15th Triennial World Congress, Barcelona, Spain

GENERALIZED $S(C, A, B)$-PAIRS FOR INFINITE-DIMENSIONAL SYSTEMS

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Abstract: In this paper, the generalized $S(C, A, B)$-pairs which is an extension of generalized (C, A, B)-pair investigated by the present author is introduced for infinite-dimensional systems and its properties are investigated. And a parameter-insensitive disturbance-rejection problem with dynamic compensator is formulated and then its solvability conditions are presented.

Keywords: Geometric approaches, Disturbance rejection, invariance, uncertain dynamical systems, dynamic output feedback

1. INTRODUCTION

In the framework of the so-called geometric approach, many control problems with state feedback or incomplete-state feedback (e.g., decoupling problems, disturbance-rejection problems etc) have been studied for finite-dimensional systems, see (Wonham, 1984). Further, the notion of (C, A, B)-pairs was first introduced by Schumacher (Schumacher, 1980) and this concept has been used successfully to design dynamic compensators. After that Curtain extended the geometric concepts to infinite-dimensional systems and various control problems have been studied, see e.g. ((Curtain, 1984), (Curtain, 1986a), (Curtain, 1986b), (Inaba and Otsuka, 1989), (Otsuka *et al.*, 1990), (Otsuka, 1991), (Otsuka *et al.*, 1994), (Zwart, 1988)). On the other hand, from the practical viewpoint Ghosh (Ghosh, 1986) and Otsuka (Otsuka, 1999) studied the concepts of simultaneous (C, A, B)-pairs and of generalized (C, A, B)-pairs for finite-dimensional systems, respectively, and the parameter-insensitive disturbance-rejection problems for uncertain linear systems were studied. Further, Otsuka and Inaba (Otsuka and Inaba, 1997), Otsuka and Inaba (Otsuka and Inaba, 1998) and Otsuka and Inaba (Otsuka and Inaba, 1999) extended the concepts of simultaneous invariant subspaces and of (C, A, B)-pairs to infinite-dimensional systems. Further, Otsuka and Hinata (Otsuka and Hinata, 2000) studied the generalized invariant subspaces for infinite-dimensional systems.

The objective of this paper is to investigate the notion of generalized $S(C, A, B)$-pairs which is an extension of (C, A, B)-pairs investigated by Otsuka (Otsuka, 1999) to infinite-dimensional systems, and to study the parameter insensitive disturbance-rejection problem with dynamic compensator for uncertain linear systems in the sense that system's operators depend linearly on uncertain parameters.

This paper is organized as follows. Section 2 gives the concept of generalized $S(C, A, B)$-pairs and its properties. In Section 3, the parameter-insensitive disturbance-rejection problem with dynamic compensator is formulated and its solvability conditions are presented. Finally, Section 4 gives some concluding remarks.

2. GEMERALIZED $S(C, A, B)$-PAIRS

First, some notations are used throughout this investigation. Let $\boldsymbol{B}(X;Y)$ denote the set of all bounded linear operators from a Hilbert space X into another Hilbert space Y; for notational simplicity, we write $\boldsymbol{B}(X)$ for $\boldsymbol{B}(X;X)$. For a linear operator A the domain, the image, the kernel and the C_0-semigroup generated by A are denoted by $\mathrm{D}(A)$, $\mathrm{Im}A$, $\mathrm{Ker}A$ and $\{S_A(t); t \geq 0\}$, respectively. Further, the dimension and the orthogonal complement of a closed subspace V are denoted by $\dim(V)$ and $(V)^{\perp}$, respectively.

Next, consider the following linear systems defined in a Hilbert space X :

$$S(\alpha,\beta,\gamma):\begin{cases}\dfrac{d}{dt}x(t)=A(\alpha)x(t)+B(\beta)u(t),\\ y(t)=C(\gamma)x(t),\end{cases}$$

where $x(t)\in X$, $u(t)\in U:=\boldsymbol{R}^m$, $y(t)\in Y:=\boldsymbol{R}^\ell$ are the state, the input and the measurement output, respectively. And operators $A(\alpha)$, $B(\beta)$ and $C(\gamma)$ are unknown in the sense that they are represented as the forms :

$$A(\alpha)=A_0+\alpha_1A_1+\cdots+\alpha_pA_p:=A_0+\Delta A(\alpha),$$

$$B(\beta)=B_0+\beta_1B_1+\cdots+\beta_qB_q:=B_0+\Delta B(\beta),$$

$$C(\gamma)=C_0+\gamma_1C_1+\cdots+\gamma_rC_r:=C_0+\Delta C(\gamma),$$

where $\alpha:=(\alpha_1,\cdots,\alpha_p)\in\boldsymbol{R}^p$, $\beta:=(\beta_1,\cdots,\beta_q)\in\boldsymbol{R}^q$, $\gamma:=(\gamma_1,\cdots,\gamma_r)\in\boldsymbol{R}^r$, A_0 is the infinitesimal generator of a C_0-semigroup $\{S_{A_0}(t);t\geq 0\}$ on X, $A_i\in\boldsymbol{B}(X)$ $(i=1,\cdots,p)$, $B_i\in\boldsymbol{B}(\boldsymbol{R}^m;X)$ $(i=1,\cdots,q)$ and $C_i\in\boldsymbol{B}(X;\boldsymbol{R}^\ell)$ $(i=1,\cdots,r)$.

Here, in system $S(\alpha,\beta,\gamma)$ (A_0,B_0,C_0) and $(\Delta A(\alpha),\Delta B(\beta),\Delta B(\beta))$ mean the nominal system model and a specific uncertain perturbation, respectively.

Since A_i $(i=1,\cdots,p)$ are bounded linear operators, it remarsk that $A(\alpha)$ always generates a C_0-semigroup and has the domain $\mathrm{D}(A(\alpha))=\mathrm{D}(A_0)$ for all $\alpha\in\boldsymbol{R}^p$. Further, from the practical viewpoint it remarks that the dimensions of input and output are finite.

Now, introduce a compensator (K,L,M,N) defined in a Hilbert space W of the form :

$$\begin{cases}\dfrac{d}{dt}w(t)=Nw(t)+My(t),\\ u(t)=Lw(t)+Ky(t),\end{cases}\tag{1}$$

where N is the infinitesimal generator of a C_0-semigroup $\{S_N(t);t\geq 0\}$ on a Hilbert space $W, M\in\boldsymbol{B}(\boldsymbol{R}^\ell;W), L\in\boldsymbol{B}(W;\boldsymbol{R}^m)$ and $K\in\boldsymbol{B}(\boldsymbol{R}^\ell;\boldsymbol{R}^m)$.

If a compensator of the form (1) is applied to system $S(\alpha,\beta,\gamma)$, the resulting closed-loop system $S_{cl}(\alpha,\beta,\gamma)$ with the extended state space $X^e:=X\oplus W$ is easily seen to be

$$\frac{d}{dt}\begin{bmatrix}x(t)\\ w(t)\end{bmatrix}=\begin{bmatrix}A(\alpha)+B(\beta)KC(\gamma) & B(\beta)L\\ MC(\gamma) & N\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\end{bmatrix},$$

where $X\oplus W$ means the direct sum of X and W. For the closed-loop system $S_{cl}(\alpha,\beta,\gamma)$, define

$$x^e(t):=\begin{bmatrix}x(t)\\ w(t)\end{bmatrix}\ \text{and}$$

$$A^e_{\alpha\beta\gamma}:=\begin{bmatrix}A(\alpha)+B(\beta)KC(\gamma) & B(\beta)L\\ MC(\gamma) & N\end{bmatrix}$$

with domain $\mathrm{D}(A^e_{\alpha\beta\gamma})$ $(=\ \mathrm{D}(A_0)\oplus W)$.

For the system $S(\alpha,\beta,\gamma)$, the following invariant subspaces are introduced.

Definition 1. Let V be a closed subspace of X.

(i) V is said to be a generalized (A,B)-invariant if there exists an $F\in\boldsymbol{B}(X;\boldsymbol{R}^m)$ such that

$$(A(\alpha)+B(\beta)F)(V\cap\mathrm{D}(A_0))\subset V\ \text{for all}\ \alpha,\beta.$$

$\boldsymbol{F}(V):=\{F\in\boldsymbol{B}(X;\boldsymbol{R}^m)\mid(A(\alpha)+B(\beta)F)(V\cap\mathrm{D}(A_0))\subset V$ for all $\alpha,\beta.\}$.

(ii) V is said to be a generalized $S(A,B)$-invariant if there exists an $F\in\boldsymbol{B}(X;\boldsymbol{R}^m)$ such that

$$S_{A(\alpha)+B(\beta)F}(t)V\subset V\ \text{for all}\ t\geq 0\ \text{and all}\ \alpha,\beta.$$

$\boldsymbol{V}(A,B;\Lambda):=\{V|\ V$ is a generalized $S(A,B)$-invariant and is contained in a given closed subspace $\Lambda\}$

$\boldsymbol{F}_s(V):=\{F\in\boldsymbol{B}(X;\boldsymbol{R}^m)\mid S_{A(\alpha)+B(\beta)F}(t)V\subset V$ for all $t\geq 0$ and all $\alpha,\beta.\}$.

(iii) V is said to be a generalized (C,A)-invariant if there exists a $G\in\boldsymbol{B}(\boldsymbol{R}^\ell;X)$ such that

$$(A(\alpha)+GC(\gamma))(V\cap\mathrm{D}(A_0))\subset V\ \text{for all}\ \alpha,\gamma.$$

$\boldsymbol{G}(V):=\{G\in\boldsymbol{B}(\boldsymbol{R}^\ell;X)\mid(A(\alpha)+GC(\gamma))(V\cap\mathrm{D}(A_0))\subset V$ for all $\alpha,\gamma.\}$.

(iv) V is said to be a generalized $S(C,A)$-invariant if there exists a $G\in\boldsymbol{B}(\boldsymbol{R}^\ell;X)$ such that

$$S_{A(\alpha)+GC(\gamma)}(t)V\subset V\ \text{for all}\ t\geq 0\ \text{and all}\ \alpha,\gamma.$$

$\boldsymbol{V}(\varepsilon;C,A):=\{V|\ V$ is a generalized $S(C,A)$-invariant and contains a given closed subspace $\varepsilon.\}$.

$\boldsymbol{G}_s(V):=\{G\in\boldsymbol{B}(\boldsymbol{R}^\ell;X)\mid S_{A(\alpha)+GC(\gamma)}(t)V\subset V$ for all $t\geq 0$ and all $\alpha,\gamma.\}$. ■

Remark 2.

(i) For the system $S(\alpha,\beta,\gamma)$ a generalized $S(A,B)$-invariant subspace V has the property that if an arbitrary initial state $x(0)\in V$ then there exists a state feedback $u(t)=Fx(t)$ which is independent of α and β such that the state trajectory $x(t)\in V$ for all $t\geq 0$.

(ii) If A_0 is a bounded linear operator on X (i.e., $A_0\in\boldsymbol{B}(X)$), then the statements (i) and (ii), and (iii) and (iv) in Definition 2.1 are equivalent, respectively. Further, in this case $\boldsymbol{F}_s(V)=\boldsymbol{F}(V)$ and $\boldsymbol{G}_s(V)=\boldsymbol{G}(V)$. ■

For finite-dimensional systems, Schumacher (Scumacher, 1980) first introduced the concept of (C,A,B)-

pair. The following definition is a generalized and infinite-dimensional version of (C, A, B)-pair.

Definition 3. Let V_1 and V_2 be closed subspaces of X. A pair (V_1, V_2) of subspaces is said to be a generalized $S(C, A, B)$-pair if the following three conditions hold.

(i) V_1 is a generalized $S(C, A)$-invariant.

(ii) V_2 is a generalized $S(A, B)$-invariant.

(iii) $V_1 \subset V_2$. ■

For closed-loop system $S_{cl}(\alpha, \beta, \gamma)$, the following definition is given.

Definition 4. Let V^e be closed subspace of X^e.

(i) V^e is said to be a generalized A^e-invariant if

$$A^e_{\alpha,\beta,\gamma}(V^e \cap \mathrm{D}(A^e_{\alpha,\beta,\gamma})) \subset V^e \text{ for all } \alpha, \beta, \gamma.$$

(ii) V^e is said to be a generalized $S_{A^e}(t)$-invariant if

$$S_{A^e_{\alpha,\beta,\gamma}}(t)V^e \subset V^e \text{ for all } t \geq 0 \text{ and all } \alpha, \beta, \gamma. \blacksquare$$

The following lemma is an extension of the results of Otsuka (Otsuka, 1999) to infinite-dimensional systems and is used to prove Lemmas 2.6 and 2.7.

Lemma 5. If a pair (V_1, V_2) of subspaces of X is a generalized $S(C, A, B)$-pair such that

$$\sum_{i=1}^{q} \mathrm{Im}B_i \subset V_1 \subset V_2 \subset \bigcap_{i=1}^{r} \mathrm{Ker}C_i \text{ and } A_i V_2 \subset V_1$$

$(i = 1, \cdots, p)$, then there exist $G \in \boldsymbol{G}(V_1)$, $G(\beta) \in \boldsymbol{B}(\boldsymbol{R}^\ell; X)$, $F(\gamma) \in \boldsymbol{F}(V_2)$, $F_0 \in \boldsymbol{B}(X; \boldsymbol{R}^m)$ and $K \in \boldsymbol{B}(\boldsymbol{R}^\ell; \boldsymbol{R}^m)$ such that

$$G = B(\beta)K + G(\beta), \ \ \mathrm{Im}G(\beta) \subset V_2,$$

$$F(\gamma) = KC(\gamma) + F_0 \text{ and } \mathrm{Ker}F_0 \supset V_1$$

for all $(\beta, \gamma) \in \boldsymbol{R}^q \times \boldsymbol{R}^r$. ■

The following two lemmas play important role to prove the main Theorems in Section 3.

Lemma 6. If a pair (V_1, V_2) of subspaces of X is a generalized $S(C, A, B)$-pair such that

$$\sum_{i=1}^{q} \mathrm{Im}B_i \subset V_1 \subset V_2 \subset \bigcap_{i=1}^{r} \mathrm{Ker}C_i, \ \ A_i V_2 \subset V_1$$

$(i = 1, \cdots, p)$ and $V_2 \subset \mathrm{D}(A_0)$, then there exist a compensator (K, L, M, N) on $W := (V_2 \cap V_1^\perp)$ and a subspace V^e of X^e such that $V_1 = S_1$, $V_2 = S_2$ and V^e is generalized $S_{A^e}(t)$-invariant. ■

Lemma 7. Assume that $B_1 = \cdots = B_q = 0$. If a pair (V_1, V_2) of subspaces of X is a generalized $S(C, A, B)$-pair such that

$$V_2 \subset \left\{ \bigcap_{i=1}^{r} \mathrm{Ker}C_i \cap \bigcap_{i=1}^{p} \mathrm{Ker}A_i, \right\},$$

then there exist a compensator (K, L, M, N) on $W := (V_2 \cap V_1^\perp)$ and a subspace V^e of X^e such that $V_1 = S_1, V_2 = S_2$ and V^e is generalized $S_{A^e}(t)$-invariant. ■

3. PARAMETER-INSENSITIVE DISTURBANCE-REJECTION

In this section, the parameter insensitive disturbance-rejection problem with dynamic compensator for uncertain linear systems in the sense that system's operators depend linearly on uncertain parameters.

Consider the following uncertain linear system $S(\alpha, \beta, \gamma, \delta, \sigma)$ defined in a Hilbert space X.

$$\begin{cases} \dfrac{d}{dt}x(t) = A(\alpha)x(t) + B(\beta)u(t) + E(\sigma)\xi(t), \\ y(t) = C(\gamma)x(t), \\ z(t) = D(\delta)x(t) \end{cases}$$

where $x(t) \in X, u(t) \in U := \boldsymbol{R}^m, y(t) \in Y := \boldsymbol{R}^\ell, z(t) \in Z := \boldsymbol{R}^\mu$ and $\xi(t) \in L_1^{loc}((0, \infty); Q)$ are the state, the input, the measurement output, the controlled output and the disturbance which is a Hilbert space Q valued locally integrable function, respectively. It is assumed that coefficient operators have the following unknown parameters.

$$\begin{aligned} A(\alpha) &= A_0 + \alpha_1 A_1 + \cdots + \alpha_p A_p := A_0 + \Delta A(\alpha), \\ B(\beta) &= B_0 + \beta_1 B_1 + \cdots + \beta_q B_q := B_0 + \Delta B(\beta), \\ C(\gamma) &= C_0 + \gamma_1 C_1 + \cdots + \gamma_r C_r := C_0 + \Delta C(\gamma), \\ D(\delta) &= D_0 + \delta_1 D_1 + \cdots + \delta_s D_s := D_0 + \Delta D(\delta), \\ E(\sigma) &= E_0 + \sigma_1 E_1 + \cdots + \sigma_t E_t := E_0 + \Delta E(\sigma), \end{aligned}$$

where A_i, B_i, C_i are the same as system $S(\alpha, \beta, \gamma)$ in Section 2, $D_i \in \boldsymbol{B}(X; \boldsymbol{R}^\mu)$, $E_i \in \boldsymbol{B}(Q; X)$ and $\alpha := (\alpha_1, \cdots, \alpha_p) \in \boldsymbol{R}^p$, $\beta := (\beta_1, \cdots, \beta_q) \in \boldsymbol{R}^q$, $\gamma := (\gamma_1, \cdots, \gamma_r) \in \boldsymbol{R}^r$, $\delta := (\delta_1, \cdots, \delta_s) \in \boldsymbol{R}^s$, $\sigma := (\sigma_1, \cdots, \sigma_t) \in \boldsymbol{R}^t$.

In system $S(\alpha, \beta, \gamma, \delta, \sigma)$, $(A_0, B_0, C_0, D_0, E_0)$ and $(\Delta A(\alpha),\ \Delta B(\beta),\ \Delta C(\gamma),\ \Delta D(\delta),\ \Delta E(\sigma))$ represent the nominal system model and a specific uncertain perturbation, respectively.

If a compensator of the form (1) is applied to system $S(\alpha, \beta, \gamma, \delta, \sigma)$, the resulting closed-loop system with the extended state space $X^e := X \oplus W$ is easily obtained as

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} x(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} A(\alpha)+B(\beta)KC(\gamma) & B(\beta)L \\ MC(\gamma) & N \end{bmatrix} \\ \qquad \cdot \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} E(\sigma) \\ 0 \end{bmatrix} \xi(t), \\ z(t) = \begin{bmatrix} D(\delta) & 0 \end{bmatrix} \begin{bmatrix} E(\sigma) \\ 0 \end{bmatrix}. \end{cases}$$

For convenience, we set

$$x^e(t) := \begin{bmatrix} x(t) \\ w(t) \end{bmatrix},$$

$$A^e_{\alpha,\beta,\gamma} := \begin{bmatrix} A(\alpha)+B(\beta)KC(\gamma) & B(\beta)L \\ MC(\gamma) & N \end{bmatrix},$$

$$E^e(\sigma) := \begin{bmatrix} E(\sigma) \\ 0 \end{bmatrix} \text{ and } D^e(\delta) := \begin{bmatrix} D(\delta) & 0 \end{bmatrix}.$$

Then, our disturbance-rejection problem with dynamic compensator is to find a compensator (K, L, M, N) of (1) such that

$$D^e(\delta)\int_0^t S_{A^e_{\alpha,\beta,\gamma}}(t-\tau)E^e(\sigma)\xi(\tau)d\tau = 0$$

for all $\xi(\cdot) \in L_1^{loc}(0,\infty;Q)$, all $t \geq 0$ and all parameters $\alpha,\beta,\gamma,\delta,\sigma \in \boldsymbol{R}$.

This problem can be formulated as follows.

Parameter Insensitive Disturbance Rejection Problem with Dynamic Compensator (PIDRPDC) Given A_i, B_i, C_i, D_i, E_i, find (if possible) a compensator (K, L, M, N) of (1) such that

$$< S_{A^e_{\alpha,\beta,\gamma}}(\cdot)|\mathrm{Im}E^e(\sigma) > := \overline{L\left(\bigcup_{t\geq 0} S_{A^e_{\alpha,\beta,\gamma}}(t)(\mathrm{Im}E^e(\sigma))\right)} \subset \mathrm{Ker}D^e(\delta)$$

for all parameters $\alpha,\beta,\gamma,\delta,\sigma$, where $L(\Omega)$ and the over bar indicate the linear subspace generated by the set Ω and the closure in X^e, respectively.

The following results are extensions of the results of Otsuka (1999) to infinite-dimensional systems.

Theorem 8. If there exists a generalized $S(C, A, B)$-pair (V_1, V_2) such that

$$\left\{\sum_{i=1}^{q}\mathrm{Im}B_i + \sum_{i=0}^{t}\mathrm{Im}E_i\right\} \subset V_1 \subset V_2 \subset$$

$$\left\{\bigcap_{i=1}^{r}\mathrm{Ker}C_i \cap \bigcap_{i=0}^{s}\mathrm{Ker}D_i\right\}, A_iV_2 \subset V_1\ (i=1,\cdots,p)$$

and $V_2 \subset \mathrm{D}(A_0)$, then the PIDRPDC is solvable.

Sketch of Proof. Suppose that the stated above conditions are satisfied. Then, it follows from Lemma 2.6 that there exist a compensator (K, L, M, N) on $W := (V_2 \cap V_1^{\perp})$ and a subspace V^e of X^e such that $V_1 = S_1$, $V_2 = S_2$ and V^e is generalized $S_{A^e}(t)$-invariant. Further, it can be easily shown that $\mathrm{Im}E^e(\sigma) \subset V^e \subset \mathrm{Ker}D^e(\delta)$. Then,

$$< S_{A^e_{\alpha,\beta,\gamma}}(\cdot)|\mathrm{Im}E^e(\sigma) > \subset < S_{A^e_{\alpha,\beta,\gamma}}(\cdot)|V^e > = V^e \subset \mathrm{Ker}D^e(\delta)$$

for all parameters $\alpha,\beta,\gamma,\delta,\sigma$ which imply the PIDRPDC is solvable. ■

Corollary 9. Assume that $\boldsymbol{V}(\sum_{i=0}^{t}\mathrm{Im}E_i; C, A)$ and $\boldsymbol{V}(A, B; \bigcap_{i=1}^{r}\mathrm{Ker}D_i)$ have the minimal element V_{1*} and the maximal element V_2^*, respectively. If $\sum_{i=1}^{q}\mathrm{Im}B_i \subset V_{1*} \subset V_2^* \subset \bigcap_{i=1}^{r}\mathrm{Ker}C_i$, $A_iV_2^* \subset V_{1*}$ $(i = 1,\cdots,p)$ and $V_2^* \subset \mathrm{D}(A_0)$, then the PIDRPDC is solvable. ■

The following theorem can be obtained from Lemma 2.7.

Theorem 10. Assume that $B_1 = \cdots = B_q = 0$. If there exists a generalized $S(C, A, B)$-pair (V_1, V_2) such that

$$\sum_{i=1}^{t}\mathrm{Im}E_i \subset V_1 \subset V_2 \subset$$

$$\left\{\bigcap_{i=1}^{r}\mathrm{Ker}C_i \cap \bigcap_{i=0}^{s}\mathrm{Ker}D_i \cap \bigcap_{i=1}^{p}\mathrm{Ker}A_i\right\},$$

then the PIDRPDC is solvable. ■

Corollary 11. Assume that $B_1 = \cdots = B_q = 0$. And suppose that $\boldsymbol{V}(\sum_{i=0}^{t}\mathrm{Im}E_i; C, A)$ and $\boldsymbol{V}(A, B; \bigcap_{i=1}^{r}\mathrm{Ker}D_i)$ have the minimal element V_{1*} and the maximal element V_2^*, respectively. If

$$V_{1*} \subset V_2^* \subset \left\{\bigcap_{i=1}^{r}\mathrm{Ker}C_i \cap \bigcap_{i=1}^{p}\mathrm{Ker}A_i\right\},$$

then the PIDRPDC is solvable. ■

4. CONCLUDING REMARKS

In this paper, from the mathematical viewpoint the infinite-dimensional version of generalized

$S(C, A, B)$-pair investigated by Otsuka (Otsuka, 1999) for finite-dimensional systems was studied, and then its properties were investigated. Further, a parameter insensitive disturbance-rejection problem with dynamic compensator for uncertain linear systems in the sense that system's operators depend linearly on uncertain parameters was formulated and its solvability conditions were studied. However, the conditions $V_2 \subset \mathrm{D}(A_0)$ for unbounded operator A_0 in the main Theorem 3.1 is restrictive one from the viewpoint of applications. Therefore, it is necessary to investigate the solvability conditions without assuming the conditions $V_2 \subset \mathrm{D}(A_0)$ as future studies.

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