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IDENTIFICATION OF PERFORMANCE LIMITATIONS IN CONTROL USING ARX-MODELS

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Abstract: Non-minimum phase zeros and poles of a process put upper and lower constraints on the bandwidth of a closed loop system. It is thus of great interest to be able to identify these quantities. In this contribution it is shown that non-minimum phase zeros and unstable poles can be identified using high order ARX-models without the standard $o(n)$ (n is the model order) variance penalty for over modeling. An asymptotic, in the model order and the number of data, expression for the variance of non-minimum phase zeros is derived. This result shows that the problem of determining the performance limits of a system from experimental data is considerably easier than identifying the complete system.

Keywords: System identification, poles and zeros, performance limitations

1. INTRODUCTION

Identification for control has received considerable interest in recent years. See (Gevers, 1993) for overviews of the activity in the early 1990's. Much of the attention has been on closed loop identification, see (Van den Hof and Schrama, 1995) and (Forssell and Ljung, 1999) and more recently on model validation/unfalsification (Bombois *et al.*, 1999), (Woodley *et al.*, 1998).

Experiment design in the context of control design has also received renewed interest, see e.g. (Hjalmarsson *et al.*, 1996), (Forssell and Ljung, 1998), (Lindqvist and Hjalmarsson, 2000), (Cooley *et al.*, 1998) and (Lindqvist, 2001).

An intrinsic problem in experiment design is that the optimal design depends on the system which is to be identified. Hence, even though these designs may be used to get intuition for how to design the identification experiment, they are in general infeasible.

Hence suboptimal methods must be developed. It is generally acknowledged that an accurate model is needed around the cross-over frequency *of the loop gain*. Since the loop gain depends on the, yet to be designed controller, this frequency region is in generally unknown. However, for non-minimum phase systems, the non-minimum phase zeros restricts the achievable bandwidth, see e.g. (Freudenberg and Looze, 1998) or (Skogestad and Postlethwaite, 1996). A real single non-minimum phase zero at z restricts the bandwidth to approximately $z/2$. Hence, if the non-minimum phase zeros were known, the experiment design problem would be simplified considerably. Knowledge of the performance limits would also ease the task of deciding on model structure, model order, noise model and pre-filters since one then knows the important frequency range.

From the discussion above it should be clear that knowledge of the non-minimum phase zeros is very useful in system identification for control design. Typically these zeros are not known so the question arises how difficult it is to get this information from a preliminary identification procedure. This is the theme of this paper.

We assume that the model order is unknown. Hence, the first problem is to determine a suitable model order. Usually it is desirable that the model is not more complex than the system to be identified since a too high model order results in a, due to variance errors, less accurate model. Asymptotically, the variance of e.g. the frequency function estimate is proportional to the model order (Ljung, 1999). In fact, the optimal model complexity should be less than the complexity of the system itself (this is the so called bias/variance trade-off), (Ninness and Goodwin, 1995). Thus much attention in the identification for control literature has been given to reduced order models, see e.g. (Zang *et al.*, 1995) and (Lee *et al.*, 1993).

In this contribution we show that identification of non-minimum phase zeros is not subject to the variance penalty referred to above. We show that the variance for identified non-minimum phase zeros converges to a finite value as the model order tends to infinity. A similar expression can be derived for unstable poles. Thus the model order issue is not critical when non-minimum phase zeros are estimated. *A high order ARX-model will do the job!* On the contrary, the variance of minimum-phase zeros and stable poles grows geometrically with the model order for models parameterized by transfer function coefficients. A result that shows that control design based on estimates of these quantities should be avoided.

These results then lead to a simple experiment design procedure when it is known that the system is non-minimum phase:

- Do an initial experiment design which is aimed at identifying non-minimum phase zeros and unstable poles of the system. This design is based on the asymptotic variance expression for that is derived in this paper.
- Then identify a high order ARX-model using the data from the initial experiment.
- Use the identified unstable non-minimum phase zeros of this model to determine the interesting frequency range for control design. For example, an identified real non-minimum phase zero at z implies that the maximum bandwidth is $z/2$. Use this information to do a new experiment design. This design can be more or less elaborate.
- Identify a new model based on data obtained with the new experiment design. Here, the knowledge of the desired bandwidth will help when deciding on model structure, model order, noise model and so on.

Notice that dual results can be derived for unstable poles.

In Section 2, the assumptions on which the following results rely on are stated. In Section 3, the covariance of the parameter estimates is given. Following, in Section 4 and 5, the asymptotic (in the number of data) covariance of the zeros is derived. Section 6 proposes a simplified expression for the asymptotic (in the number of data and in the model order) covariance of the ARX-model zeros and contain examples/simulations of the expression. Finally, conclusions can be found in Section 7.

2. ASSUMPTIONS

The conditions under which the identification will be performed must be defined carefully.

The following assumptions on the system and identification procedure are used throughout this paper

A1: The system is described by the ARX-structure, i.e.,

$$y(t) + \sum_{k=1}^{n_o^a} a_k^o y(t-k) = \sum_{k=0}^{n_o^b} b_k^o u(t-k) + e_o(t) \tag{1}$$

where $n_o^a \geq n_o^b$. Thus, the system is of order n_o^a. For ease of presentation, we will assume that $n_o^a - 1 = n_o^b = n_o$. It can be easily shown that the results holds even if this is not the case. Further, we assume that the system is stable, i.e. all poles lie inside the unit circle.

A2: The input is generated as $u(t) = F(q)v(t)$ where $F(q)$ is a minimum phase filter with no zeros on the unit circle and $v(t)$ is zero mean white Gaussian noise with variance 1.

A3: The system noise $e_o(t)$ is zero mean white Gaussian noise with variance σ^2.

A4: The model is described by

$$y(t) + \sum_{k=1}^{n^a} a_k y(t-k) = \sum_{k=1}^{n^b} b_k u(t-k) + e(t) \tag{2}$$

where $e(t)$ is white noise and $n^a \geq n_o^a$, $n^b \geq n_o^b$. This means that the system is in the model set. For ease of presentation, the modeling will be assumed to use the same number of a and b parameters i.e., $n = n^a - 1 = n^b$. The notation

$$\theta_o^n = \begin{bmatrix} a_1^o & \cdots & a_{n_0+1}^o & 0 & \cdots & 0 & b_0^o & \cdots & b_{n_0}^o & 0 & \cdots & 0 \end{bmatrix}^T$$

will also be used to denote the true parameter vector when the model is over-parametrized, i.e. when $n > n_o$.

A5: The least squares estimate

$$\widehat{\theta}_N^n = \left(\frac{1}{N} \sum_{k=0}^{N-1} \phi_n(k)\phi_n(k)^T \right)^{-1} \times \frac{1}{N} \sum_{k=0}^{N-1} \phi_n(k)y(k) \tag{3}$$

is used, where

$$\phi_n(t) = \begin{bmatrix} \left[-y(t-1) \ \cdots \ -y(t-n-1)\right]^T \\ \left[u(t) \ \cdots \ u(t-n)\right]^T \end{bmatrix}.$$

3. ESTIMATION ACCURACY

Under Assumptions A1 – A5, the optimal one step predictor for the output signal can be formed linearly in the parameters as

$$\widehat{y}(t|t-1,\theta_o^n) = \phi_n^T(t)\theta_o^n$$

Thus the parameter estimate is unbiased and the asymptotic covariance of the estimate is given by

$$\lim_{N\to\infty} N\mathbf{E}\{(\widehat{\theta}_N^n - \theta_o^n)(\widehat{\theta}_N^n - \theta_o^n)^T = \sigma^2\left(\mathbf{E}\{\phi_n(k)\phi_n(k)^T\right)^{-1}$$

See e.g. (Ljung, 1999) for details on estimation and asymptotic covariance expressions.

4. SYSTEM AND MODEL ZEROS

The linear parameterization is convenient since the identification procedure is easily performed. However, the objects of most interest are the zeros (and poles) of the system. Therefore, the measure of interest is not only the variance of the estimated parameters $\widehat{\theta}_N^n$, but also the variance of the corresponding zeros.

Introduce the polynomial

$$p(z,\theta^n) = b_0 z^n + b_1 z^{n-1} \cdots + b_n \qquad (4)$$

The system zeros are then defined as the solutions z_i^o, $i = 1, \cdots, n_o$ to the equation

$$p(z,\theta_o^{n_o}) = 0$$

For a *model* represented by the parameter θ^n, the zeros $z_i(\theta^n)$, $i = 1, \cdots, n$ are the solutions to

$$p(z,\theta^n) = 0$$

When the model is over-parametrized, i.e. when $n > n_o$, more parameters than necessary will be estimated, resulting in larger variance for the estimated parameters. It is therefore of great interest to see how the variance of the model zeros is affected by the over-modeling.

5. ASYMPTOTIC COVARIANCE EXPRESSION FOR THE MODEL ZEROS

For the purpose of evaluating the accuracy of the estimated parameters, the covariance and mean of the parameter estimates are usually used. The asymptotic expression show what happens for large sample data i.e., when the data used for identification is very long. These results are useful tools in determining parameter accuracy , since finite-data expressions are often very hard to compute. See (Ljung, 1999) for more background on the use of asymptotic expressions in the area of system identification.

Over-modeling implies that more zeros than necessary will be estimated. The asymptotic covariance for the zeros can be calculated using first order approximations (over line denotes complex conjugate). Below, let $z_k(\widehat{\theta}_N^n)$ be one of the estimated zeros and let z_k^o be the corresponding system zero $z_k(\theta_o^n)$. Then, as a natural extension of the previous results for FIR systems reported in (Lindqvist, 2001; Hjalmarsson and Lindqvist, 2001) (with a slight correction with the factor $|b_0^o|^2$), we have

$$\lim_{N\to\infty} N\mathbf{E}|z_k(\widehat{\theta}_N^n) - z_k^o)|^2 = \frac{\sigma^2|z_k^o|^2\left[1 \ \cdots \ (z_k^o)^{-n}\right]}{|b_0^o|^2\prod_{i\neq k}^{n_o}|1-\frac{z_i^o}{z_k^o}|^2} \times \left[\mathbf{E}\{\phi_n(t)\phi_n(t)^T\}^{-1}\right]_{(2,2)} \begin{bmatrix} 1 \\ \vdots \\ (\overline{z_k^o})^{-n} \end{bmatrix} \qquad (5)$$

where the subscript $(2,2)$ is used to extract the $(n+1)\times(n+1)$ lower right sub matrix.

Allowing the model order to increase together with the number of samples, we can formulate a simplified expression for the asymptotic covariance of non-minimum phase zeros. In previous contributions (Hjalmarsson and Lindqvist, 2001; Lindqvist, 2001), the asymptotic results was derived for the FIR case. In the presentation here, this model corresponds to the choice $n_a = 0$ and $n = n_b$ in the system structure (1) as well as the model structure (2). In the contributions (Hjalmarsson and Lindqvist, 2001; Lindqvist, 2001), the following theorem was the key result for deriving the asymptotic covariance expression in the FIR case.

Theorem 5.1. Let $u(t) = F(q)v(t)$ where $F(q)$ is minimum phase stable filter and $v(t)$ is zero mean Gaussian white noise with variance 1. Further, let R_{uu}^n be the Toeplitz matrix built up by the elements $r_u(k) = \mathbf{E}u(t)u(t-k)$, i.e., that the (j,k)th element of R_{uu}^n is $r_u(j-k)$. Further let z be such that $|z| > 1$. Then it holds that

$$\lim_{n\to\infty} \left[1 \ \cdots \ (z)^{-n}\right]\left(R_{uu}^n\right)^{-1}\begin{bmatrix} 1 \\ \vdots \\ (\overline{z})^{-n} \end{bmatrix} = \frac{1}{(1-|z|^{-2})\,|F(z)|^2}$$

Proof: see (Lindqvist, 2001) □.

From this result, it follows that the asymptotic (in n) limit of (5) for the FIR-system and model case is

$$\lim_{n\to\infty}\lim_{N\to\infty} N\mathbf{E}\left|z_k(\widehat{\theta}_N^n)-z_k^o\right|^2 = \frac{\sigma^2|z_k^o|^2}{|b_0^o|^2\left(1-|z_k^o|^{-2}\right)\prod_{i\neq k}^{n_o}\left|1-\frac{z_i^o}{z_k^o}\right|^2\left|F(z_k^o)\right|^2} \quad (6)$$

under the same assumptions as Theorem 5.1. This follows directly as $\mathbf{E}\{\phi_n(t)\phi_n(t)^T\} = \left(R_{uu}^n\right)^{-1}$ as defined in Theorem 5.1.

This theorem shows the interesting (and somewhat surprising) result that for non-minimum phase zeros, the variance does not grow unbounded with n, but converges to the limit (6). Contrary, for minimum phase zeros, it can be shown that the variance will grow geometrically with the number (n) of estimated parameters.

For details on the FIR case, see (Hjalmarsson and Lindqvist, 2001; Lindqvist, 2001).

6. ORDER-ASYMPTOTIC COVARIANCE OF THE ARX MODEL ZEROS

The most commonly used model structure in system identification is the ARX-model. The reason is that it gives a closed form solution to the identification problem, see (Ljung, 1999), and that it is very flexible; by choosing sufficiently high orders of the A and B polynomials any linear system can be modeled with arbitrary accuracy. However, if the true system is of ARMAX or Box-Jenkins structure, the orders of A and B may have to be chosen much higher than the true order of the system. It is thus of great interest to analyze how this over-modeling will affect the accuracy of pole and zero estimates.

The following theorem is an extension of the result in (Lindqvist, 2001; Hjalmarsson and Lindqvist, 2001) to ARX-models.

Theorem 6.1. Consider the ARX-model structure (1) and the least squares estimate (3). Assume that all *system* zeros are unique, i.e. that $z_k^o \neq z_i^o$ for $k \neq i$, and that Assumptions A1 – A5 are satisfied. The asymptotic covariance for the estimate $z_k(\widehat{\theta}_N^n)$ of a non-minimum phase zero at z_k^o is then

$$\lim_{n\to\infty}\lim_{N\to\infty} N\mathbf{E}\left|z_k(\widehat{\theta}_N^n)-z_k^o\right|^2 = \frac{\sigma^2|z_k^o|^2}{|b_0^o|^2\left(1-|z_k^o|^{-2}\right)\prod_{i\neq k}^{n_o}\left|1-\frac{z_i^o}{z_k^o}\right|^2\left|F(z_k^o)\right|^2} \quad (7)$$

Proof: see Appendix A □

Note that this result shows that the accuracy of the estimated non minimum phase zero is the same as for FIR modeling. This is interesting, especially in view of the extra a-parameters of the dynamics that are estimated. However, there is some intuition in the result as the signal to noise ratio of the output is independent of the (common) A-polynomial.

Notice that similar expressions can be derived for unstable poles of systems (identified in closed loop), however in this paper focus is on the identification of non-minimum phase zeros.

The implication of Theorem 6.1 is that there (asymptotically in the model order) is only a small penalty for the over-modeling when the aim is to identify non-minimum phase zeros.

6.1 *Numerical Example*

In this section, we will exemplify Theorem 6.1 and the implications of this result to high order estimation of zeros.

6.1.1. *Simulated Colored Noise* Consider the ARX system

$$\begin{aligned} y(t) - 1.93y(t-1) + 0.942y(t-2) = \\ -0.0242u(t) + 0.0073u(t-1) + \\ 0.96u(t-2) + e(t) \end{aligned} \quad (8)$$

which has a minimum phase zero in -0.9 and a non-minimum phase zeros in 1.2. The input for the identification experiment is generated as

$$u(t) = F(q)v(t) = \frac{q^2 + 1.273q + 0.81}{q^2 - 1.131q + 0.64}v(t) \quad (9)$$

where $v(t)$ is white Gaussian noise with variance 1.

The non minimum phase zero is identified using a model of order n, see Assumption A1, where n is varied from small to very large. Figure 1 show the results of Monte Carlo simulated covariance and the asymptotic expression (7) plotted versus the model order n.

The next example clarifies the difference between the variance for minimum phase and non-minimum phase zeros as the model order increases. For the same system (8) as previously we show the resulting zeros from 100 identifications superimposed in Figure 2 for different model orders. Notice how the locations of the non-minimum phase zeros are much less affected by the increase in model orders than the minimum phase zero.

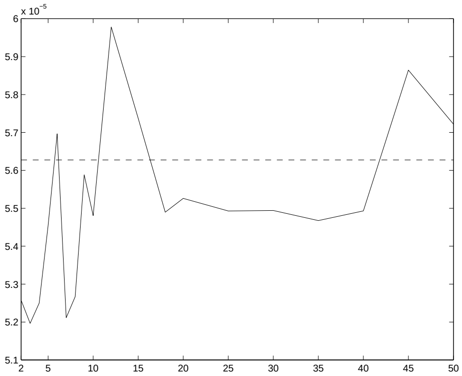

Fig. 1. Asymptotic covariance for the non-minimum phase zero plotted versus the model order. Solid line: Monte Carlo (1000 runs) simulated covariance. Dashed line: Asymptotic covariance expression using Theorem 6.1

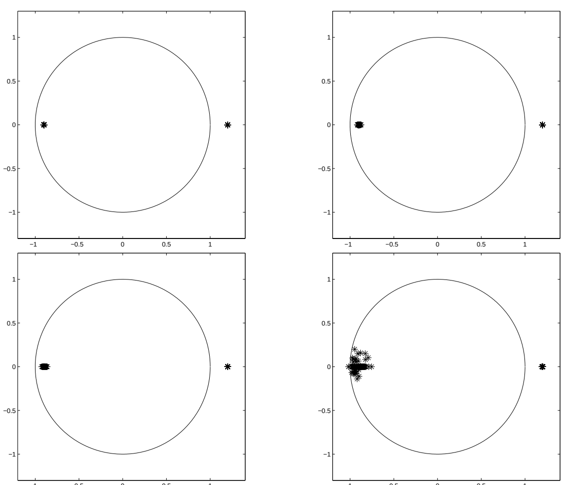

Fig. 2. Estimated zeros (at -0.9 and 1.2)) from 100 experiments for model orders $n = 2, 5, 10$ and 20. Notice the drastic increase in the variance for the minimum phase zero.

7. CONCLUSIONS

We have in this contribution shown the rather surprising result that the variance of estimated non-minimum phase zeros depends very little on the model order. Further, we have shown that there is only a small penalty for introducing the ARX-model structure (even if the model is of FIR-type) when considering the non-minimum phase zeros. This simplifies the problem of identifying a system's performance limitations considerably. A numerical example was used to illustrate the results. It has also been argued that these results can be explored in experiment design in identification for control.

APPENDIX A : PROOF OF THEOREM 6.1

Let $\Gamma_n(z) = [1 \cdots z^{-n}]$. The nontrivial part of the proof of Theorem 6.1 is to show that

$$\lim_{n\to\infty} \Gamma_n(z)\left[\mathbf{E}\{\phi_n(t)\phi_n(t)^T\}^{-1}\right]_{(2,2)} \Gamma_n^T(\overline{z}) = \frac{1}{\left(1-|z|^{-2}\right)\left|F(z_k^o)\right|^2} \tag{10}$$

where the subscript $(2,2)$ is used to denote the $n \times n$ lower right sub matrix.

For this purpose we define the impulse response of the system from the input $u(t)$ to the output $y(t)$ as the sequence $\{g_t\}_{t=0}^{\infty}$. The impulse response from the noise to output is defined as $\{h_t\}_{t=0}^{\infty}$.

Thus, the correlations $r_{uu}(k), r_{yu}(k), r_{yy}(k)$ can be derived as:

$$\begin{aligned} r_{uu}(k) &= \mathbf{E}u(t)u(t+k), \\ r_{yu}(k) &= \mathbf{E}y(t)u(t+k) = \sum_{m=0}^{\infty} g_m r_{uu}(m), \\ r_{yy}(k) &= \mathbf{E}y(t)y(t+k) = r_{yy}^u + r_{yy}^e, \\ r_{yy}^u(k) &= \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} g_m g_l r_{uu}(m-l), \\ r_{yy}^e(k) &= \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} h_m h_l \sigma^2 \end{aligned}$$

where $r_{yy}^e(k)$ corresponds to the part of $r_{yy}(k)$ that comes from the noise and r_{yy}^u corresponds to the part of $r_{yy}^u(k)$that comes from the input.

Let $R_{uu}^n, R_{yu}^n, R_{uy}^n, R_{yy}^n$ be the corresponding $n+1 \times n+1$ correlation matrices such that

$$\begin{aligned} &[R_{uu}^n]_{(j,k)} = r_{uu}(j-k), \quad \left[R_{yy}^n\right]_{(j,k)} = r_{yy}(j-k), \\ &\left[R_{yu}^n\right]_{(j,k)} = r_{yu}(j-k+1), \\ &\left[R_{uy}^n\right]_{(j,k)} = r_{yu}(k-j-1). \end{aligned}$$

Then, the matrix $\mathbf{E}\{\phi_n(t)\phi_n(t)^T\}$ can be composed as

$$\mathbf{E}\left\{\phi_n(t)\phi_n(t)^T\right\} = \begin{bmatrix} R_{yy}^n & -R_{yu}^n \\ -R_{uy}^n & R_{uu}^n \end{bmatrix}. \tag{11}$$

Now, using block matrix inversion formulas, the lower sub matrix of the inverse of 11 can be derived as

$$\left[\mathbf{E}\{\phi_n(t)\phi_n(t)^T\}^{-1}\right]_{(2,2)} = \left(R_{uu}^n\right)^{-1} + \left(R_{uu}^n\right)^{-1} \times R_{uy}^n \left(R_{yy}^n - R_{yu}^n\left(R_{uu}^n\right)^{-1}R_{uy}^n\right) R_{yu}^n\left(R_{uu}^n\right)^{-1} \tag{12}$$

Using Theorem 5.1, we have that

$$\lim_{n\to\infty} \Gamma_n(z)\left(R_{uu}^n\right)^{-1}\Gamma_n(z) = \frac{1}{\left(1-|z|^{-2}\right)|F(z)|^2} \tag{13}$$

where $F(q)$ is the minimum phase input signal shaping filter as defined in A2.

The remaining issue of the proof is to determine that the second part of (12) vanishes as $n \to \infty$. To this end, we note that there exists a finite c and a $|\lambda| < 1$ such that

$$|g_n| < c\lambda^n \quad \text{and} \quad |r_{uu}(n)| < c\lambda^n$$

holds. This follows from assumption A2.

Using this, it can be shown that

$$\begin{aligned}
&\left[R^n_{yu}\left(R^n_{uu}\right)^{-1}\right]_{(k,j)} = \\
&\sum_{m=1}^{n+1}\sum_{l=0}^{\infty} g_l r_{uu}(k-m+l)\left[\left(R^n_{uu}\right)^{-1}\right]_{(m,j)} = \\
&\sum_{l=0}^{n-k} g_l \sum_{m=1}^{n+1} [R^n_{uu}]_{(k+l,m)}\left[\left(R^n_{uu}\right)^{-1}\right]_{(m,j)} + \\
&\sum_{l=n-k+1}^{\infty} g_l r_{uu}(k-m+l)\left[\left(R^n_{uu}\right)^{-1}\right]_{(m,j)} = \\
&g_{j-k} + o(\lambda^{n-k})
\end{aligned}$$

holds (since $\|\left(R^n_{uu}\right)^{-1}\|$ is limited, see (Grenander and Szegö, 1958)) . As a consequence, we obtain

$$\begin{aligned}
\Gamma_n(z)\left[R^n_{yu}\left(R^n_{uu}\right)^{-1}\right]_{(:,j)} &= \\
z^{-j}G(z) + o(\max\{z^{-n}, \lambda^n\})
\end{aligned} \tag{14}$$

where $(:, j)$ is used to denotes the jth column of the corresponding matrix and $G(z) = \sum_{l=0}^{\infty} z^{-l} g_l$.

Now, continuing with $R^n_{yu}\left(R^n_{uu}\right)^{-1}R^n_{uy}$, we have

$$\left[R^n_{yu}\left(R^n_{uu}\right)^{-1}R^n_{uy}\right]_{(k,j)} = r^u_{yy}(j-k) + o(\lambda^n) \tag{15}$$

using the arguments as before.

Thus, combining (14) and (15) and applying Theorem 5.1 we have that

$$\begin{aligned}
\lim_{n\to\infty} &\Gamma^T_n(z)\left(R^n_{uu}\right)^{-1}R^n_{uy} \times \\
&\left(R^n_{yy} - R^n_{yu}\left(R^n_{uu}\right)^{-1}R^n_{uy}\right) \times \\
&R^n_{yu}\left(R^n_{uu}\right)^{-1}\Gamma_n(\overline{z}) = \\
&\frac{|G(z)|^2}{\sigma^2 |H(z)|^2 (1 - |z|^{-2})} = 0
\end{aligned} \tag{16}$$

since $G(z) = 0$ (z is a zero of $G(z)$).

Thus, by combining (13), (14) and (15), we have that (10) holds.

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