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REAL-TIME MANAGEMENT OF MULTI-RESERVOIR HYDRAULIC SYSTEMS USING H_∞ OPTIMIZATION

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Abstract: The article considers the problem of designing a robust controller for an open-channel hydraulic system with multiple dams for controlling discharge in rivers. The open-channel system considered is made of a network of river reaches controlled by two dams located upstream, one large dam far upstream, and one closer but smaller. As shown by the authors previously, the performance of such systems is structurally limited. The present paper shows how to recover the performance of the system controlled by the small dam, without emptying it too soon in the season. The problem is casted into the H_∞ framework, using frequency weighting functions to specify design requirements. Note that a strong interest of H_∞ controllers is to ensure robustness margin and performance.

Keywords: Open-channel system, irrational system, optimal performance, H_∞ control, rational approximation, Time Delay Systems.

1. INTRODUCTION

Water is becoming a scarce resource, which needs to be managed efficiently. Automatic control techniques are used by hydraulic engineers in order to obtain a better performance in real-time operation of open-channel systems. The problem considered in this paper is inspired from a real system, the Dropt, a controlled river located in southwestern France. This river is used to supply water to water users (mainly farmers who irrigate crops), who can take water out of the river without notice to the manager. We will focus on the upstream part of the river, which is 26 km long, with two dams, a large one located upstream of the river, and a smaller one, located on a affluent close to the downstream controlled point. This problem has already been considered using classical LQG controllers (Litrico, 1999), but the solution could not explicitly solve the allocation problem: how to take into account in real-time the constraint on the volume of each dam? The literature on multi-reservoirs hydraulic systems focuses generally on the tactical management of the reservoirs (*i.e.* at a higher level than the real-time control, in order to tackle the water allocation in terms of volume, not discharge) in a stochastic context, as the precipitations are not easily predictable. Many authors have contributed to this line of research, either with an empirical method (Lebdi et al., 1997), or using optimal control strategies (Ozelkan et al., 1997; Mousavi and Ramamurthy, 2000). These authors seek a control politics which tries to optimize the volume on a time horizon of typically one week or one month. The main contribution of the

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present paper is to provide a methodology which takes into account the dynamics of the river. To the best of the authors' knowledge, there are no other published work considering the robust real-time control of MIMO dam-river systems with constraints on the volume of the reservoirs.

As the systems considered are subject to large unknown perturbations (water withdrawals), one main objective for real-time management is that the controller rejects such perturbations. We will evaluate the "performance" of the controllers by determining the maximal frequency of sinusoidal perturbations that can be attenuated by the closed-loop. This corresponds to a measure of the bandwidth of the closed-loop system.

In previous papers, the authors characterized the maximal achievable performance of an open-channel system when only the regulated variable is measured and perturbations are assumed unmeasured (Litrico and Fromion, 2001); an H_∞ scheme allowing was developed to achieve this maximum performance for the SISO case, and extended this approach to the case where other intermediate variables of the open-channel system are measured and some perturbations are known (Litrico et al., 2001) (SIMO case). In this paper, we put a practical problem encountered by hydraulic engineers into a general H_∞ framework allowing to provide a Computer Aided Design method.

The problem to be considered here is the fact that the two dams have a different available volume of water: the large dam is far upstream from the controlled point and the small one is close to the controlled point. Therefore, the best achievable performance (in terms of rejection of perturbations, which is directly linked to the sensitivity bandwidth) could be obtained by using only the closest dam. But, since its available volume is limited, it cannot deliver water continuously to satisfy all water needs. The water demand can be satisfied by the large upstream dam, but with a much lower real-time performance. The paper presents a way to recover the performance obtained with the small dam by using a combination of the two dams, allowing the small dam to compensate only for high frequency perturbations.

The paper will focus on the analysis of a controlled river of total length $X = X_1 + X_2 + X_3$ with two dams upstream, delivering discharges u_1 and u_2. We suppose that $X_1 \gg X_2$, and the controlled discharge is located downstream of the third reach (see fig. 1).

The discharge is measured at different locations along the river reach and there is a finite number of intermediate pumping stations distributed along the reach which provide water to consumers. The objective of the controller is to act on the

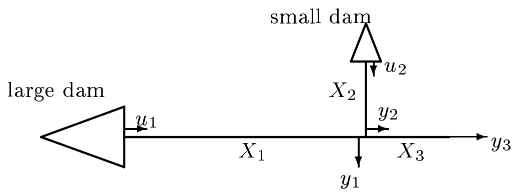

Fig. 1. System considered with 2 dams

upstream discharges u_1 and u_2 in order to keep the measured downstream discharge y_3 close to a target despite unmeasured users' withdrawals w_i, using intermediate discharge measurements y_i.

This paper is organized as follows. The problem under consideration is presented in section 2. In section 3, some results previously obtained on the achievable performance for open-channel hydraulic systems are recalled and two controllers for each SIMO subsystem are designed. In the next section, the MIMO controller design is presented, as a natural extension of the obtained SIMO controllers: the proposed design naturally mixes the advantages of each controller.

Notations:
$\mathcal{RH}_\infty$ represents the set of proper stable rational transfer matrices.
The acronyms MIMO and SIMO are respectively for Multiple Inputs Multiple Outputs and Single Input, Multiple Outputs systems.

2. CONSIDERED PROBLEM

2.1 System modelling

The system considered in this paper is a controlled river where the action variables are the upstream discharges noted u_1 and u_2 and the controlled variable is the downstream discharge (see fig. 1). The river is used to deliver water from the upstream dam to various consumers pumping water along the reach (farmers irrigating their fields, industries, etc.).

The dynamics of these open-channel hydraulic systems are classically represented by the Saint-Venant equations, a system of partial derivative equations involving the discharge $q(x,t)$ and the water elevation $z(x,t)$. This model is well suited to irrigation canals where the geometry is well-known, but it is too detailed for natural rivers where the geometry is much more complicated. A simplified linearized model, called Hayami model, is usually preferred in this case, involving only the discharge q.

The Hayami equation is a linear partial derivative equation representing the discharge transfer in a river reach around an equilibrium point:

Table 1. Physical parameters of reaches

	length X	C	D
reach 1	23.1 km	0.32 m/s	98 m^2/s
reach 2	1 km	0.43 m/s	108 m^2/s
reach 3	3.3 km	0.51 m/s	128 m^2/s

$$\frac{\partial q}{\partial t} + C\frac{\partial q}{\partial x} - D\frac{\partial^2 q}{\partial x^2} = 0 \qquad (1)$$

where

- q is the discharge [m^3/s],
- C the celerity coefficient [m/s] and
- D the diffusion coefficient [m^2/s].

The relation between upstream and downstream discharge can also be expressed as a transfer function using Laplace transform. Thus, fixing a downstream limit condition of the type $\lim_{x\to\infty}\frac{\partial q}{\partial x} = 0$, one gets the Hayami transfer function:

$$G(x,s) = e^{(\frac{C-\sqrt{C^2+4Ds}}{2D})x} \qquad (2)$$

with x the length of the considered reach [m] and s the Laplace variable.

This model can be identified using input-output data (Litrico, 2001), which simplifies the modelling step.

In order to use classical $\mathcal{H}\infty$ control, we need a rational model. We will use a rational approximation of the transfer function (2) obtained by frequency least squares minimization.

The numerical applications of this paper are done for a model of the upstream part of the river Dropt. The river exhibits the physical parameters given in table 1 (obtained from measurement and identification).

2.2 Control objectives

The control system should reach the following objectives:

- use the measures on the system y_i in order to act on the control variables u_1 and u_2 to reject unknown perturbations w_i acting on the system,
- follow a reference signal r,
- achieve the above objectives without using too much water from the second dam.

In other terms, the control objective is to use the upstream discharges u_1 and u_2 in order to keep the downstream discharge y_3 as constant as possible, which means that the control should attenuate the perturbations w_i. This is a problem of regulation or desensitivity. As the “real” system may be different from the model used for controller design, the controller design should incorporate some robustness requirements.

3. PRELIMINARY RESULTS

3.1 Performance limitations

As shown in a previous paper by the authors, the maximal achievable performance of an open-channel system where only the regulated variable is measured and perturbations are assumed unmeasured is structurally limited, as in the case of a time-delay system (Litrico and Fromion, 2001). This implies that for a given river reach, the perturbation rejection cannot be done on an infinite bandwidth.
More specifically, the performance is here considered in terms of attenuation of perturbations acting on the output of the system. This requirement can be formalized by direct constraints on the output sensitivity function. The performance considered is the maximum frequency ω_s such that the output sensitivity function ($S = (1+GK)^{-1}$ for a classical SISO controller) stays below one

$$\omega_s = \max\{\omega_1 \ : \ |S(j\omega)| < 1, \ \forall\omega < \omega_1\}$$

We evaluated this maximal achievable performance, taking into account the necessary robustness margins : with a phase margin of $\pi/4$ and a slope of -1 for $G(j\omega)K(j\omega)$ near the bandwidth, the limit obtained for the Hayami model is (Litrico and Fromion, 2001):

$$\omega_s = \frac{2\alpha_m C}{X}\sqrt{1+\frac{16\alpha_m^2 D^2}{C^2X^2}}$$

with $\alpha_m = \pi/8$.

Using an equivalent Hayami model (obtained with the moment matching method (Litrico, 1999)) for a series of models, one can evaluate the bandwidths ω_{si} for the two dams. The values obtained in this case are $\omega_{s1} = 10^{-5}$ rad/s and $\omega_{s2} = 9.10^{-5}$ rad/s. These values will be used for the controllers design.

We also considered in another paper (Litrico et al., 2001) the possible improvement given by additional information, like additional discharge measurement in the river, perturbation measurement or future perturbations prediction. We showed that the performance in terms of downstream perturbation rejection is not improved by additional measurement points in the river, or by perturbation measurement, and that the knowledge of future perturbations (leading to a predictive controller) can greatly improve performance.

3.2 H_∞ controller design in the SIMO case

Following the approach used by Pognant-Gros et al. (2001), design specifications can be formulated using a H_∞ 4-blocks type criteria. As a matter of fact, let us consider that the system is described by:

$$y = Gu + \widetilde{G}w$$

The closed-loop system which links the reference, r, and the perturbation, w to the tracking error, e and the controlled input, u is given by

$$\begin{bmatrix} e \\ u \end{bmatrix} = \begin{pmatrix} S & S\widetilde{G} \\ KS & KS\widetilde{G} \end{pmatrix} \begin{bmatrix} r \\ w \end{bmatrix} \tag{3}$$

where $S = (I + GK)^{-1}$ (sensitivity function).

The design specifications are then formulated using the following 4-blocks criteria, where the goal is to find the smallest $\gamma > 0$ and the stabilizing controller K such that

$$\left\| \begin{matrix} W_1 S & W_1 \tilde{G} S W_3 \\ W_2 K S & W_2 \tilde{G} K S W_3 \end{matrix} \right\|_\infty \leq \gamma$$

Following Pognant-Gros et al. (2001), $W_1, W_1^{-1} \in \mathcal{RH}_\infty$ is used to specify tracking performances, perturbation rejection and modulus margin.
$W_2, W_2^{-1} \in \mathcal{RH}_\infty$ is used to specify high frequencies constraints on the controlled input. This allows to constrain command effort and effects of sensor noise command. W_3 is a scaling factor acting on the perturbation.

The weighting functions W_1 and W_2 are chosen of the first order. In order to facilitate the frequency tuning, we use the form proposed by Font (1995):

$$W_i(s) = \frac{G_{i,\infty}\sqrt{|G_{i,0}^2 - 1|}s + G_{i,0}\omega_{i,c}\sqrt{|G_{i,\infty}^2 - 1|}}{\sqrt{|G_{i,0}^2 - 1|}s + \omega_{i,c}\sqrt{|G_{i,\infty}^2 - 1|}}$$

where

$$\begin{cases} |W_i(j0)| & = G_{i,0} \\ \lim_{\omega \to \infty} |W_i(j\omega)| & = G_{i,\infty} \\ |W_i(j\omega_{i,c})| & = 1 \end{cases}$$

We first present two SIMO H_∞ designs corresponding respectively to solution associated to the long and short river reaches.

Design for water saving: controller SIMO ♯1 We first present the classical SIMO controller, taking water from the first (large) dam, which has the larger time-delay.

The chosen weighting functions parameters are given in table 2. The optimization resulted in $\gamma = 1$, with a gain margin of $7.5\,dB$.

This controller takes water from the first dam, but reacts with a low performance to an unpredicted withdrawal w_3 (see fig. 2).

Design for high performance: controller SIMO ♯2 We now present the "best" controller with respect to perturbations rejection; this controller takes water from the second (small) dam, which is the one that can react quickly to a unmeasured perturbation. A withdrawal in w_3 is countered very quickly, as shown in fig. 3. However, this solution is not satisfactory, as the second dam has a limited resource. The chosen weighting functions parameters are given in table 3. The optimization resulted in $\gamma = 1$, with a gain margin of $8\,dB$.

Table 2. Parameters for weighting functions, controller SIMO ♯1

	W_1	W_2
G_0	1000	0.7
G_∞	0.2	100
ω_c	10^{-5} rad/s	10^{-4} rad/s

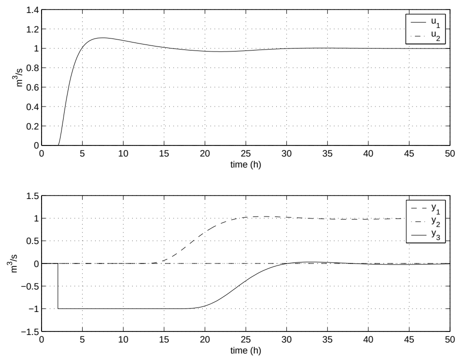

Fig. 2. SIMO controller ♯1 (large dam): reaction to an unmeasured withdrawal w_3

Table 3. Parameters for weighting functions, controller SIMO ♯2

	W_1	W_2
G_0	1000	0.1
G_∞	0.2	100
ω_c	9.10^{-5} rad/s	3.10^{-4} rad/s

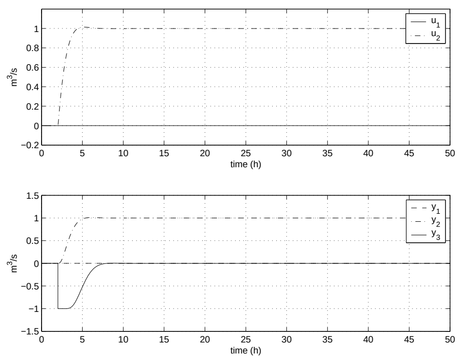

Fig. 3. SIMO controller ♯2 (small dam): reaction to an unmeasured withdrawal w_3

Summary The preliminary results presented show that it is possible to design performing H_∞ SIMO controllers for each dam, ensuring robustness margins and performance. However, our purpose is to take advantage of the two dams in order to recover the performance of the small one, without taking all the water from this one. This MIMO controller design will be detailed in the second part of the paper.

4. PROBLEM ANALYSIS

We will first consider a simplified model of the system in order to clarify the MIMO design philosophy, then the design specifications will be written in terms of H_∞ constraints.

4.1 Simplified model

Below is a simplified representation of the control problem, supposing only the output discharge y is measured and there is only an output perturbation w. This simplified representation enables to clearly explain the design trade-offs. With this hypothesis, the system can be written as the sum of two sub-systems, as in fig. 4. The controller is also the sum of two controllers, that can be tuned differently. The system $G_1(s)$ is the one with the larger time-delay, whereas the system G_2 exhibits a lower time-delay. G_3 is the common part, corresponding to the reach X_3.

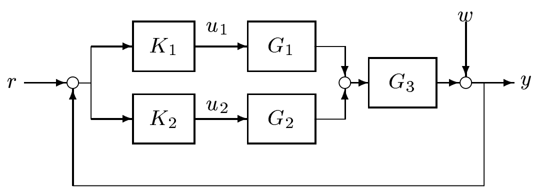

Fig. 4. Simplified representation of the control system

The objective is to recover the performance of the second dam (corresponding to the transfer G_2) with the sum of the two systems, one acting in low frequencies (K_1G_1) and the other one in high frequencies (K_2G_2). A naive solution would be to choose a perfect transfer function from a given desired closed-loop response (giving the best possible performance), and then find the corresponding open-loop transfers enabling to realize this perfect transfer. Let $K_{perfect}G_2$ be the desired open-loop transfer for the system. Then, one will try to identify K_1 and K_2 such that

$$K_{perfect}G_2G_3 = (K_1G_1 + K_2G_2)G_3$$

The idea is to tune K_1 in order to fit to $K_{perfect}G_2G_3$ in low frequencies, and to tune K_2 in order to fit to $K_{perfect}G_2G_3$ in high frequencies. However, this is difficult to do "manually" (by trial and error) and to simultaneously ensure the required robustness margins for the controlled system.

This is why the design is done using a systematic tool, by putting the design specifications in terms of H_∞ constraints.

4.2 MIMO H_∞ design

We now go back to the original system depicted in fig. 1, with three measurement points (denoted by the vector y), three perturbations (w) and a reference for the output discharge (r). The augmented system is represented in fig. 5.

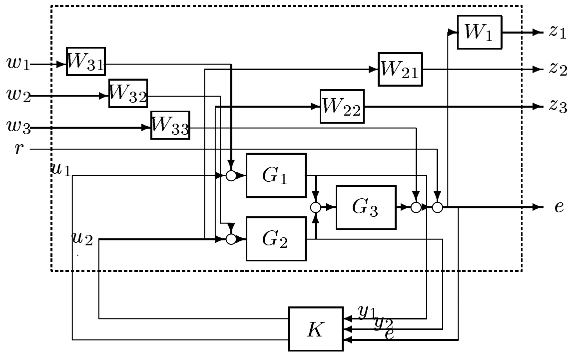

Fig. 5. Augmented system for H_∞ optimization

The transfer matrix between inputs $[r(s)\ w(s)]$ and outputs $[z(s)\ e(s)]$ is:

$$M = \begin{pmatrix} W_1S & W_1\tilde{G}SW_3 \\ W_2KS & W_2\tilde{G}KSW_3 \end{pmatrix}$$

The proposed design is developed based on the chosen weighting functions for the SIMO controllers. This design consists in mixing the previous controllers in the frequency domain. The first controller (the rapid one) is used in high frequencies, while the slow controller is used to deliver water in low frequencies.

This is done by specifying two weighting functions for the transfer KS:

- The constraint on KS_1 (from r to u_1) is specified by a weighting function W_{21} identical to the one used for the SIMO controller ♯1 design.
- The constraint on KS_2 (from r to u_2) is specified by a weighting function W_{22}, keeping the high frequency constraint defined in the SIMO case ♯2 and adding a supplementary constraint in low frequencies, in order to impose a quasi zero mean value for the discharge u_2 (see fig. 6).

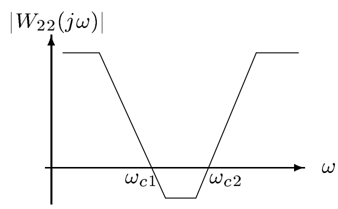

Fig. 6. Shape of frequency weighting function W_{22} for the MIMO controller

We use a band filter, as given by Font (1995), for the weighting function W_{22}:

$$W_{22}(s) = \frac{G_\infty s^2 + G_{min}\xi\omega_{min}s + G_\infty\omega_{min}^2}{s^2 + \xi\omega_{min}s + \omega_{min}^2}$$

where ξ is used to tune the bandwidth, G_{min} is the minimum gain, and ω_{min} the corresponding frequency. Let ω_{c1} and ω_{c2} be the cutting frequencies of this filter, one then has, for any $k > 0$ (specifying the filter width): $\omega_{c1} = \frac{1}{k}\omega_{min}$, $\omega_{c2} = k\omega_{min}$ and $\xi = \frac{|k^2-1|}{k}\frac{\sqrt{1-G_\infty^2}}{\sqrt{G_{min}^2-1}}$. The filter W_{22} is therefore completely determined if one chooses G_∞, G_{min}, ω_{c1} and ω_{c2}.

The weighting functions parameters were chosen equal to $\omega_{c1} = 10^{-5}$ rad/s, $\omega_{c2} = 3.10^{-4}$ rad/s, $G_\infty = 100$ and $G_{min} = 0.3$. The results of H_∞ optimization (transfer functions and corresponding constraints) are given in fig. 7.

The optimization resulted in $\gamma = 1.1$, with an input gain margin of $6.8\,dB$ and an output gain margin of $7.6\,dB$.

The simulation in fig. 8 shows that the desired behavior is obtained (compare to fig. 2): the second dam reacts quickly to a perturbation in w_3, and while the water released by the first dam reaches the controlled point, the water delivered by the second dam diminishes and goes back to zero. Therefore, the MIMO controller enables to recover the performance obtained by the SIMO controller ♯2 (see fig. 3), but with a constraint on the available volume of each dam.

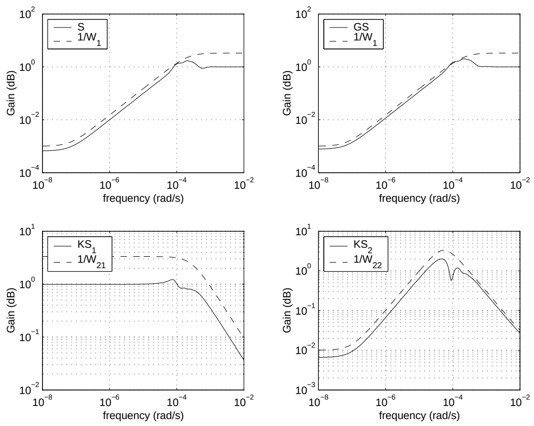

Fig. 7. Transfer functions and corresponding constraints for the MIMO controller

5. CONCLUSION

This paper ends a series of three papers on the control of open-channel systems. After having shown that the performance of such systems in terms of perturbation rejection is limited (Litrico and Fromion, 2001), we considered the possible improvement provided by additional information (Litrico et al., 2001). This paper focuses on the

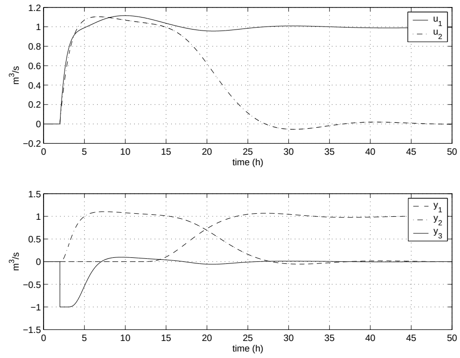

Fig. 8. Proposed MIMO controller: reaction to an unmeasured withdrawal w_3

problem of designing controllers for MIMO systems (river controlled by 2 dams) with a constraint on the available volume of each dam. This problem could not be solved easily with other classical approaches like LQG control (Litrico, 1999). This paper shows that the best achievable performance in terms of output perturbation rejection can be obtained with the two dams while constraining the small dam to have a mean discharge equal to zero (this constraint is specified by frequency weighting functions in a H_∞ framework). The mixed sensitivity controller exhibits good robustness margins, which are needed in an application perspective. The proposed control design method is flexible and can easily incorporate additional information when available.

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