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BLOCK DESIGN OF THE TRACKING PROBLEM WITH RESPECT TO THE OUTPUT

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Abstract: This paper is introduced the new approach to synthesis of the tracking problem with respect to the output on the basis of transformations of linear systems into the block-canonical form of the controllability. The decomposition approach to a solution of the tracking problem with a given accuracy is offered, that allows one to loosen conditions imposed on the class of demanded signals. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The paper is studied the structural properties of control systems in the tracking problem with respect to demanding values of the output and introduced synthesis algorithms of such problems on the basis of the block-control principle (Drakunov, *et al.*, 1990) with reference to linear stationary systems of a general view. The necessary and sufficient conditions of a solution of the tracking problem are obtained on the basis of the transformation of the initial system into the block-canonical form of the controllability with respect to the output (Utkin V.A., 2001). It is essential, that the given algorithms of synthesis of the tracking problem with a given accuracy do not superimpose the padding requirements on smoothness of driving functions and guess only their restricted modulo together with the first derivatives.

2. PRIOR RESULTS AND PROBLEM FORMULATION

Consider a time-invariant linear system described by

$$\dot{x} = Ax + Bu, \quad y_1 = Dx, \tag{1}$$

where $x \in X \subset R^n$ is the state, $u \in U \subset R^m$ is the control input, $y_1 \in R^p$ is the output, A, B, D are know constant matrices of appropriate dimensions. The tracking problem with respect to demanded values of the output $y_{d1}(t) \in R^p$ is reduced to the stabilization problem of tracking errors

$$\Delta y_1 = y_1 - y_{d1}(t), \; \lim_{t\to\infty} \Delta y_1 = 0. \tag{2}$$

Standard approach to a solution of the problem is the translation one to the stabilization problem of system (1), rewritten with respect to tracking errors of the state $\Delta x = x - x_d(t)$:

$$\begin{aligned} &\Delta\dot{x} = A\Delta x + Bu + \eta(t), \; y = D(\Delta x + x_d), \\ &\eta = (Ax_d + \dot{x}_d), \end{aligned} \tag{3}$$

where the demanded values of the state components are searched from the following relation

$$Dx_d(t) = y_{d1}(t). \tag{4}$$

Let us note some problems originating at usage of the approach.

1. The stabilization problem with respect to all components of the state of system (3), in which the variables $\eta(t)$ are considered as the external disturbances, has a solution with necessity and sufficiency only in the case, when the disturbing actions are affixed only on the input of the control plant (Drazenovic, 1969). In particular, the stabilization problem of system (3) has a solution always, if the dimension of the control input is equal or more to the dimension of the state and $\text{rank}B = n$. In a case, when $\dim y_1 = \text{rank}D = n$, equation (4) has an alone solution, and there is no possibility to affect the circumscribed above situation. Otherwise, if $\dim y_1 = \text{rank}D < n$, the equation (4) has not an alone solution and there is a multivariate possibility of a choice of demanded values only for a part of components (or their linear combination) of the state. For example, if the state of system (1) is divided into two groups so, that in the expression $y_1 = D_1 x_1 + D_2 x_2$ condition $\text{rank}D_1 = \dim y_1 = p$ is satisfied, the choice of demanded values $\bar{x}_1 = D_1^{-1}(\bar{y}_1(t) - D_2 x_2)$ ensures equality (4), and components of the state x_2 remain free. Accordingly, the dimension of the disturbances in system (3) is reduced and conditions of a solution of the stabilization problem are loosened. Thus, for translation of the tracking problem with respect to output (2) to stabilization problem (3) is necessary to found any compromise between a choice of a minor of the complete rank of the matrix D and evaluation of demanded values of the relevant components of the state, and possibility of a solution of the stabilization problem of system (3) with respect to these components. As a whole, the problem of translation of the tracking problem on the output to the stabilization problem of all state vector (or definition of the demanded values with respect to state components under the demanded output) with provision of it's solution is put.

2. The representation of values of the output can be not correct in the sense, that they can not be implemented in a closed-loop system. For example, in a system of the second order described by $\dot{x}_1 = ax_1 + x_2, \dot{x}_2 = u$, by virtue of a physical relation between variables of the type "position – derivative" is not possible to implement arbitrary given functions of these variables. Let us rewrite this system with respect to mismatches as follows

$$\Delta\dot{x}_1 = a\Delta x_1 + \Delta x_2 + (a_1 x_{d1} + x_{d2} - \dot{x}_{d1}),\ \Delta\dot{x}_2 = u - \dot{x}_{d2}$$

and consider the static equations $\Delta\dot{x}_1 = \Delta\dot{x}_2 = 0$. So we shall receive the unique optional version of implementation of demanded values $a_1 x_{d1} + x_{d2} - \dot{x}_{d1} = 0$, where one of a variable completes a demanded value, and another obeys to the equation. On the other hand, there can be only the problem of engineering implementation of a tracking system, bound with its interior stability. For example, if in the system of the second order described above to give the tracking problem with respect to only the second variable $\Delta x_2 = x_2 - x_{d2}(t) \to 0, t \to \infty$, a solution of the tracking problem will be correct, if the proper motions of the first variable (zero of a closed-loop system) are stabilized, i.e. $a < 0$.

Summarizing this point, let us allocate necessity of solving of two following problems: first, inspection of a formulation of the tracking problem on correctness of forming of demanded values; second, functional test of a closed-loop system on an interior stability (transmission zero).

For a solution of the tracking problem with respect to the output, in present paper the known approach based on the transformations of the initial linear model of the control plant into the block-canonical form of the controllability (Utkin Victor A., 2001) is utilized to a solution of the stabilization problem. The block-canonical form of controllability described by

$$\begin{aligned}
\Delta\dot{x}_r &= A_{r,r}\Delta x_r + B_r x_{r-1} + \eta_1 + Q_r f(t),\\
\dot{x}_{r-1} &= A_{r-1,r}x_r + A_{r-1,r-1}x_{r-1} + B_{r-1}x_{r-2} + Q_{r-1} f(t),\\
&\ldots\\
\dot{x}_i &= A_{i,r}x_r + \ldots + A_{i,i}x_i + B_i x_{i-1} + Q_i f(t),\\
&\ldots\\
\dot{x}_1 &= A_{1,r}x_r + \ldots + A_{1,1}x_1 + B_1 x_0 + Q_1 f(t),\\
\dot{x}_0 &= A_{0,r}x_r + \ldots + A_{0,0}x_0 + B_0 u + Q_0 f(t),
\end{aligned} \tag{5}$$

where $\dim x_i = \text{rank}B_i = m_i,\ \sum_{i=0}^{r} m_i = n,\ i = \overline{1,r}$, A_{ij}, B_i, Q_i are known constant matrices of appropriate dimensions, vector $f(t)$ is arbitrary restricted modulo exterior disturbances, the vector $\eta_1 = (A_{rr}x_{dr} - \dot{x}_{dr}) \in R^{m_r}$ occurs in connection with an entry of the first equation in (5) with respect to tracking errors $\Delta x_r = x_r - x_{dr}$ in the tracking problem on variables x_r considered as the output. In the framework of systems synthesis with divided motions with respect to system (5), following theorem is proved.

Theorem 1. (Utkin V. A., 2001). In system (5) the invariance conditions with respect to the output $y = D_r x_r$ are fulfilled.

Thus, in the supposition, that the disturbances are not accessible to measuring and restricted modulo functions in time, the stabilization problem of the output with a given accuracy is resolved. In particular, vector η_1 can be referred to external disturbances, if to assume, that its components are restricted modulo functions, that allows to not utilize derivatives of demanded values in a feedback circuit. For a case, when the disturbances are accessible to measuring and in the supposition about existence their derivative up to the order $(r-1)$, the problem of asymptotic convergence to zero of the output is resolved. Designed step-by-step procedure (Utkin V. A., 2001) of the stabilization problem of system (5) allows sequentially to select variables $x_{r-1},...x_0$, considered as the virtual controls, and purely the control u.

Unfortunately, the block-canonical form of a controllability (5) does not allow to solve a tracking problem on the output, as, in generally, the output in the terms of system (5) looks like $y = D_r x_r + ... + D_0 x_0$ and does not coincide with the vector x_r. In following section, with reference to linear systems, the tracking problem with respect to the output is solved on the basis of the transformations of the initial system into the block-canonical form of a controllability with respect to the output.

3. MAIN RESULT

Consider the tracking problem on the output for the linear system described by (1), (2). Let us introduce the step-by-step procedure of the transformations of system (1)-(2) into the block-canonical form of the controllability with respect to the output.

Step 1. (i) Let us assume, that $\text{rank}D = p_1 \neq 0$ and divide the state components of system (1) so that in the equation $y_1 = D_{10}^1 x_0^1 + D_{10} x_0$ the conditions $\dim D_{10}^1 = p_1 \times p_1$, $\dim D_{10} = p_1 \times (n - p_1)$, $\det D_{10}^1 \neq 0$ are met. Let us rewrite the system given by equation (1) with respect to variables y_1, x_0 as follows

$$\begin{aligned} \dot{y}_1 &= A_1 y_1 + A_{10} x_0 + B_{10} u, \\ \dot{x}_0 &= A_{01} y_1 + A_0^* x_0 + B_0 u, \\ y_1 &\in R^{p_1}, x_0 \in R^{n-p_1}, u \in R^m. \end{aligned} \tag{6}$$

(ii) If in system (6) the conditions $\text{rank}B_1 = \hat{p}_1$ and $\text{rank}\{B_1, A_{10}\} = \hat{p}_1 + p_2$, $p_2 \neq 0$ are met, the first equation in (6), after series transformations to the canonical form of the controllability, at first, with respect to the control u and then, with respect to the virtual control x_0, can be transformed into the following form

$$\begin{aligned} \dot{\bar{y}}_1 &= \bar{A}_1 y_1, \\ \dot{y}_2^* &= A_{21}^* y_1 + A_{20}^* x_0, \\ \dot{\hat{y}}_1 &= \hat{A}_1 y_1 + \hat{A}_{10} x_0 + \hat{B}_1 u, \end{aligned} \tag{7}$$

where $y_1 = (\bar{y}_1^T, y_2^{*T}, \hat{y}_1^T)^T \in R^{p_1}$, $\hat{y}_1 \in R^{\hat{p}_1}$, $y_2^* \in R^{p_2}$, $\bar{y}_1 \in R^{\bar{p}_1}$, $\text{rank}\hat{B}_1 = \hat{p}_1$, $\text{rank}A_{20}^* = p_2$. Generally, some equations in (7) can miss, if dimensions of their states are equal to zero. If $p_2 = 0$ or $\hat{p}_1 = 0$, then the second or the third equation is missed. A situation $\hat{p}_1 + p_2 = 0$ is impossible, as contradicts a requirement of the controllability of the initial system.

(iii) Let us make non-singular substitution of control components as follows

$$\begin{vmatrix} \hat{B}_{11} & \hat{B}_{12} \\ 0 & I \end{vmatrix} \begin{vmatrix} u_0 \\ u_1 \end{vmatrix} = \begin{vmatrix} v_0 \\ u_1 \end{vmatrix},$$

where $u^T = (u_0^T, u_1^T)$, $\hat{B}_1 = (\hat{B}_{11}, \hat{B}_{12})$, $\dim \hat{B}_{11} = (\hat{p}_1 \times \hat{p}_1)$, $\det \hat{B}_{11} \neq 0$, $\dim I = (m - \hat{p}_1) \times (m - \hat{p}_1)$.

As a result of the transformations on the step 1, the initial system, by virtue of the control $v_0 = -\hat{A}_{10} x_0 + v_0^*$ selected on the basis of (6)-(7), will be rewritten as follows

$$\begin{aligned} \dot{\bar{y}}_1 &= \bar{A}_1 y_1, \\ \dot{y}_2^* &= A_2^* y_1 + A_{20}^* x_0, \\ \dot{\hat{y}}_1 &= \hat{A}_1 y_1 + v_0^*, \\ \dot{x}_0 &= A_{01} y_1 + A_0 x_0 + B_{00} v_0^* + B_{01} u_1, \end{aligned} \tag{8}$$

where $v_0^* \in R^{\hat{p}_1}$, $u_1 \in R^{m - \hat{p}_1}$, $\text{rank}B_{01} = m - \hat{p}_1$. The procedure is finished, if the second equation in (8) ($p_2 = 0$) is missed. Thus, the pair of matrixes $[(\bar{A}_1^T, \hat{A}_1^T)^T, I_{\hat{p}_1 \times \hat{p}_1}]$ is controllable. An engineering specification is stabilizability of a pair of matrixes (A_0, B_{01}). Otherwise ($p_2 \neq 0$) we go to the step 2, where the transformations of the step 1 are also applied to the system

$$\begin{aligned} \dot{x}_0 &= A_{01} y_1 + A_0 x_0 + B_{00} v_0^* + B_{01} u_1, \\ \dot{y}_2^* &= A_2^* y_1 + A_{20}^* x_0. \end{aligned} \tag{9}$$

Step 2. (i) Let us assume, that $\text{rank}A_{20}^* = p_2$ and divide the state components of system (9) so that in the equation $A_{20}^* x_0 = A_{11}^1 x_1^1 + A_{11} x_1$ the conditions $\dim A_{11}^1 = p_2 \times p_2$, $\det A_{11}^1 \neq 0$,

$\dim A_{11} = p_2 \times (n - p_2)$ are met. Let us rewrite the system given by equation (9) with respect to variables $\dot{y}_2^*, x_1^*$ as follows

$$\ddot{y}_2^* = A_{21}^* y_1 + A_{22}^* \dot{y}_2^* + A_{21} x_1^* + B_{20} v_0^* + B_{21} u_1,$$
$$\dot{x}_1^* = A_{11}^* y_1 + A_{12}^* \dot{y}_2^* + A_1 x_1^* + B_{10} v_0^* + B_1^* u_1, \quad (10)$$
$$y_2^* \in R^{p_2},\ x_1^* \in R^{n_1 - p_1 - p_2},\ u_1 \in R^{m - \hat{p}_2}.$$

Change of the variables described by

$$\dot{y}_2 = \dot{y}_2^* - B_{20} \hat{y}_1 - A_{22}^* y_2^*,\ x_1 = x_1^* - B_{10} \hat{y}_1 - A_{12}^* y_2^*$$

in valid (8) allows to eliminate the second and fourth items in both equations (10), that allows to rewrite system (10) as follows

$$\ddot{y}_2 = A_{21} y_1 + A_{21} x_1 + B_{21} u_1,$$
$$\dot{x}_1 = A_{11} y_1 + A_1 x_1 + B_{11} u_1. \quad (11)$$

As a result of the transformations of points (ii) and (iii), the system given by equation (11) can be transformed into the following form

$$\ddot{\bar{y}}_2 = \bar{A}_2 y_1,$$
$$\ddot{y}_3^* = A_3^* y_1 + A_{31}^* x_1,$$
$$\ddot{\hat{y}}_2 = \hat{A}_2 y_1 + v_1^*, \quad (12)$$
$$\dot{x}_1 = A_{11} y_1 + A_1 x_1 + B_{11}^* v_1^* + B_{12} u_2,$$

where $\bar{y}_2 \in R^{\bar{p}_2}$, $y_3^* \in R^{p_3}$, $y_2 \in R^{\hat{p}_2}$, $\bar{p}_2 = p_2 - \hat{p}_2 - p_3$, $x_1 \in R^{n_1 - p_1 - p_2}$, $u_2 \in R^{m - \hat{p}_1 - \hat{p}_2}$, $\text{rank} A_{31}^* = p_3$. The procedure is finished, if the second equation in (12) ($p_3 = 0$) misses. Otherwise ($p_2 \neq 0$), we go to the step 3, where the transformations of the step 1 are also applied to the system $\dot{x}_1 = A_{11} y_1 + A_1 x_1 + B_{11} v_1^* + B_{12} u_2$, $\ddot{y}_3^* = A_3^* y_1 + A_{31}^* x_1$, etc.

Thus, on each i-th step of the procedure, the dimension of the not transformed state x_{i-1} decreases and the procedure is finished for a finite number of steps. Let the procedure is finished on the step ν. As a result, system (1) is transformed into the block-canonical form of controllability with respect to the output $y_1^T = (\bar{y}_1^T, ..., \bar{y}_\nu^T, \hat{y}_1^T, ..., \hat{y}_\nu^T) \in R^p$ designated as follows $\bar{y}_i = \bar{y}_{i1}$, $\hat{y}_i = \hat{y}_{i1}, i = \overline{1, \nu}$, which described by

$$\dot{\bar{y}}_{11} = \bar{A}_1 y_1,$$
$$\dot{\bar{y}}_{21} = \bar{y}_{22},\ \dot{\bar{y}}_{22} = \bar{A}_{21} y_1 \quad (13)$$
$$\ldots$$
$$\dot{\bar{y}}_{\nu 1} = \bar{y}_{\nu 2},\ \ldots,\ \dot{\bar{y}}_{\nu\nu} = \bar{A}_{\nu 1} y_1.$$

$$\dot{\hat{y}}_{11} = \hat{A}_1 y_1 + v_0^*,$$
$$\dot{\hat{y}}_{21} = \hat{y}_{22},\ \dot{\hat{y}}_{22} = \hat{A}_{21} y_1 + v_1^*, \quad (14)$$
$$\ldots$$
$$\dot{\hat{y}}_{\nu 1} = \hat{y}_{\nu 2}, \ldots, \dot{\hat{y}}_{\nu\nu} = \hat{A}_{\nu 1} y_1 + v_{\nu-1}^*.$$

$$\dot{x}_{\nu-1} = A_{\nu-1,1} y_1 + A_{\nu-1} x_{\nu-1} + B_{\nu-1} u_\nu. \quad (15)$$

The structure of system (13)-(15) is shown in fig. 1.

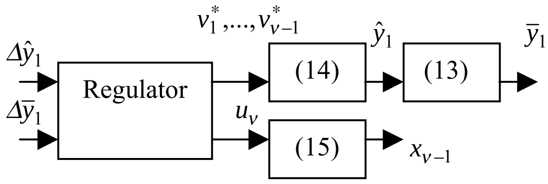

Figure 1.

The aggregate dimension of system (13)-(15) is equal to the dimension of initial system (1) and $\dim \bar{y}_1 + \dim \hat{y}_1 = \dim y_1$. Sub-system (15) is not observable with respect to the states of sub-systems (13)-(14). If initial system (1) is controllable, then sub-system (13) is also controllable by the input $\hat{y}_1$. With reference to system (13)-(15), necessary and sufficient conditions of a solution of the tracking problem with respect to the output are the following.

1. Allowing, that in further the stabilization problem of systems (13)-(14) is considered, stabilizability of system (13) or pair $(A_{\nu-1}, B_{\nu-1})$ is the necessary condition.

2. If system (13) misses ($\dim \bar{y}_1 = \dim(\bar{y}_{11}^T, ..., \bar{y}_{\nu 1}^T)^T = 0$), the expression $y_1 = H\hat{y}_1$, $\hat{y}_1^T = (\hat{y}_{11}^T, ..., \hat{y}_{\nu 1}^T)$, where matrix $H\,(\det H \neq 0)$ determined by described above procedure, meets. Sub-system (14) has structure of the block-canonical form of the controllability (5) and, therefore, the tracking problem has a solution for arbitrary demanded values y_{d1}. Let us note, that in this case requirements of autonomous control (Morse A. S and Wonham W.M.,1091) are not met. These reasons allow to formulate the following theorem.

Theorem 2. The tracking problem with respect to output (1)-(2) has a solution if and only if, in the terms of system (13)-(15), $\dim \bar{y}_1 = \dim(\bar{y}_{11}^T, ..., \bar{y}_{\nu 1}^T)^T = 0$ and pair of matrixes $(A_{\nu-1}, B_{\nu-1})$ is stabilizable.

The proof of theorem 2. The necessity immediately follows from the fact, that if even one i - th equation in system (13) is present, a part of variables $\hat{y}_1$

completes demanded values $\bar{y}_{di}(t)$. Really, let us consider without loss of generality a solution of the tracking problem $\Delta y_1 = y_1 - y_{d1}(t) \to 0$ in the last subsystem of system (13) described by

$$\Delta\dot{\bar{y}}_{v1} = \bar{y}_{v2} + \dot{y}_{v1d}, ..., \dot{\bar{y}}_{vv} = \bar{A}_{v1}(\bar{y}_1^T, \hat{y}_1^T)^T, \quad (16)$$

where variables $\hat{y}_1$ are considered as virtual variables. Then, variables $\hat{y}_1$ become functions in mismatches Δy_1, and the tracking problem with respect to arbitrary demanded values of these variables can not be solved. The sufficiency follows from the following design procedure. Without loss of generality, let us consider sub-system (16), rewritten with respect to tracking errors $\Delta\hat{y}_{v1} = \hat{y}_{v1} - \hat{y}_{dv1}$ designated as follows $x_1 = \Delta\hat{y}_{v1}$, $x_2 = \hat{y}_{v2}, ..., x_v = \hat{y}_{vv}$ and $v = v_{v-1}^* + \hat{A}_{v1} y_1$:

$$\begin{aligned} \dot{x}_1 &= x_2 + \eta, \\ \dot{x}_i &= x_{i+1}, i = \overline{2, v-1}, \\ \dot{x}_v &= v, \end{aligned} \quad (17)$$

where $\eta = \hat{y}_{dv1} - \dot{\hat{y}}_{dv1}$, $|\eta(t)| \le N = \text{const}$. A solution of the stabilization problem of system (17) with given accuracy $|x_1| \le \delta_1 = \text{const}$ is grounded on the basis of the backstepping procedure and the second method by Lyapunov. Rewrite system (17) as follows

$$\begin{aligned} \dot{\bar{x}}_1 &= -k_1\bar{x}_1 + \bar{x}_2 + \eta, \\ \dot{\bar{x}}_i &= -k_i\bar{x}_i + \bar{x}_{i+1} + k_1...k_{i-1}\eta, \; i = \overline{2, v-1}, \\ \dot{\bar{x}}_v &= -k_v\bar{x}_v + z + v + k_1...k_{v-1}\eta \end{aligned} \quad (18)$$

via the step-by-step procedure of non-special transformations described as

$$\begin{aligned} \bar{x}_1 &= x_1, \\ \bar{x}_i &= \sum_{j=1}^{i-1} p_{ij}\bar{x}_j + x_i, \; i = \overline{2, v}, \\ z &= \sum_{j=1}^{v} p_{v+1,j}\bar{x}_j, \end{aligned}$$

where $p_{21} = k_1$, $p_{i1} = -k_1 p_{i-1,1}$, $i = \overline{3, v+1}$, $p_{ij} = p_{i-1,j-1} - p_{i-1,j}k_j$, $j = \overline{2, i-2}$, $i = \overline{4, v+1}$; $p_{i,i-1} = p_{i-1,i-2} + k_{i-1}$, $i = \overline{3, v+1}$; $k_i > 0$, $i = \overline{1, v}$ are feedback gains, which are being a subject to definition. Variable z in system (18) is compensated by the controls selected as follows $v = -z$. Then, the last equation in system (18) designated as follows

$$\dot{\bar{x}}_v = -k_v\bar{x}_v + k_1...k_{v-1}\eta. \quad (19)$$

Let us consider a quadratic form $V = V_1 + ... + V_v$ as a sum of quadratic forms described by

$$V_i = 1/2(\bar{x}_i^T \bar{x}_i), \; i = \overline{1, v}. \quad (20)$$

The common estimation of derivative of the quadratic form $\dot{V}$ can be received by sequentially estimation of derivative of each item (20) with the following step-by-step procedure.

Step 1. For derivative of the first item of a quadratic form (20), taking into account of (18), the following estimation is valid:

$$\dot{V}_1 = \bar{x}_1(-k_1\bar{x}_1 + \bar{x}_2 + \eta) \le \|\bar{x}_1\|(-k_1\|\bar{x}_1\| + \|\bar{x}_2\| + \|\eta\|).$$

The inequality $\dot{V}_1 < 0$ is met outside of a neighbourhood $\|\bar{x}_1\| \le (\|\bar{x}_2\| + \|\eta\|)/k_1 \le \delta_1$ under fulfillment of the condition $k_1 > (\|\bar{x}_2\| + \|\eta\|)/\delta_1$. Taking into account, that on step 2 of the procedure by choice of feedback gain k_2 (under fixed gain k_1) the inequality $\|\bar{x}_2\| \le \delta_2$ will be valid, a lower estimation for choice of feedback gain k_1 is defined from the following expression

$$k_1^* = k_1 > (\delta_2 + \|\eta\|)/\delta_1.$$

The procedure of a choice of gain described by step 1 is also applied to the following items in (20) on steps i, where $i = \overline{2, v-1}$.

Step i. For derivative of the i - th item of a quadratic form (20), taking into account of (18), the following estimation is valid:

$$\begin{aligned} \dot{V}_i &= \bar{x}_i(-k_i\bar{x}_i + \bar{x}_{i+1} + k_1...k_{i-1}\eta) \le \\ &\le \|\bar{x}_i\|(-k_i\|\bar{x}_i\| + \|\bar{x}_{i+1}\| + k_1^*...k_{i-1}^*\|\eta\|). \end{aligned}$$

The inequality $\dot{V}_i < 0$ is met outside of a neighbourhood $\|\bar{x}_i\| \le (\|\bar{x}_{i+1}\| + k_1^*...k_{i-1}^*\|\eta\|)/k_i \le \delta_i$ under fulfillment of the condition $k_i > (\|\bar{x}_{i+1}\| + k_1^*...k_{i-1}^*\|\eta\|)/\delta_i$. Taking into account, that on step $(i+1)$ of the procedure the inequality $\|\bar{x}_{i+1}\| \le \delta_{i+1}$ will be valid, a lower estimation for choice of feedback gain k_i is defined from the following expression

$$k_i^* = k_i > (\delta_{i+1} + k_1^*...k_{i-1}^*\|\eta\|)/\delta_i.$$

Step v. For derivative of the v -th item of a quadratic form (20), taking into account of (19), the following estimation is valid:

$$\dot{V}_{\nu} = \bar{x}_{\nu}(-k_{\nu}\bar{x}_{\nu} + k_1 ... k_{\nu-1}\eta) \le$$
$$\le \|\bar{x}_{\nu}\|(-k_{\nu}\|\bar{x}_{\mu}\| + k_1^* ... k_{\nu-1}^* \|\eta\|) .$$

The inequality $\dot{V}_{\nu} < 0$ is met outside of a neighbourhood $\|\bar{x}_{\nu}\| \le k_1^* ... k_{\nu-1}^* \|\eta\| / k_{\nu} \le \delta_{\nu}$ under fulfillment of the condition

$$k_{\nu}^* = k_{\nu} > k_1^* ... k_{\nu-1}^* \|\eta\| / \delta_{\nu} .$$

The given procedure of feedback synthesis can be also applied to a solve of the tracking problem in the last sub-system in (14). Behaviour of variables in the closed-loop system can be shows by following logic line-up:

$$\|\bar{x}_{\nu}\| \le \delta_{\nu} \Rightarrow \|\bar{x}_{\nu-1}\| \le \delta_{\nu-1} \Rightarrow$$
$$\Rightarrow \|\bar{x}_{\nu-2}\| \le \delta_{\nu-2} \Rightarrow \ldots \Rightarrow \|\bar{x}_1\| = \|x_1\| \le \delta_1 .$$

The given procedure of feedback synthesis can be also applied to choice of feedback gains in other subsystems of system (14). As a result, the given tracking problem with respect to the output will be solved.

Note 1. The last requirement of theorem 1 with respect to stabilizability of matrixes pair $(A_{\nu-1}, B_{\nu-1})$ (or stability of transmission zero of the closed-loop system (Wonham, 1979)) is the requirement of engineering implementation of a solution of the stabilization problem.

Note 2. In the given procedure of synthesis of the tracking problem with a given accuracy with respect to the output, only tracking errors with respect to the output are utilized for forming of the control (unlike the using of derivatives of tracking errors in the problem of ensure of asymptotic convergence to zero). Moreover, in formulation of the tracking problem with a given accuracy the limitations on smoothness of demanded functions are not superimposed, and restrictions modulo of demanded functions and their derivative of the first order are required only.

Note 3. In the framework of the geometrical approach (Wonham, 1979; Willems, 1982) the necessity and sufficient requirements of a solution of a tracking problem with respect to the output were obtained. Given in present paper the direct synthesis of the tracking problem on the basis of the block-canonical form of the controllability with respect to the output, at first, allows to eliminate an analysis stage of existence conditions of a solution and immediately to initiate to a solution of the design problem. Secondly, usage of the motions separation method (Drakunov, *el al.,* 1990) (or backstepping procedure (Krstic., *et al.,* 1995)) allows to decompose the design problem of high dimension on separately solved sub-problems of smaller dimensions.

4. CONCLUSION

In this paper, the decomposition design procedure of the tracking problem with respect to the output with given accuracy on the basis of the transformation of a linear system into the block-canonical form of the controllability is introduced. Necessary and sufficient conditions of a solution of the given problem is obtained. Unlike standard approaches, in the given approach, restrictions modulo of demanded functions and their derivative of the first order are required only, and, therefore, only tracking errors with respect to the output is utilized for forming of the control.

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