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ROBUST DISSIPATIVE CONTROL FOR UNCERTAIN NONLINEAR DYNAMICAL SYSTEMS

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Abstract: The problem of robust dissipative control for uncertain nonlinear systems is investigated in this paper. The uncertainty is described in the form of bounded norm. Both state feedback control and output feedback control are designed to achieve quadratic dissipativeness for the systems. The robust dissipative control problem can be resolved for all admissible uncertainties, if there exist nonnegative solutions of Hamilton-Jacobi inequalities.
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1. INTRODUCTION

Since the theory of dissipative system was introduced, it has played an important role in circuit, system and control. Dissipativeness is a generalization of the concept of passivity in electrical networks. The notion of dissipativeness and its application in the stability analysis for dynamical systems were first discussed by Willems [Willems, 1972]. Then, Hill investigated the stability analysis of nonlinear systems based on dissipative theory [Hill et.al., 1976]. It is accepted that the theory of dissipative system generalizes the basic theorems of control systems, such as the passivity theorem, bounded real lemma, Kalman-Yakubovich lemma, etc [Fu, 2000].

In recent years, considerable results have obtained on the synthesis problem of passivity or L_2-gain analysis for dynamical system [Van der Schaft A J, 1992]. Since the general concept of dissipativity for nonlinear system includes positive realness, passivity and L_2-gain as special cases, many nonlinear system control design problems can be regarded as dissipative synthesis problem.

In this paper, we considered the problem of robust dissipative control for uncertain nonlinear system. In the considered system, the uncertainty is described in the form of bounded norm. Both state feedback control and output feedback control are designed to achieve quadratic dissipativeness. The robust dissipative control problem can be resolved for all admissible uncertainties, if there exist nonnegative solutions of Hamilton-Jacobi inequalities.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the nonlinear dynamical system:

$$\begin{aligned} \dot{x}(t) &= f(x(t), w(t)) \\ z(t) &= g(x(t), w(t)) \\ x(0) &= x_0 \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $w(t) \in R^m$ is the input, $z(t) \in R^p$ is the output, $f(\cdot)$ and $g(\cdot)$ are smooth real vector functions.

The following definition is the extension of Definition 3.1.2 of Ref [Van der Schaft A J, 1996].

Definition 1: The state space system (2.1) is passive if it is disspative with respect to the supply rate $S(w,z) = w^T Rz$. System (2.1) is strictly input passive if there exists a symmetric positive matrix $P > 0$ such that (2.1) is dissipative with respect to $S(w,z) = w^T Rw - z^T Pz$. System (2.1) is strictly output passive if there exist symmetric positive matrices $Q > 0$ and $R > 0$ such that (2.1) is dissipative with respect to the quadratic supply rule $S(w,z) = w^T Rz - w^T Qw$.

In our paper, the quadratic supply rate is set as

$$S(w(t), z(t)) = \frac{1}{2} w^T Qw - \frac{1}{2} z^T Rz \tag{2.2}$$

where Q and R are symmetric positive define matrices.

3. MAIN RESULTS

3.1 *Dissipativeness Analysis*

Consider the nonlinear system

$$\dot{x} = f(x) + g_1(x)w$$
$$z = h(x) \qquad (3.1)$$

where $x \in R^n$ is the state vector, $z \in R^p$ is the output vector, $w \in R^m$ is input vector. $f(x)$, $g_1(x)$ and $h(x)$ are known smooth functions.

Theorem 1: If the following Hamilton-Jacobi inequality

$$V_x f + \frac{1}{2}V_x g_1 Q^{-1} g_1^T V_x^T + \frac{1}{2}h^T R h \le 0 \qquad (3.2)$$

has a nonnegative definite solution $V(x) \in C^1$ with $V(0) = 0$, then system (3.1) with storage function $V(x)$ is quadratic dissipative with respect to supply rate $S(w,z)$.

Proof: Along the solution of system (3.1), we have

$$\frac{dV(x)}{dt} - S(w,z)$$
$$= V_x[f(x) + g_1(x)w] - \frac{1}{2}w^T Q w + \frac{1}{2}z^T R z$$
$$= V_x f + \frac{1}{2}V_x g_1 Q^{-1} g_1^T V_x^T + \frac{1}{2}h^T R h$$
$$- \frac{1}{2}(Q^{\frac{1}{2}} w - Q^{-\frac{1}{2}} g_1^T V_x^T)^T (Q^{\frac{1}{2}} w - Q^{-\frac{1}{2}} g_1^T V_x^T)$$

From condition (3.2), we obtain $\dot{V}(x) \le S(w,z)$.

Remark 1: If Q and R are identity matrices, Theorem 1 is reduced to

$$V_x f + \frac{1}{2}V_x g_1 g_1^T V_x^T + \frac{1}{2}h^T h \le 0 \qquad (3.3)$$

This is Theorem 2 in [Van der Shafter A. J. 1992].

3.2 State Feedback Dissipative Control

Consider the nonlinear system

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u$$
$$z = h(x) + k(x)u \qquad (3.4)$$

where $x \in R^n$ is the state vector, $z \in R^p$ is the output vector, $w \in R^m$ is exogenous input, $u \in R^r$ is control input. $f(x)$, $g_1(x)$, $g_2(x)$, $k(x)$ and $h(x)$ are known smooth functions.

Theorem 2: If the following Hamilton-Jacobi inequality

$$V_x f + \frac{1}{2}V_x g_1 Q^{-1} g_1^T V_x^T + \frac{1}{2}h^T R h$$
$$- \frac{1}{2}(V_x g_2 + h^T R k)\overline{R}^{-1}(V_x g_2 + h^T R k)^T \le 0 \qquad (3.5)$$

has a nonnegative definite solution $V(x) \in C^1$ with $V(0) = 0$, where $\overline{R} = k^T R k$, then under the control of

$$u = \alpha(x) = -\overline{R}^{-1}(V_x g_2 + h^T R k)^T \qquad (3.6)$$

system (3.4) with storage function $V(x)$ is quadratic dissipative with respect to supply rate $S(w,z)$.

Proof: Along the solution of system (3.1), we have

$$\frac{dV(x)}{dt} - S(w,z)$$
$$= V_x[f(x) + g_1(x)w + g_2(x)u]$$
$$- \frac{1}{2}w^T Q w + \frac{1}{2}z^T R z$$
$$= V_x f(x) + V_x g_1(x)w + V_x g_2(x)u - \frac{1}{2}w^T Q w$$
$$+ \frac{1}{2}h^T R h + h^T R k u + \frac{1}{2}u^T k^T R k u$$
$$= V_x f + \frac{1}{2}V_x g_1 Q^{-1} g_1^T V_x^T + \frac{1}{2}h^T R h$$
$$- \frac{1}{2}(Q^{\frac{1}{2}} w - Q^{-\frac{1}{2}} g_1^T V_x^T)^T (Q^{\frac{1}{2}} w - Q^{-\frac{1}{2}} g_1^T V_x^T)$$
$$- \frac{1}{2}(V_x g_2 + h^T R k)\overline{R}^{-1}(V_x g_2 + h^T R k)^T$$
$$+ \frac{1}{2}(\overline{R}^{\frac{1}{2}} u + \overline{R}^{-\frac{1}{2}}(V_x g_2 + h^T R k)^T)^T$$
$$\cdot (\overline{R}^{\frac{1}{2}} u + \overline{R}^{-\frac{1}{2}}(V_x g_2 + h^T R k)^T)$$

when $u = \alpha(x) = -\overline{R}^{-1}(V_x g_2 + h^T R k)^T$ and the condition (3.5) holds, we have $\dot{V}(x) \le S(w,z)$.

Remark 2: If Q and R are identity matrices, and there exists $k^T[h \quad k] = [0 \quad I]$, then theorem 2 is reduced to

$$V_x f + \frac{1}{2}V_x g_1 g_1^T V_x^T - \frac{1}{2}V_x g_2 g_2^T V_x^T + \frac{1}{2}h^T h \le 0$$
$$u = -g_2^T V_x^T \qquad (3.7)$$

This is Theorem 16 in [Van der Shafter A. J. 1992].

3.3 Robust State Feedback Dissipative Control

Consider the uncertain dynamical system

$$\dot{x} = f(x) + \Delta f(x) + g_1(x)w + g_2(x)u$$
$$z = h(x) + k(x)u \qquad (3.8)$$

where $x \in R^n$ is the state vector, $z \in R^p$ is the output vector, $w \in R^m$ is exogenous input, $u \in R^r$ is control input. $f(x)$, $g_1(x)$, $g_2(x)$, $k(x)$ and $h(x)$ are known smooth functions. The uncertainty $\Delta f(x)$ is assumed as norm bounded function that satisfies as follows:

$$\Delta f(x) \in \Omega$$
$$= \{e(x)\delta(x) : \delta^T(x)\delta(x) \le m^T(x)m(x)\} \quad (3.9)$$

where $e(x)$ and $\delta(x)$ are known smooth mappings.

Theorem 3: If the following Hamilton-Jacobi inequality

$$V_x f + \frac{1}{2} V_x g_1 Q^{-1} g_1^T V_x^T + \frac{1}{2} h^T R h$$
$$-\frac{1}{2}(V_x g_2 + h^T R k)\overline{R}^{-1}(V_x g_2 + h^T R k)^T$$
$$+\frac{1}{4}\lambda_f V_x e e^T V_x^T + \frac{1}{\lambda_f} m^T m \le 0 \qquad (3.10)$$

has a nonnegative definite solution $V(x) \in C^1$ with $V(0) = 0$, where $\overline{R} = k^T R k$, then under the control of

$$u = \alpha(x) = -\overline{R}^{-1}(V_x g_2 + h^T R k)^T \qquad (3.11)$$

system (3.4) with storage function $V(x)$ is quadratic dissipative with respect to supply rate $S(w,z)$.

Proof: Along the system (3.8), using the inequality

$$V_x \Delta f(x) = V_x e(x)\delta(x)$$
$$\le + \frac{1}{4}\lambda_f V_x e e^T V_x^T + \frac{1}{\lambda_f} m^T m \qquad (3.12)$$

where λ_f is some given real constant.

Then, we can obtain Theorem 3.

Remark 3: If Q and R are identity matrices, and there exists $k^T[h \quad k] = [0 \quad I]$, then theorem 2 is reduced to

$$V_x f + \frac{1}{2} V_x (g_1 g_1^T - \lambda_f^2 e e^T - g_2 g_2^T) V_x^T$$
$$+\frac{1}{2}(h^T h + \frac{1}{\lambda_f^2} m^T m) \le 0$$
$$u = -g_2^T V_x^T \qquad (3.13)$$

This is Theorem 2 in [Shen T.L. 1995].

3.4 Output Feedback Dissipative Control

Consider the nonlinear dynamical system

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u$$
$$z = h_1(x) + k_{12}(x)u$$
$$y = h_2(x) + k_{21}(x)w \qquad (3.14)$$

where $x \in R^n$ is the state vector, $y \in R^p$ is the measurement output vector, $z \in R^s$ is the penalty variavble ,$w \in R^m$ is exogenous input, $u \in R^r$ is control input. $f(x)$, $g_1(x)$, $g_2(x)$, $h_1(x)$, $h_2(x)$, $k_{12}(x)$ and $k_{21}(x)$ are smooth functions defined on the neighborhood of origin of R^n. We assume that $f(0) = 0$, $h_1(0) = 0$, $h_2(0) = 0$.

In this paper, we design an output feedback control law of the form:

$$\dot{\xi} = \eta(\xi, y)$$
$$u = \alpha(\xi) \qquad (3.15)$$

where $\xi(\cdot) \in R^v$ is the state variable of the controller; $\eta(\cdot)$ and $\alpha(\cdot)$ are smooth functions, satisfying $\eta(0,0) = 0$, $\alpha(0) = 0$.

The structure of output feedback controller is of the following form:

$$\dot{\xi} = f_c(\xi) + g_c(\xi)y$$
$$u = h_c(\xi) \qquad (3.16)$$

Then, we have the following results:

Theorem 4: If there exist smooth positive definite function $V(x) > 0$ and smooth semi-positive definite function $W(x,\xi) \ge 0$, $\forall \xi \in R^v$, $\forall x \in R^n$, which satisfy the following inequalities:

$$\mathrm{I}: \quad V_x f(x) + \frac{1}{2} V_x g_1(x) Q^{-1} g_1^T(x) V_x^T$$
$$-[V_x g_2(x) + h^T(x) R k_{12}(x)]$$
$$\cdot R^{-1}[V_x g_2(x) + h^T(x) R k_{12}(x)]^T$$
$$+\frac{1}{2} h_1^T(x) R h_1(x) \le 0 \qquad (3.17)$$

$$\Pi: \quad [W_x, W_\xi] f_e(x,\xi) + \frac{1}{2}\Phi^T(x,\xi) Q^{-1} \Phi(x,\xi)$$
$$+\frac{1}{2} h_e^T(x,\xi) Q^{-1} h_e(x,\xi) \le 0 \qquad (3.18)$$

where

$$f_e(x,\xi) = \begin{bmatrix} f(x) + g_2(x)\alpha(\xi) + g_1(x)Q^{-1}g_1(x)V_x^T \\ f_c(\xi) + Gh_2(x) + Gk_{21}(x)Q^{-1}g_1(x)V_x^T \end{bmatrix}$$

$$h_e(x,\xi) = R^{-\frac{1}{2}}[V_x g_2(x) + h^T(x) R k_{12}(x)]$$
$$+ R^{\frac{1}{2}}\alpha(\xi)$$

$$\Phi^T(x,\xi) = [W_x, W_\xi]\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix}$$

$$g_c(\xi) = G$$
$$u = \alpha(\xi)$$

then under the control of u, system (3.14) is quadratic dissipative with respect to supply rate $S(w,z)$.

Proof: The closed-loop system can be expressed as

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \tilde{f}(x,\xi) + \tilde{g}(x,\xi)w$$
$$z = \tilde{h}(x,\xi)$$

where

$$u=\alpha(\xi)$$

$$\tilde{f}(x,\xi)=\begin{bmatrix} f(x)+g_2(x)\alpha(\xi) \\ f_c(\xi)+Gh_2(x) \end{bmatrix}$$

$$\tilde{g}(x,\xi)=\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix}$$

$$\tilde{h}(x,\xi)=h_1(x)+k_{12}(x)\alpha(\xi)$$

Let storage function

$$U(x,\xi)=V(x)+W(x,\xi)>0$$

By using theorem 1, we have

$$\begin{aligned}
&[U_x,U_\xi]\tilde{f}(x,\xi)+\frac{1}{2}\tilde{h}^T(x,\xi)R\tilde{h}(x,\xi) \\
&+\frac{1}{2}[U_x,U_\xi]\tilde{g}(x,\xi)Q^{-1}\tilde{g}^T(x,\xi)\begin{bmatrix} U_x^T \\ U_\xi^T \end{bmatrix} \\
&=\{[V_x,0]+[W_x,W_\xi]\}\begin{bmatrix} f(x)+g_2(x)\alpha(\xi) \\ f_c(\xi)+Gh_2(x) \end{bmatrix} \\
&+\frac{1}{2}\{[V_x,0]+[W_x,W_\xi]\}\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix} \\
&\cdot Q^{-1}[g_1^T(x),k_{21}^T(x)G^T]\left\{\begin{bmatrix} V_x^T \\ 0 \end{bmatrix}+\begin{bmatrix} W_x^T \\ W_\xi^T \end{bmatrix}\right\} \\
&+\frac{1}{2}[h_1^T(x)+\alpha^T(\xi)k_{12}^T(x)]R \\
&\cdot[h_1(x)+k_{12}(x)\alpha(\xi)] \\
&=V_xf(x)+V_xg_2(x)\alpha(\xi) \\
&+\frac{1}{2}V_xg_1(x)Q^{-1}g_1^T(x)V_x^T \\
&+[W_x,W_\xi]\begin{bmatrix} f(x)+g_2(x)\alpha(\xi) \\ f_c(\xi)+Gh_2(x) \end{bmatrix} \\
&+[W_x,W_\xi]\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix} \\
&\cdot Q^{-1}[g_1^T(x),k_{21}^T(x)G^T]\begin{bmatrix} V_x^T \\ 0 \end{bmatrix} \\
&+\frac{1}{2}[W_x,W_\xi]\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix} \\
&\cdot Q^{-1}[g_1^T(x),k_{21}^T(x)G^T]\begin{bmatrix} W_x^T \\ W_\xi^T \end{bmatrix} \\
&+\frac{1}{2}h_1^T(x)Rh_1(x)+\alpha^T(\xi)k_{12}^T(x)Rh_1(x) \\
&+\frac{1}{2}\alpha^T(\xi)k_{12}^T(x)Rk_{12}(x)\alpha(\xi) \\
&=V_xf(x)+\frac{1}{2}V_xg_1(x)Q^{-1}g_1^T(x)V_x^T \\
&+\frac{1}{2}h_1^T(x)Rh_1(x) \\
&+[W_x,W_\xi]f_e(x,\xi) \\
&+\frac{1}{2}\Phi^T(x,\xi)Q^{-1}\Phi(x,\xi) \\
&+\frac{1}{2}h_e^T(x,\xi)Q^{-1}h_e(x,\xi) \\
&-[V_xg_2(x)+h^T(x)Rk_{12}(x)] \\
&\cdot R^{-1}[V_xg_2(x)+h^T(x)Rk_{12}(x)]^T \\
&\le \mathrm{I}+\Pi
\end{aligned}$$

when the conditions (3.17) and (3.18) hold, we have the theorem 4.

3.5 Robust Output Feedback Dissipative Control

Consider the uncertain nonlinear dynamical system

$$\begin{aligned}
\dot{x}&=f(x)+\Delta f(x)+g_1(x)w+g_2(x)u \\
z&=h_1(x)+k_{12}(x)u \\
y&=h_2(x)+k_{21}(x)w \qquad (3.19)
\end{aligned}$$

where $x\in R^n$ is the state vector, $y\in R^p$ is the measurement output vector, $z\in R^s$ is the penalty variavble , $w\in R^m$ is exogenous input, $u\in R^r$ is control input. $f(x)$, $g_1(x)$, $g_2(x)$, $h_1(x)$, $h_2(x)$, $k_{12}(x)$ and $k_{21}(x)$ are smooth functions defined on the neighborhood of origin of R^n. We assume that $f(0)=0$, $h_1(0)=0$, $h_2(0)=0$. The uncertainty $\Delta f(x)$ is assumed as norm bounded function that satisfies as follows:

$$\begin{aligned}
&\Delta f(x)\in\Omega \\
&=\{e(x)\delta(x):\delta^T(x)\delta(x)\le m^T(x)m(x)\} \qquad (3.20)
\end{aligned}$$

where $e(x)$ and $\delta(x)$ are known smooth mappings.

Theorem 5: If there exist smooth positive definite function $V(x)>0$ and smooth semi-positive definite function $W(x,\xi)\ge 0$, $\forall\xi\in R^v$, $\forall x\in R^n$, which satisfy the following inequalities:

I:

$$\begin{aligned}
&V_xf(x)+\frac{1}{2}V_x[g_1(x)Q^{-1}g_1^T(x)+\frac{1}{4}\lambda_f ee^T]V_x^T \\
&-[V_xg_2(x)+h^T(x)Rk_{12}(x)] \\
&\cdot R^{-1}[V_xg_2(x)+h^T(x)Rk_{12}(x)]^T \\
&+\frac{1}{2}h_1^T(x)Rh_1(x)+\frac{1}{\lambda_f}m^Tm\le 0 \qquad (3.21)
\end{aligned}$$

Π:

$$\begin{aligned}
&[W_x,W_\xi]f_e(x,\xi)+\frac{1}{2}\Phi^T(x,\xi)Q^{-1}\Phi(x,\xi) \\
&+[W_x,W_\xi]\tilde{E}\begin{bmatrix} W_x^T \\ W_\xi^T \end{bmatrix}+\frac{1}{\tilde{\lambda}_f}m^Tm
\end{aligned}$$

$$+\frac{1}{2}h_e^T(x,\xi)Q^{-1}h_e(x,\xi)\le 0 \qquad (3.22)$$

where λ_f and $\tilde{\lambda}_f$ are some given real constants,

$$f_e(x,\xi)=\begin{bmatrix} f(x)+g_2(x)\alpha(\xi)+g_1(x)Q^{-1}g_1(x)V_x^T \\ f_c(\xi)+Gh_2(x)+Gk_{21}(x)Q^{-1}g_1(x)V_x^T \end{bmatrix}$$

$$h_e(x,\xi)=R^{-\frac{1}{2}}[V_x g_2(x)+h^T(x)Rk_{12}(x)]$$

$$+R^{\frac{1}{2}}\alpha(\xi)$$

$$\Phi^T(x,\xi)=[W_x,W_\xi]\begin{bmatrix} g_1(x) \\ Gk_{21}(x) \end{bmatrix}$$

$$\tilde{E}=\begin{bmatrix} \frac{\tilde{\lambda}_f ee^T}{4} & 0 \\ 0 & 0 \end{bmatrix}$$

$$g_c(\xi)=G$$

$$u=\alpha(\xi)$$

then under the control of u, uncertain nonlinear system (3.19) is quadratic dissipative with respect to supply rate $S(w,z)$.

4 CONCLUSIONS

In this paper, we considered the problem of robust dissipative control for uncertain nonlinear systems. Both state feedback control and output feedback control are designed to achieve quadratic dissipativeness. The robust dissipative control problem can be resolved for all admissible uncertainties, if there exist nonnegative solutions of Hamilton-Jacobi inequalities.

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