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IMPROVED LMI REPRESENTATION FOR THE ANALYSIS OF CONTINUOUS STATE DELAYED SYSTEMS WITH POLYTOPIC UNCERTAINTY

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Abstract: This paper considers the problem of robust stability for linear systems with a constant time-delay in the state and subject to real convex polytopic uncertainty. First of all, for robust stability problem, new matrix inequalities characterization of delay-dependent robust stability results are exploited, which demonstrat that it allows the use of parameter-dependent Lyapunov functions in comparison to the classical one, whose drawback stands in the use of a single Lyapunov function to assess the stability over the whole uncertainty domain. Next, as for delay dependent case, the problem of determining the maximum time-delay under which the system will remain stable is cast into a generalized eigenvalue problem and thus solved by LMI techniques. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: Time-delay systems; Uncertain systems; Robust stability; LMI.

1. INTRODUCTION

Time-delay occurs in many dynamical systems such as biological systems, chemical systems, metallurgical processing systems, nuclear reactor, long transmission lines in pneumatic, hydraulic systems and electrical networks. Frequently, it is a source of the generation of oscillation, instability and poor performance. There has been a large amount of literature during the last couple decades dedicated to systems called Smith predictor theories (Smith, 1959; Matausek and Micic, 2000; Furutani and Araki, 1998), in which the time delay is known. For a system with unknown delays, however, the concepts of robust control theories have been recently introduced. If an unknown delay term is constant but unlimited, researchers have provided several delay-independent stability criteria (Lee *et al.*, 1994; Chen and Lathchman, 1994; Chen *et al.*, 1995). If an unknown delay term is constant but bounded, delay-dependent stability criteria in (Niculesu *et al.*, 1995; Kolmanovskii and Niculescu, 1999; de Souza and Li, 1999; Huang and Zhou, and references therein, 2000; Zhang *et al.*, 2001) improve stability margins compared to delay-independent criteria.

Recently, much effort has been devoted to developing frequency-domain and time-domain based techniques which may be extendable to the problems of robust stability of uncertain linear time-delay systems with delay-dependent Lyapunov functions. The results obtained in the frequency-domain give less conservative stability criteria (Huang and Zhou, and references therein, 2000; Zhang *et al.*, 2001), Unfortunately, these methods can not be applied to deal with the delayed systems with ploytopic uncertainty. In the time-domain Lyapunov-Krasovskii or Lyapunov-Razumikhin-based functionals are used as Lyapunov function candidate for quadratical stabilization. One of the main drawbacks of the quadratic approach comes from the fact that it make use of a single constant Lyapunov matrix to simultaneously test the stability over the whole uncertain domain. Recently in the study of linear systems with polytopic uncertainty, a few attempts have been made to reduce the conservatism of the approach by defining and using parameter dependent Lyapunov functions for continuous linear uncertain systems (Tuan *et al.*, 2000; Apkarian *et al.*, 2000; Shaked, 2001), and discrete uncertain systems (Bachelier *et al.*, 1999; Geromel *et al.*, 1999). In these papers, a new stabilization condition for linear uncertain systems has been derived. Besides being quite simple LMI obtained expanding the Lyapunov relation by introducing a new matrix variable, it enable to obtain parameter dependent Lyapunov function which, of course, is a step ahead the quadratic approach. Moreover, in this new condition, the Lyapunov matrix is not involved in any product term with the dynamic matrix. With this property, the determination of the Lyapunov matrix are, in some sense, independent, unlike in the quadratic approach. However, few results have been reported to deal with uncertain time-delay systems using parameter-dependent Lyapunov function methods.

In this paper, the problem of robust stability for linear systems with a delayed state and subject to polytopic uncertainty in the state-space model is addressed. The case of a single, constant time delay is considered. Delay-dependent results are investigated through a new parameter-dependent Lyapunov function. When time-delay is constant but unknown, the problem of determining the maximum time-delay under which the system will remain stable in spite of polytopic uncertainty is cast into a generalized eigenvalue problem and thus solved by LMI techniques.

2. PROBLEM FORMULATION

Consider the following linear uncertain system with time-delay

$$\begin{array}{l} \dot{x}(t) = Ax(t) + A_d x(t-\tau), \\ x(t) = \psi(t), \;\; \forall t \in [-\tau,\ 0] \end{array} \qquad (1)$$

where $x(t) \in R^n$ is is the state, $\tau > 0$ is a given constant. The state-space data are subject to uncertainties and obey the real convex polytopic model

$$\begin{aligned} \left[\, A \;\; A_d \,\right] \in \Omega := \Bigg\{ \left[\, A(\xi) \;\; A_d(\xi) \,\right] \\ = \sum_{i=1}^{p} \xi_i \left[\, A_i \;\; A_{di} \,\right], \sum_{i=1}^{p} \xi_i = 1, \xi_i \geq 0 \Bigg\}. \end{aligned} \qquad (2)$$

Definition 1. The system (1) is said to be robustly stable if the equilibrium solution $x(t) = 0$ of the functional differential equation associated to system (1) is globally uniformly asymptotically stable for all $\left[\, A \;\; A_d \,\right] \in \Omega$.

The goal of this paper is to address the following problems:

- How to judge whether a given uncertain system with time-delay is stable or not.
- When the time delay is unknown, for a given system with time-delay to be stable how long can the time delay be?

The following Lemmas are needed to derive main results of this paper.

Lemma 1. (Reciprocal Projection Lemma, see, (Apkarian *et al.*, 2000)). Let U be any given positive definite matrix. The following statements are equivalent:

$$(i) : \Psi + S + S^T < 0. \qquad (3)$$

(ii) : the LMI problem

$$\begin{bmatrix} \Psi + U - (W + W^T) & S^T + W^T \\ S + W & -U \end{bmatrix} < 0 \quad (4)$$

is feasible with respect to W.

Lemma 2. The following statements are equivalent.

I. There exist positive definite matrices $W \in R^{n\times n}$, $P \in R^{n\times}$ and $Q \in R^{n\times n}$ such that

$$-W + PA_d Q^{-1} A_d^T P < 0 \qquad (5)$$

II. There exist positive matrices $Y \in R^{n\times n}$, $Z \in R^{n\times n}$ and general matrix $G^{n\times n}$ such that

$$\begin{bmatrix} -Y & A_d G \\ G^T A_d^T & -G - G^T + Z \end{bmatrix} < 0 \qquad (6)$$

Proof. $I \Rightarrow II$. Assuming (5) is satisfied, using Shur complement formula, it is equivalent to

$$\begin{bmatrix} -W & PA_d Q^{-1} \\ Q^{-1} A_d^T P & -Q^{-1} \end{bmatrix} < 0 \qquad (7)$$

Let $Z = Q^{-1}$, and multiplying the latter inequality, from both sides, by diag$\{P^{-1}, I\}$ yields

$$\begin{bmatrix} -P^{-1} W P^{-1} & A_d Z \\ Z A_d^T & -Z \end{bmatrix} < 0 \qquad (8)$$

Let $Y = P^{-1} W P^{-1}$, $G = G^T = Z$, yields (6).

$II \Rightarrow I$. Assuming (6) is satisfied. Since the matrix $\left[\, I \;\; A_d \,\right]$ has full rank, (6) implies that

$$\left[\, I \;\; A_d \,\right] \begin{bmatrix} -Y & A_d G \\ G^T A_d^T & -G - G^T + Z \end{bmatrix} \begin{bmatrix} I \\ A_d^T \end{bmatrix} < 0 \quad (9)$$

which is

$$-Y + A_d Z A_d^T < 0 \qquad (10)$$

Let $W = PYP, Q = Z^{-1}$ and multiply (10), from both sides, by P, yields (5).

Lemma 3. Gien a scalar $\tau > 0$. The following matrix inequality conditions $(5) - (6)$, with positive definite matrix variables Y, X, Z, and general matrix variable V, are equivalent.

$$YA^T + AY + \tau(A + A_d)W^{-1} \times(A + A_d)^T + \tau Q < 0. \quad (11)$$

$$\begin{bmatrix} -\Omega_1 & \Omega_2^T & V^T & \Omega_3^T & V^T \\ \Omega_2 & -X & 0 & 0 & 0 \\ V & 0 & -\tau^{-1}Z & 0 & 0 \\ \Omega_3 & 0 & 0 & -\tau^{-1}W & 0 \\ V & 0 & 0 & 0 & -X \end{bmatrix} < 0. \quad (12)$$

where $\Omega_1 = V + V^T$, $\Omega_2 = A^TV + X$, $\Omega_3 = (A + A_d)^TV$

Proof. (11)$\Rightarrow$ (12). From (11), using Lemma 1 with $\Psi := \tau(A + A_d)Y(A + A_d)^T + \tau Q$, $S = AY$ and any given positive definite matrix U yields

$$\begin{bmatrix} \Omega_4 & AY + W^T \\ YA^T + W & -U \end{bmatrix} < 0 \quad (13)$$

where $\Omega_4 = \tau(A + A_d)W^{-1}(A + A_d)^T + \tau Q + U - (W + W^T)$.

By Shur complement operation with respect to the term Q and the congruence transformation

$$\begin{bmatrix} V & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & I \end{bmatrix}, \text{with } X := Y^{-1}, V := W^{-1}, Z := Q^{-1}$$

the inequality above in turn becomes

$$\begin{bmatrix} \Omega_5 & V^TA + X & V^T \\ A^TV + X & -XUX & 0 \\ V & 0 & -\tau^{-1}Z \end{bmatrix} < 0 \quad (14)$$

where $\Omega_5 = \tau V^T(A + A_d)W^{-1}(A + A_d)^TV + V^TUV - (V + V^T)$

By Shur complement operation with respect to the terms $V^TA_dW^{-1}A_d^TV$ and V^TUV, the above inequality becomes

$$\begin{bmatrix} -\Omega_1 & \Omega_2^T & V^T & \Omega_3^T & V^T \\ \Omega_2 & -XUX & 0 & 0 & 0 \\ V & 0 & -\tau^{-1}Z & 0 & 0 \\ \Omega_3 & 0 & 0 & -\tau^{-1}W & 0 \\ V & 0 & 0 & 0 & -U^{-1} \end{bmatrix} < 0 \quad (15)$$

The above inequality implies (6) with $U := X^{-1}$.

(12)$\Rightarrow$ (11). If (12) holds, using Lemma 1 and Shur complement formula, we can get that (11) hold with $U := X^{-1}$ and $Y := X^{-1}$. The proof is completed.

3. MAIN RESULTS

In this section, New alternative characterizations of an important stability theorem for linear systems with time-delay are introduced. The result below constitutes the core of the development in the subsequent sections. It introduces a new transformation on the Lypunov variables which helps reduce the degree of conservatism in some delicate problems. This will appear more light for robust synthesis problems.

Theorem 1. Consider the system (1), given a scalar $\overline{\tau} > 0$. If there exist positive definite matrices $X \in R^{n\times n}$, $W \in R^{n\times n}$, $Z \in R^{n\times n}$, matrices $V \in R^{m\times n} \in R^{n\times n}$ and $G \in R^{n\times n}$ such that the following LMIs hold for all $\begin{bmatrix} A & A_d \end{bmatrix} \in \Omega$

$$\begin{bmatrix} -\Omega_1 & \Omega_2^T & V^T & \Omega_3^T & V^T \\ \Omega_2 & -X & 0 & 0 & 0 \\ V & 0 & -\overline{\tau}^{-1}Z & 0 & 0 \\ \Omega_3 & 0 & 0 & -\overline{\tau}^{-1}W & 0 \\ V & 0 & 0 & 0 & -X \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} -2X + W & A_dG \\ G^TA_d^T & -G - G^T + Z \end{bmatrix} < 0 \quad (17)$$

Then, the uncertain system (1) is robustly stable for any constant time delay τ satisfying $0 \le \tau \le \overline{\tau}$.

Proof. First, the system (1) is rewritten in the form:

$$\begin{aligned} z(t) &= x(t) + \int_{t-\tau}^{t} A_dx(w)dw \\ \dot{z}(t) &= (A + A_d)x(t) \end{aligned} \quad (18)$$

Next, consider the Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) \\ V_1(t) &= z^T(t)Pz(t) \\ V_2(t) &= \int_{t-\tau}^{t}\int_{w}^{t} x^T(v)Qx(v)dvdw \end{aligned} \quad (19)$$

where P and Q are positive definite matrices. Then, differentiating the functional along the solutions of (18) yields:

$$\begin{aligned} \dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) \quad (20) \\ \dot{V}_1(t) &= x^T(t)(A + A_d)^TPz(t) + z^T(t)P(A + A_d)x(t) \\ &= x^T(t)[(A + A_d)^TP + P(A + A_d)]x(t) + W(t) \\ \dot{V}_2(t) &= -\int_{t-\tau}^{t} x^T(w)Qx(w)dw + \int_{t-\tau}^{t} x^T(t)Qx(t)dw \end{aligned}$$

where

$$W(t) = 2\int_{t-\tau}^{t} x^T(t)(A + A_d)^TPA_dx(w)dw$$

Recalling that for any vectors u, v and any matrix $Q > 0$ of appropriate dimensions

$$2u^T v \le u^T Q u + v^T Q^{-1} v$$

Therefore, for any matrix $Q > 0$

$$W(t) \le \int_{t-\tau}^{t} x^T(t)(A + A_d)^T P A_d Q^{-1} A_d^T P$$

$$\times (A + A_d) x(t) dw + \int_{t-\tau}^{t} x^T(w) Q x(w) dw$$

Hence, it follows that

$$\dot{V}(t) \le x^T(t)[(A + A_d)^T P + P(A + A_d) + \tau Q + \tau (A + A_d)^T P A_d Q^{-1} A_d^T P(A + A_d)] x(t) \quad (21)$$

The matrix in (21) is negative definite if the following inequality:

$$(A + A_d)^T P + P(A + A_d) + \tau Q + \tau (A + A_d)^T P \times A_d Q^{-1} A_d^T P(A + A_d) < 0 \quad (22)$$

In order to turn (22) into LMI expression, it is transformed into

$$(A + A_d)^T P + P(A + A_d) + \tau Q + \times \tau (A + A_d)^T W^{-1}(A + A_d) - \tau (A + A_d)^T (W^{-1} - P A_d Q^{-1} A_d^T P)(A + A_d) < 0 \quad (23)$$

where W is a positive definite matrix which satisfy

$$W^{-1} > P A_d Q^{-1} A_d^T P \quad (24)$$

Thus, (22) is negative definite if (24) and the following inequality is satisfied for any constant time delay τ satisfying $0 \le \tau \le \overline{\tau}$

$$(A + A_d)^T P + P(A + A_d) + \overline{\tau} Q + \overline{\tau}(A + A_d)^T W^{-1}(A + A_d) < 0 \quad (25)$$

By using Lemma 2 and 3, (24) and (25) are equivalent to (16) and the following inequality for some positive definite matrices $X \in R^{n\times n}$, $W \in R^{n\times n}$, $Z \in R^{n\times n}$, matrices $V \in R^{m\times n} \in R^{n\times n}$ and $G \in R^{n\times n}$, respectively

$$\begin{bmatrix} -XW^{-1}X & A_d G \\ G^T A_d^T & -G - G^T + Z \end{bmatrix} < 0 \quad (26)$$

From the equality

$$XW^{-1}X - 2X + W = (X - W)W^{-1}(X - W) \ge 0$$

it can be shown that

$$-2X + W \ge -XW^{-1}X$$

Hence, (26) will hold if the following inequality is satisfied for some positive definite matrices X, W, Z and matrix G satisfying

$$\begin{bmatrix} -2X + W & A_d G \\ G^T A_d^T & -G - G^T + Z \end{bmatrix} < 0 \quad (27)$$

then (16) and (17) will imply that (25) and (27) hold for any constant time delay τ satisfying $0 \le \tau \le \overline{\tau}$, respectively. Thus, the LMIs (16-17) will guarantees the negativeness of $\dot{V}(t)$ for any non-zero $x(t) \in R^n$, which immediately implies the robust stability of the system (1). The proof is completed.

Remark 1. It is clear that for considered problem a natural assumption is the asymptotical stability of the uncertain system free of delay $\tau = 0$:

$$\dot{z}(t) = (A + A_d) z(t). \quad (28)$$

This yields to a $A + A_d$ Hurwitz stable matrix for A, A_d as defined in (2).

It is interesting to observe that the new LMI conditions: (16) and (17) do not exhibit the product $(A + A_d X)$ and $(A + A_d)^T P A_d Q^{-1} A_d^T P(A + A_d)$. In this sense a new degree of freedom has been created, which may be explored to provided a new robust stability. The following theorem generalizes the concept of quadratic stability of uncertain systems with time-delay(Boyd *et al.*, 1994).

Theorem 2. Consider the system (1). Then given a scalar $\overline{\tau} > 0$, the uncertain system (1) is robustly stable for any constant time delay τ satisfying $0 \le \tau \le \overline{\tau}$ if there exist positive definite matrices $\{X_i\}_{i=1,\cdots,p}$, $\{W_i\}_{i=1,\cdots,p}$, $\{Z_i\}_{i=1,\cdots,p}$, matrices V and G such that

$$\begin{bmatrix} -\Omega_1 & \Omega_{2i}^T & V^T & \Omega_{3i}^T & V^T \\ \Omega_{2i} & -X_i & 0 & 0 & 0 \\ V & 0 & -\overline{\tau}^{-1} Z_i & 0 & 0 \\ \Omega_{3i} & 0 & 0 & -\overline{\tau}^{-1} W_i & 0 \\ V & 0 & 0 & 0 & -X_i \end{bmatrix} < 0 \quad (29)$$

$$\begin{bmatrix} -2X_i + W_i & A_{di} G \\ G^T A_{di}^T & -G - G^T + Z_i \end{bmatrix} < 0 \quad (30)$$

where $\Omega_{2i} = (A_i + A_{di})V + X_i, \Omega_{3i} = (A_i + A_{di})V$.

Proof. By virtue of the properties of convex combination, and using the characterization (16) and (17), it is easy to prove the uncertain system (1) is stable under the conditions (31) and (30).

Based on the assumption that the time delay $\overline{\tau}$ is constant but unknown, Theorem 3 establishes an LMI-based stability condition for system (1). Now, we come to consider the problem of determining the upper bound for the time-delay τ. Obviously, when τ is constant but unknown, (31) is nonlinear in $\overline{\tau}$. Let $\nu = \frac{1}{\overline{\tau}}$. Then, (31) can be rewritten as follows:

$$
\begin{bmatrix} -\Omega_1 & \Omega_{2i}^T & V^T & \Omega_{3i}^T & V^T \\ \Omega_{2i} & -X_i & 0 & 0 & 0 \\ V & 0 & 0 & 0 & 0 \\ \Omega_{3i} & 0 & 0 & 0 & 0 \\ V & 0 & 0 & 0 & -X_i \end{bmatrix}
< \nu \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_i & 0 & 0 \\ 0 & 0 & 0 & W_i & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (31)
$$

Note that when using gevp of LMI toolbox to solve the generalized eigenvalue problem (GEVP)

$$
\min_x \nu
$$

$$
A(x) < \nu B(x)
$$

the matrix $B(x)$ must be positive-definite. Thus, to cast the problem of maximizing the time-delay $\overline{\tau}$, i.e., minimizing ν, into the framework of GEVP, let us introduce some auxiliary matrices $W_{i1} > 0$, $W_{i2} > 0$ and rewrite (31) as follows:

$$
\begin{bmatrix} -\Omega_1 & \Omega_{2i}^T & V^T & \Omega_{3i}^T & V^T \\ \Omega_{2i} & -X_i & 0 & 0 & 0 \\ V & 0 & -W_{i1} & 0 & 0 \\ \Omega_{3i} & 0 & 0 & -W_{i2} & 0 \\ V & 0 & 0 & 0 & -X_i \end{bmatrix} < 0 \quad (32)
$$

$$
\begin{bmatrix} W_{i1} & 0 \\ 0 & W_{i2} \end{bmatrix} < \nu \begin{bmatrix} Z_i & 0 \\ 0 & Wi \end{bmatrix} \quad \text{for } i = 1, \cdots, p \quad (33)
$$

The optimization problem that determines the upper bound of the delay can be formulated as:

$$
\min_{X_i, W_i, Z_i, W_{i1}, W_{i2}, V, G (i=1,\cdots,p)} \nu
$$

$$
\text{s.t. } (17), (32), (33)
$$

Remark 2. The above optimization problem consists of minimizing a generalized eigenvalue problem which is a quasi-convex optimization problem. It is important to notice that this algorithm can be numerically solved with very efficient methods (Boyd *et al.*, 1994).

The importance of the Theorem 2 is apparent. First it generalizes the concept quadratic stability of uncertain systems where a single couple of Lyapunov matrices P and Q are used, for example, the Lyapunov functional

$$
V(t) = z^T(t) P z(t) + \int_{t-\tau}^{t} \int_{w}^{t} y^T(v) Q y(v) dv dw \quad (34)
$$

where $z(t) = y(t) + \int_{t-\tau}^{t} A_d y(w) dw$

However, as indicated in the proof of Theorem 1, the stability of family of matrices $\sum_{i=1}^{p} \xi_i [A_i \; A_{di}]$ is tested by the parameter dependent Lyapunov function

$$
V(t) = z^T(t) \sum_{i=1}^{p} \xi_i P_i z(t) + \int_{t-\tau}^{t} \int_{w}^{t} x^T(v) \sum_{i=1}^{p} \xi_i Q_i x(v) dv dw \quad (35)
$$

where $z(t) = x(t) + \int_{t-\tau}^{t} A_d x(w) dw$. In other words our new stability condition includes as a particular case the quadratic stability condition.

4. EXAMPLES

In this section, we shall illustrate the results by using one example.

Example. Consider uncertain time-delay system (1) with

$$
A_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},
$$
$$
A_2 = \begin{bmatrix} -2 & -1 \\ 0 & -2 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},
$$
$$
A_3 = \begin{bmatrix} -1.9 & 0 \\ 0 & -1 \end{bmatrix}, A_{d3} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.1 \end{bmatrix}
$$

Note that the third vertex of the above uncertain system has been considered in (de Souza and Li, 1999) which is not asymptotically stable independent of the size of the delay, i.e. the stability is delay-dependent. Applying the method of (de Souza and Li, 1999), the above uncertain system is robustly stable for any τ satisfying $0 \le \tau \le 0.0853$. Based on our result, we obtain that the uncertain system is robustly stable for any τ satisfying $0 \le \tau \le 0.6829$. Hence , for this example, the robust stability criterion of this paper gives a less conservative result than that obtained by the methods of (de Souza and Li, 1999).

5. CONCLUSION

In this paper, different techniques and tools for robust stability and control problems have been developed. A new robust stability criterion is provided with parameter- as well as delay-dependent Lyapunov functions when convex polytopic uncertainty is present on the dynamic matrices. This work is based on new robust stability condition which presents a kind of separation between the Lypunov matrix and the matrices of the dynamic model. When the time-delay is assumed to be constant but unknown, the robust delay-dependent stability problem are cast into the framework of generalized eigenvalue problem and thus the delay bound is provided by using the LMI technique.

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6. REFERENCES

Apkarian, P., H. D. Tuan, J. Bernussou. Analysis, eigenstructure assignment and H_2 multi-channel synthesis with enhanced LMI characterizations. *In Pro. 39th Conf. Decision and Control*, Australia, 2000.

Bachelier, O., J. Bernussou, M. C. de Oliveira and J. C. Geromel. Parameter dependent Lyapunov control design: numerical evaluation. *In Pro. 38th Conf. Decision and Control* USA. 1999, pp. 293-297.

Boyd, S., L. El Ghaou, E. Feron, and V. Balakrishnan. Linear matrix inequalities in systems and control theory. *Studies in Applied Mathematics.* Philadelphia: SIAM, 1994.

Chen, J. and H. A. Lathchman. Frequency sweeping tests for stability independent of delay. *IEEE Trans. Automat. Contr.*, vol. 39, pp. 159-162, 1994.

Chen, J., D. Xu, and B. Shafai. On sufficient conditions for stability independent of delay. *IEEE Trans. Automat. Contr.,* vol. 40, pp. 1675-1680, 1995.

de Souza, C. E. and X. Li. Delay-dependent robust H_∞ control of uncertain linear state-delayed systems. *Automatica* vol. 35, pp. 1313-1321, 1999.

Furutani, E. and M. Araki. Robust stability of state-predictive and Smith control systems for plants with a pure delay. *Int. J. Robust Nonlinear Control,* vol. 8, pp. 907-919, 1998.

Geromel, J. C., M. C. de Oliveira and J. Bernussou. Robust filtering of discrete-time linear systems with parameter dependent Lyapunov functions. *In Pro. 38th Conf. Decision and Control* USA. 1999, pp. 570-575.

Hale, J. and S. M. V. Lunel. Introduction to functional differential equations. New York: Springer. 1993.

Huang, Y. and K. Zhou. Robust stability of uncertain time-delay systems. *IEEE Trans. Automat. Contr.*, vol. 45, pp. 2169-2173, 2000.

Kolmanovskii, V. B. and S. -I. Niculescu. On the Liapunov-Krasovskii fuctionals for stability analysis of linear delay systems. *Int.J.Control*, vol.72, pp.374-384, 1999.

Lee, J. H., S. W. Kim, and W. H. Kwoon. Memoryless H_∞ controllers for state delayed systems. *IEEE Trans. Automat. Contr.*, vol. 39, pp. 159-162, 1994.

Matau$\hat{s}$ek, M. R. and A. D. Micic. A modified Smith predictor controlling a process with an integrator and long dead-time. *IEEE Trans. Automat. Contr.*, vol. 41, pp. 1363-1369, 2000.

Niculesu, S. -I., E. I. Verriest, L. Dugard and J.-M. Dion. Delay-dependent stability of linear systems with delayed state: An LMI approach. *In Proc. 34th IEEE Conf. Decision and Control*, 1995, pp. 1495-1496.

Shaked, U. Improved LMI Representations for the analysis and the design of continous-time systems with polytopic type uncertainty. *IEEE Trans. Automat. Contr.*, vol. 46, pp. 652-656, 2001.

Smith, O. J. A controller to overcome dead time. *ISA J.*, vol. 6, pp. 28-33, Feb. 1959.

Tuan, H. D., P. Apkarian, and T. Q. Nguyen. Robust and reduced-order filtering: new characterizations and methods. *In Proceedings of the American Control Conference* Chicago,pp.1327-1331, 2000.

Zhang, J., C. R. Knopse, and P. Tsiotras. Stability of Time-delay Systems: Equivalence between Lyapunov and Scaled Small-Gain Conditions. *IEEE Trans. Automat. Contr.*, vol. 46, pp. 482-486, 2001.