to the value of coefficient $c_4$. It is due to the relation shown in (13).

The actual value of adapted parameter was $\rho = \frac{1}{260} = 0.00384$.

Two experiments were carried out. In the first, the transient was caused by a step change in power demand from 0.8 [per unit] to 0.75 [p.u]. In this case the initial value of adapted parameter was $\hat{\rho}_0 = 0.004$.

In the second experiment the transient was caused by a step change in power demand from 0.8 [per unit] to 0.85 [p.u]. In this case, the initial value of adapted parameter was equal $\hat{\rho}_0 = 0.003$.

In both experiments two simulations were carried out. In one of them the adaptation was switched on. In the second simulation, there was no adaptation and the control system worked with a wrong value of time constant $T_w$.

It turned out that the presence of adaptation procedure does not affect greatly the quality of power transient. In this case it looks like the response of multi-inertia plant. The difference between curves with and without adaptation would be hard to distinguish with the resolution of the presented figures. Therefore, the transients of power are presented by single curve in Fig. 2 and Fig. 5.

The price paid for good power performance, are the oscillations of pressures in the boiler. In Fig. 3 and Fig. 6 appropriate transients of steam pressure at the turbine inlet are given. In the case without adaptation procedure, the oscillation amplitude is higher.

The transients of adapted parameters are given in Fig. 4 and Fig. 7

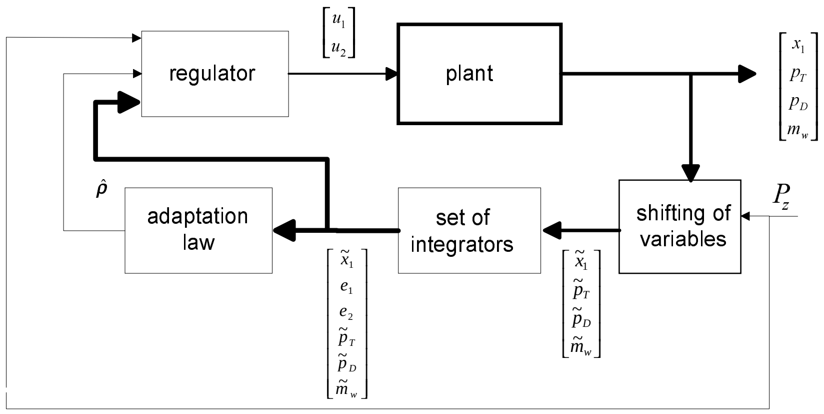


Fig. 1. The block diagram of non-linear, adaptive control system.

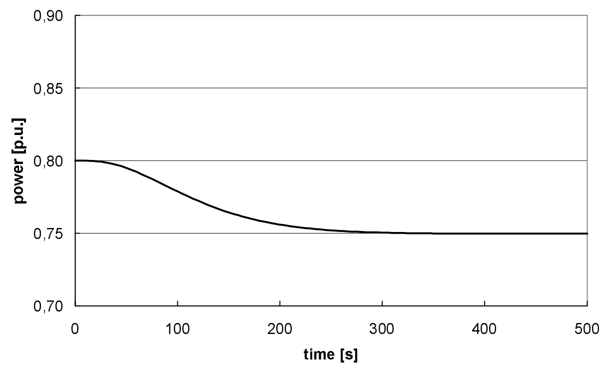


Fig. 2. Transient of power in first experiment

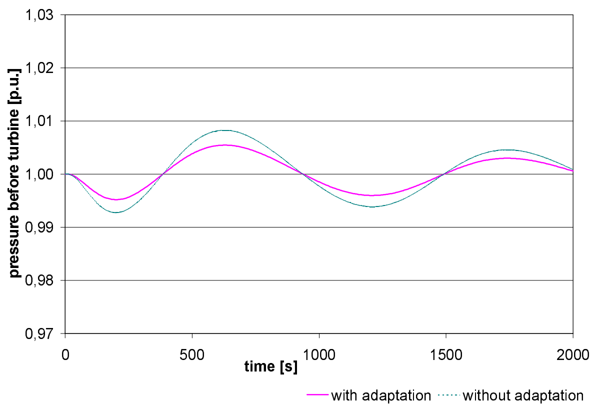


Fig. 3. Transients of $\tilde{p}_T$ in first experiment

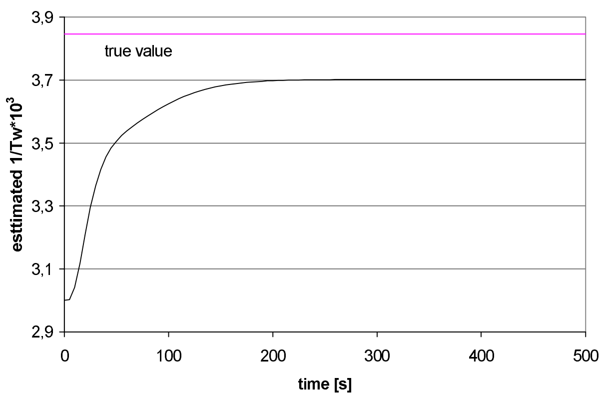


Fig. 4. Transients of estimate in first experiment

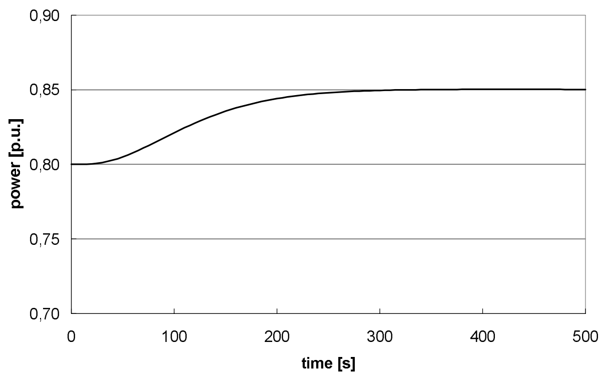


Fig. 5. Transient of power in second experiment

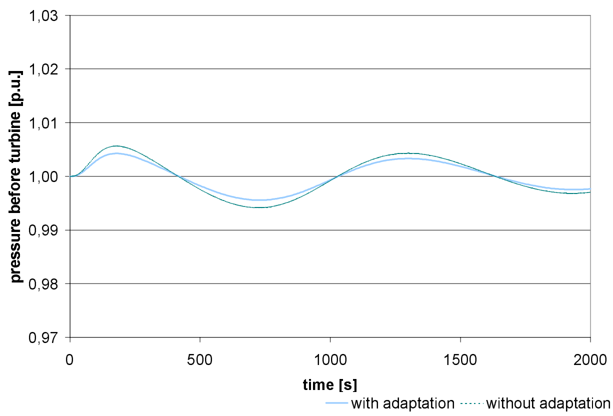


Fig. 6. Transients of $\tilde{p}_T$ in second experiment

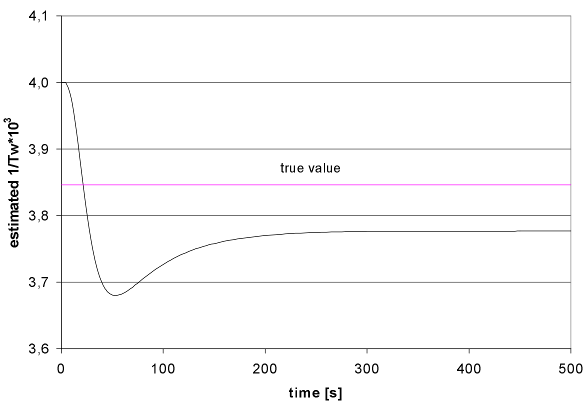


Fig. 7 Transient of estimate in second experiment

## 5. CONCLUSIONS

A non-linear model of power plant station was considered. The non-linear control system was synthesised using backstepping method. The control law is well posed, even though the plant is in pure feedback form. The obtained control law is quite complex. The symbolic computation toolbox in MATLAB was used to get the final derivation. It was possible to design, the simulation model in Simulink/Matlab.

As it can be seen in Fig. 2 and Fig. 5, the transients of power exhibit similar characteristic. They reach the desired set point without any overshoot and in reasonable time. The same performance is achieved even if the exact value of boiler's time constant is not known. The control system derived by backstepping method is a turbine-lead type. The control, which acts on turbine valve, is close to the output (power). This makes the control of power relatively easy. On the other hand, the small, damped pressure oscillations occur in the boiler (Fig. 3, Fig. 6). This is an acceptable price for higher overall performance.

If adaptation procedure is introduced, then the oscillations in pressures can be reduced, which may result in longer, uninterrupted work of the boiler without any need for repair.

During the adaptation, the estimated parameter got closer to the nominal value. The transients caused by step change in power demand were too short to obtain better convergence. There is still a discrepancy between estimated and true values. However, the stability is guaranteed analytically by adaptive Lyapunov function.

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