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GLOBAL PATH-TRACKING FOR A MULTI-STEERED GENERAL N-TRAILER [1]

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Abstract: A multi-steered general n-trailer is considered in this work. A global path-tracking control strategy for the n-trailer is proposed. Three nonlinear control laws are used to solve the problem by means of a commutation scheme. This commutation scheme allows to avoid the singularities inherent to a feedback linearization scheme. Simulation results show that the proposed scheme has good performance. *Copyright © 2002 IFAC*

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1. INTRODUCTION.

The development of path tracking controls for trailer-like systems has been widely studied in the literature, see for instance, (Altafini and Gutman, 1998; Bushnell *et al.*, 1993; Lamiraux *et al.*, 1999; Nakamura *et al.*, 2000; Rouchon *et al.*, 1993). In (Rouchon *et al.*, 1993) the problem of path tracking is addressed and solved for a general 1-trailer using the notion of differential flatness. In (Bushnell *et al.*, 1993) a standard 1-trailer with actuated trailer direction is considered and controlled in open loop using sinusoids. In (Altafini and Gutman, 1998) a n-trailer with off-axle hitching is considered and the problem of path-tracking is addressed and solved using an approximate model. In (Lamiraux *et al.*, 1999) a standard and general 1-trailer systems are controlled in forward direction. The displacement in backward direction is solved by considering a virtual robot model where the trailer is treated as the tractor. In (Nakamura *et al.*, 2000) a standard n-trailer system with steered direction on the trailers is considered and a control that achives path-tracking is proposed.

This paper addresses the global path-tracking problem for a multi-steered general n-trailer system. This system is a generalization of the standard and general n-trailer previously considered in the literature. The main difference with respect to these systems is that all the trailers are provided with steering wheels. The multi-steered general n-trailer is known to be fully linearizable by dynamic state feedback (Orosco-Guerrero *et al.*, 2002). The main obstacle to apply the linearizing feedback is that it is undefined on certain submanifolds of the state space. A control scheme that overcomes this difficulty will be obtained. The proposed control scheme avoids the singularities introduced by the linearizing scheme by the commutation between three different control laws. Each control law is related to different output functions.

The paper is organized as follows. In Section 2, the kinematic model of the multi-steered general n-trailer is recalled. In Section 3, a nonlinear control strategy based on a commutation scheme is pro-

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posed. In section 4, the particular case of 1-trailer is addressed. In Section 5, simulation results of the control strategy applied to this particular case are presented. Finally, some conclusions are presented in section 6.

2. KINEMATIC MODEL OF A MULTI-STEERED GENERAL N-TRAILER.

The object of study in this paper is the multi-steered n-trailer depicted in figure 1. This system consist in a mobile robot acting as a tractor and n trailers equipped with actuated direction wheels. The kinematic model of the multi-steered general n-trailer system can be obtained using the rigid body motion equation. The kinematic model of this system can be written as a nonholonomic system of the form (Orosco-Guerrero *et al.*, 2002)

$$\dot{x} = g(x)\, u, \tag{1}$$

where

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_{n+2} \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \beta \end{bmatrix},$$

$$\theta = \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_n \end{bmatrix}, \qquad \beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix},$$

$$g(x) = \left[\begin{array}{cc|c} \cos\theta_0 & 0 & \\ \sin\theta_0 & 0 & \mathbf{0}_{2\times n} \\ \hline a_0 & b_0 & \\ \vdots & \vdots & \mathbf{0}_{(n+1)\times n} \\ a_n & b_n & \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{n\times 2}} & \mathbf{I}_{n\times n} \end{array}\right],$$

with

$$\begin{aligned}
&a_0 = 0, \quad b_0 = 1, \quad q_0 = 0, \quad r_0 = 1, \\
&a_i = \frac{r_{i-1}\sin\left(\alpha_i + \beta_{i-1}\right) - a_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i}, \\
&b_i = \frac{q_{i-1}\sin\left(\alpha_i + \beta_{i-1}\right) - b_{i-1}d'_{i-1}\cos\alpha_i}{d_i\cos\beta_i}, \\
&r_i = \frac{r_{i-1}\cos\left(\phi_i + \beta_{i-1}\right) + a_{i-1}d'_{i-1}\sin\phi_i}{\cos\beta_i}, \\
&q_i = \frac{q_{i-1}\cos\left(\phi_i + \beta_{i-1}\right) + b_{i-1}d'_{i-1}\sin\phi_i}{\cos\beta_i}, \\
&\phi_i = \theta_{i-1} - \theta_i, \qquad \alpha_i = \phi_i - \beta_i.
\end{aligned}$$

The state variables x_1 and x_2 represent the position of the tractor with respect to fixed reference axes. The components of the vector θ represent the orientation of the tractor and the trailers with respect to the horizontal axis, as shown in figure 1. The components of the vector β represent the direction of the actuated wheels of the trailers with respect to their longitudinal axes.

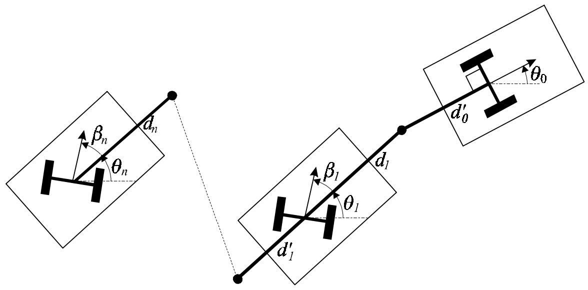

Fig. 1. A multi-steered general n-trailer system.

3. DESIGN OF A GLOBAL CONTROL LAW FOR THE MULTI-STEERED GENERAL N-TRAILER.

In this section the path-tracking problem is addressed. The control goal is that the trailer asymptotically tracks a desired path. This problem is solved using a modification of the control law proposed in (Orosco-Guerrero *et al.*, 2002). From (Orosco-Guerrero *et al.*, 2002) it is known that the system is fully linearizable by dynamic state feedback. The explicit design of the linearizing feedback is as follows. First consider the output function is,

$$\bar{y} = \left[\, x_1 \;\, x_2 \;\, \theta_1 \;\, \ldots \;\, \theta_n \,\right]^T. \tag{2}$$

The decoupling matrix associated to this output function is:

$$D(x) = \left[\begin{array}{cc|c} \cos\theta_0 & 0 & \\ \sin\theta_0 & 0 & \mathbf{0}_{2\times n} \\ \hline a_1 & b_1 & \\ \vdots & \vdots & \mathbf{0}_{n\times n} \\ a_n & b_n & \end{array}\right].$$

Clearly, $D(x)$ is noninvertible. Therefore, the noninteracting control problem has to be solved by dynamic state feedback. With this purpose, the following dynamic extension is proposed,

$$\begin{array}{lll} u_1 = \xi_1, & \dot{\xi}_1 = \xi_3, & \dot{\xi}_3 = w_1, \\ u_2 = \xi_2, & \dot{\xi}_2 = w_2, & \\ u_{i+2} = w_{i+2}, & i = 1, 2, ..., n\;. & \end{array} \tag{3}$$

where $w = (w_1, ..., w_{n+2})^T$ is the new input vector for the extended system. This dynamic extension is different from the one produced by the well known dynamic extension algorithm (Isidori, 1995). The dynamic extension (3) produces a simpler singular manifold that the one obtained by the application of the classical algorithm.

Taking successive time-derivatives of the output function (2) along the trajectories of the extended system (1)-(3) produces,

$$
\begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \\ y_3^{(2)} \\ \vdots \\ y_{n+2}^{(2)} \end{bmatrix} = \mathbf{F}(x,\xi) + \mathbf{A}(x,\xi)\, w \tag{4}
$$

where

$$
\mathbf{A} = \left[\begin{array}{cc|c} \cos\theta_0 & -\xi_1 \sin\theta_0 & \mathbf{0}_{2\times n} \\ \sin\theta_0 & \xi_1\cos\theta_0 & \\ \hline 0 & b_1 & \\ \vdots & \vdots & \mathbf{A}_{22} \\ 0 & b_n & \end{array}\right]
$$

$$
\mathbf{A}_{22} = \begin{bmatrix} \dfrac{\partial\dot\theta_1}{\partial\beta_1} & 0 & \cdots & 0 \\ \dfrac{\partial\dot\theta_2}{\partial\beta_1} & \dfrac{\partial\dot\theta_2}{\partial\beta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial\dot\theta_n}{\partial\beta_1} & \dfrac{\partial\dot\theta_n}{\partial\beta_2} & \cdots & \dfrac{\partial\dot\theta_n}{\partial\beta_n} \end{bmatrix}
$$

$$
\mathbf{F} = \begin{bmatrix} -2\xi_2\xi_3\sin\theta_0 - \xi_1\xi_2^2\cos\theta_0 \\ 2\xi_2\xi_3\cos\theta_0 - \xi_1\xi_2^2\sin\theta_0 \\ a_1\xi_3 + \displaystyle\sum_{j=0}^{1}\frac{\partial\dot\theta_1}{\partial\theta_j}\dot\theta_j \\ \vdots \\ a_n\xi_3 + \displaystyle\sum_{j=0}^{n}\frac{\partial\dot\theta_1}{\partial\theta_j}\dot\theta_j \end{bmatrix}.
$$

From equation (4) the standard noninteracting control is given by

$$
w = A(x,\xi)^{-1}\left[v - F(x,\xi)\right], \tag{5}
$$

where the function v is given by

$$
v_i = y_{id}^{(r_i)} - \sum_{j=0}^{r_i-1} k_{ij}\left(y_i^{(j)} - y_{id}^{(j)}\right),
$$

$$
\begin{aligned} r_1 &= r_2 = 3, \\ r_i &= 2, \quad i = 3,\ldots,n+2, \end{aligned}
$$

and k_{ij} are positive design parameters.

In the above equations as well as in the rest of the paper, the subscript d is used to denote the prescribed trajectories that the system should track asymptotically.

Define the tracking errors $e_i = y_i - y_{id}$, $i = 1,\ldots,n+2$. The control law (3)-(5) produces in close loop the following error dynamics,

$$
e_i^{(r_i)} + \sum_{j=0}^{r_i-1} k_{ij} e_i^{(j)} = 0.
$$

For the sake of conciceness, in the rest of the paper the control law (3)-(5) will be denoted by $\bar u(x,\xi)$.

Note that under, the dynamic state feedback $\bar u(x,\xi)$, the dimension of the zero dynamics of the extended system is zero.

Note also that, the dynamic state feedback $\bar u(x,\xi)$ is not defined when the state belongs to the following singular manifold:

$$
\bar S = \left\{ (x,\xi) \in \mathbf{R}^{2n+6} \mid \xi_1 \prod_{j=1}^{n} \frac{\partial\dot\theta_j}{\partial\beta_j} = 0 \right\}.
$$

In the rest of this section a commutation scheme that copes with this issue will be developed. First, two additional control laws associated to different output functions will be defined. Next, a commutation policy will be proposed. This commutation policy warranties that the control law applied to the system is globally defined.

Consider system (1) and the output function

$$
\tilde y = \left[\, x_1 \;\; \theta_0 \;\; \beta_1 \; \ldots \; \beta_n \,\right]^T. \tag{6}
$$

It is easy to see, that locally, system (1)-(6) has a well defined vector relative degree, since the decoupling matrix,

$$
\tilde{\mathbf{A}}(x) = \left[\begin{array}{c|c} \cos\theta_0 & \mathbf{0}_{1\times n+1} \\ \hline \mathbf{0}_{n+1\times 1} & \mathbf{I}_{n+1\times n+1} \end{array}\right]
$$

is nonsingular whenever $\cos\theta_0 \neq 0$. Consider the static state feedback

$$
\tilde u(x) = \tilde{\mathbf{A}}(x)^{-1}\tilde v, \tag{7}
$$

where

$$
\tilde v = \dot{\tilde y}_d - k_0(\tilde y - \tilde y_d).
$$

Defining $\tilde e = \tilde y - \tilde y_d$, it is possible to see that in closed loop,

$$
\dot{\tilde e} + k_0\tilde e = 0.
$$

Remark 3.1. The nonlinear static state feedback (7) has the following characteristics:

(1) It is not defined on the manifold

$$
\tilde S = \left\{ x \in \mathbf{R}^{2n+3} \mid \theta_0 = \pm\left(\frac{2m+1}{2}\right)\pi \right\}
$$

where $m = 0, 1, 2, \ldots$.

(2) The dimension of the zero dynamics under this control law is $n+1$.

Consider now the output function

$$
\hat y = \left[\, x_2 \;\; \theta_0 \;\; \beta_1 \; \ldots \; \beta_n \,\right]^T, \tag{8}
$$

together with system (1). Again, the vector relative degree is well defined. The time derivative of the output function (8) produces

$$
\dot{\hat y} = \hat{\mathbf{A}}(x)\,\hat u(x) \tag{9}
$$

where the decoupling matrix

$$\hat{\mathbf{A}}(x)=\left[\begin{array}{c|c}\sin\theta_0 & \mathbf{0}_{1\times n+1}\\ \hline \mathbf{0}_{n+1\times 1} & \mathbf{I}_{n+1\times n+1}\end{array}\right]$$

is nonsingular whenever $\sin\theta_0 \neq 0$. Therefore, the static state feedback

$$\hat{u}(x)=\hat{\mathbf{A}}(x)^{-1}\hat{v} \qquad (10)$$

with

$$\hat{v}=\dot{\hat{y}}_d-k_0(\hat{y}-\hat{y}_d),$$

produces in close loop

$$\dot{\hat{e}}+k_0\hat{e}=0,$$

where $\hat{e}=\hat{y}-\hat{y}_d$.

Remark 3.2. The nonlinear static state feedback (10) has the following characteristics:

(1) It is not defined on the manifold
$$\hat{S}=\left\{x\in\mathbf{R}^{2n+3}\mid\theta_0=\pm m\pi\right\}$$
where $m=0,1,2,\ldots$.

(2) The dimension of the zero dynamics under this control law is $n+1$.

3.1 *Global control scheme.*

As mentioned previously, the control laws $\bar{u}(x,\xi)$, $u(x)$, $u(x)$ possess different singularities. In order to obtain a globally defined control law, the following commutation scheme is proposed

$$u=\begin{cases}\bar{u}(x,\xi), & (x,\xi)\in\bar{M}\\ \tilde{u}(x), & (x,\xi)\in\tilde{M}\\ \hat{u}(x), & (x,\xi)\in\hat{M}\end{cases} \qquad (11)$$

where

$$\bar{M}=\left\{(x,\xi)\in\mathbf{R}^{2n+6}\mid\Gamma(x,\xi)\geq\varepsilon\right\}$$
$$\tilde{M}=\left\{(x,\xi)\in\mathbf{R}^{2n+6}\mid\Gamma(x,\xi)<\varepsilon,|\cos\theta_0|\geq\frac{1}{\sqrt{2}}\right\}$$
$$\hat{M}=\left\{(x,\xi)\in\mathbf{R}^{2n+6}\mid\Gamma(x,\xi)<\varepsilon,|\cos\theta_0|<\frac{1}{\sqrt{2}}\right\}$$
$$\Gamma=\xi_1\prod_{j=1}^{n}\frac{\partial\dot{\theta}_j}{\partial\beta_j}$$

and ε is a positive free parameter corresponding to the switching threshold.

The commutation scheme operates under the following principle: the control $\bar{u}(x,\xi)$ fully linearizes the extended system and allows asymptotic tracking of the desired trajectory. Therefore, it is desired that $\bar{u}(x,\xi)$ operate most of the time. This control law is disabled only when the state (x,ξ) enters in a neighborhood of the singular manifold $\bar{S}$. This occurs when $(x,\xi)\notin\bar{M}$. One of the other two control laws are enable when the control $\bar{u}(x,\xi)$ is disabled. The goal of the commutation scheme is to ensure that the enable control law is far from its corresponding singular manifold. Therefore, the control law $\tilde{u}(x)$ is enabled when $(x,\xi)\in\tilde{M}$ and is easy to see that $\tilde{S}\notin\tilde{M}$, this ensure that $\tilde{u}(x)$ does not reach its singular manifold. A similar reasoning applies for $\hat{u}(x)$. Finally, the subsets $\bar{M}$, $\tilde{M}$ and $\hat{M}$ are mutually exclusive. This ensures that only one of the three control laws is enabled.

4. A PARTICULAR CASE: 1-TRAILER MODEL.

In order to illustrate the metodology presented in previous sections, the particular case of the 1-trailer model is worked out in this section.

From (1) considering $n=1$ the 1-trailer model is obtained as,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{\theta}_0\\ \dot{\theta}_1\\ \dot{\beta}_1\end{bmatrix}=\begin{bmatrix}\cos\theta_0 & 0 & 0\\ \sin\theta_0 & 0 & 0\\ 0 & 1 & 0\\ \dfrac{\sin\alpha_1}{d_1\cos\beta_1} & -\dfrac{d_0'\cos\alpha_1}{d_1\cos\beta_1} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}.$$

From equations (3)-(5) the dynamic state feedback $\bar{u}(x,\xi)$ for this particular case is given by

$$\begin{aligned}&\bar{u}_1=\xi_1,\ \dot{\xi}_1=\xi_3,\ \dot{\xi}_3=w_1,\\ &\bar{u}_2=\xi_2,\ \dot{\xi}_2=w_2,\\ &\bar{u}_3=w_3.\end{aligned} \qquad (12)$$

$$w=A(x,\xi)^{-1}\left[v-F(x,\xi)\right], \qquad (13)$$

where

$$\mathbf{A}=\begin{bmatrix}\cos\theta_0 & -\xi_1\sin\theta_0 & 0\\ \sin\theta_0 & \xi_1\cos\theta_0 & 0\\ 0 & b_1 & \dfrac{\partial\dot{\theta}_1}{\partial\beta_1}\end{bmatrix}$$

$$\mathbf{F}=\begin{bmatrix}-2\xi_2\xi_3\sin\theta_0-\xi_1\xi_2^2\cos\theta_0\\ 2\xi_2\xi_3\cos\theta_0-\xi_1\xi_2^2\sin\theta_0\\ \xi_3a_1+\left(\dfrac{\partial\dot{\theta}_1}{\partial\theta_0}\right)\xi_2+\left(\dfrac{\partial\dot{\theta}_1}{\partial\theta_1}\right)(a_1\xi_1+b_1\xi_2)\end{bmatrix},$$

with

$$a_1=\frac{\sin\alpha_1}{d_1\cos\beta_1},b_1=-\frac{d_0\cos\alpha_1}{d_1\cos\beta_1}$$
$$\frac{\partial\dot{\theta}_1}{\partial\theta_0}=-\frac{\partial\dot{\theta}_1}{\partial\theta_1}=\frac{\xi_1\cos\alpha_1+\xi_2d_0\sin\alpha_1}{d_1\cos\beta}$$
$$\frac{\partial\dot{\theta}_1}{\partial\beta_1}=-\frac{\xi_1\cos\phi_1+\xi_2d_0\sin\phi_1}{d_1\cos^2\beta_1}$$
$$\phi_1=\theta_0-\theta_1,\quad \alpha_1=\phi_1-\beta_1.$$

From equation (7), the static state feedback $\tilde{u}$ for this particular case is

$$\tilde{u} = \begin{bmatrix} \frac{1}{\cos\theta_0} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tilde{v}, \tag{14}$$

and from equation (10), the static state feedback $\hat{u}$ for this particular case is

$$\hat{u} = \begin{bmatrix} \frac{1}{\sin\theta_0} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{v}. \tag{15}$$

The function $\Gamma(x,\xi)$ that define the singular manifold $\bar{S}$ is given by,

$$\Gamma(x,\xi) = \frac{\xi_1(\xi_1\cos\phi_1 + \xi_2 d_0 \sin\phi_1)}{d_1\cos^2\beta_1}.$$

5. SIMULATION RESULTS.

The results presented in this section were obtained using MatLab's SIMULINK. These results were obtained using the following numerical parameters:

$$\begin{aligned} &k_0 = k_{30} = 0.25,\ k_{10} = k_{20} = 0.125,\\ &k_{11} = k_{21} = 0.75,\ k_{31} = 1,\\ &k_{12} = k_{22} = 1.5\ . \end{aligned}$$

The simulations were performed assuming the following initial condition errors:

$$\begin{array}{lll} e_1(0) = -0.2, & e_2(0) = 0.5, & e_3(0) = 0.1,\\ \dot{e}_1(0) = 0, & \dot{e}_2(0) = 0, & \dot{e}_3(0) = 0,\\ \ddot{e}_1(0) = 0, & \ddot{e}_2(0) = 0. & \end{array}$$

Figures 2 and 3 display both the actual and prescribed trajectories. The prescribed trajectory crosses the singularities associated to each control law. The design of the desired trajectories $\bar{y}_{1d} = x_{1d}$ and $\bar{y}_{2d} = x_{2d}$ are obvious from figures 2 and 3. The designed trajectory $\bar{y}_{3d} = \theta_{1d}$ is a bit more involved. In order to design θ_{1d}, it was assumed that the trailer follows the same trajectory that the tractor. This is, θ_{1d} follows the same trajectory than θ_{0d}, but with a time delay in forward motion or time advance in backward motion. Therefore, the desired variable θ_{1d} is calculated as $\theta_{1d}(t) = \theta_{0d}(t-\tau)$ in forward motion and $\theta_{1d}(t) = \theta_{0d}(t+\tau)$ in backward motion. For this particular trajectory $\tau = 4.04$. The variable θ_{0d} is obtained as follows: supose that path-tracking is being achieved. Then $\dot{x}_{1d} = u_1\cos\theta_{0d}$ and $\dot{x}_{2d} = u_1\sin\theta_{0d}$. Eliminating u_1 from these expresions, one obtains $\theta_{0d} = \arctan\frac{\dot{x}_{2d}}{\dot{x}_{1d}}$.

Figure 2 display the trailer in forward motion during 90 seconds taking samples every 9 seconds. Figure 3 displays the trailer in backward motion following the same desired trajectory.

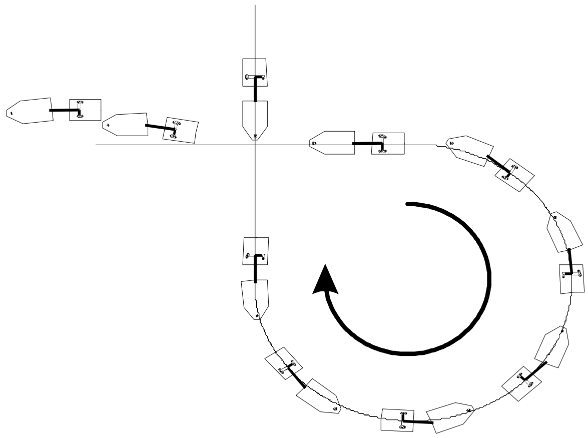

Fig. 2. Trailer system in forward motion.

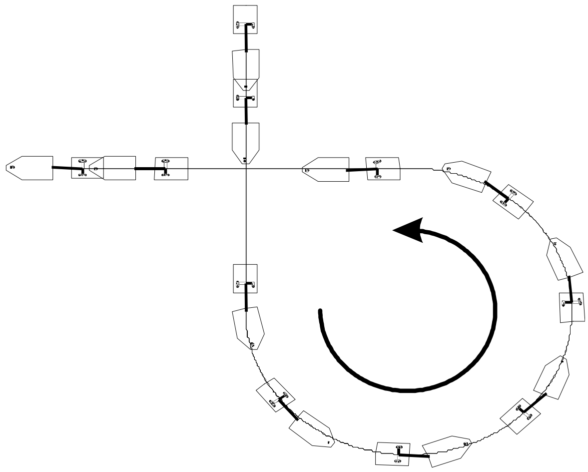

Fig. 3. Trailer system in reverse motion.

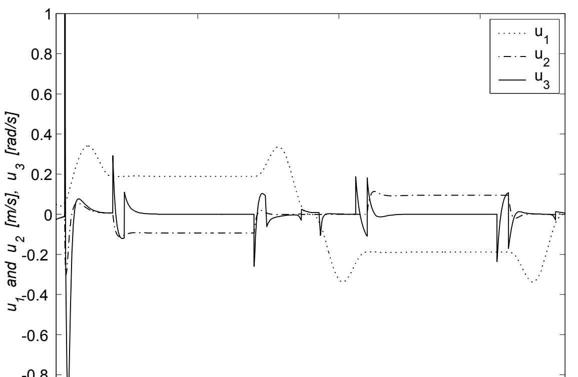

Fig. 4. Evolution of control signals.

Figure 4 correspond to the evolution of the control signals u_1, u_2 and u_3. The peaks observed in the control signal u_3 are produced by the commutation action. It should be strengthened that all the control signals remain bounded and within reasonable values.

The evolution of the tracking errors e_1, e_2 and e_3 are shown in figure 5.

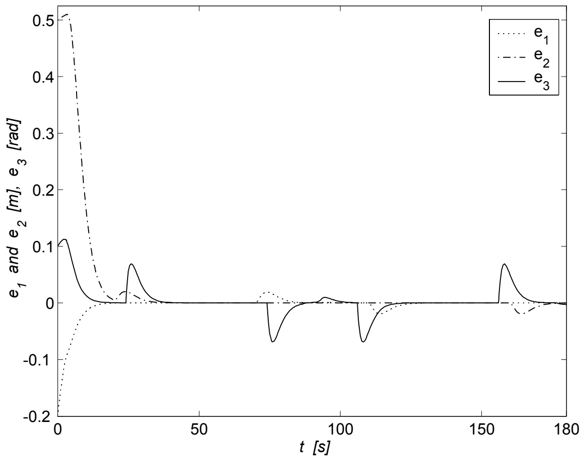

Fig. 5. Evolution of tracking errors.

6. CONCLUSIONS.

A global path-tracking control scheme for a multi-steered general n-trailer was presented. This control law employ three different nonlinear feedback controls. A commutation scheme between this feedbacks allows to avoid the singularities inherent to each control. Simulation results show that the proposed scheme exhibits an aceptable performance.

7. REFERENCES

Altafini, C. and P. O. Gutman (1998). Path following with reduced off-tracking for the n-trailer system. In: *Proc. 37th. IEEE Conf. On Decision and Control*. Tampa, Florida USA. pp. 3123–3128.

Bushnell, L.G., D.M. Tilbury and S.S. Sastry (1993). Steering three-input chained form nonholonomic systems using sinusoids: The fire truck example. In: *In Proc. ECC'93*. Groningen. pp. 1432–1437.

Isidori, A. (1995). *Nonlinear Control Systems*. third ed.. Springer-Verlag. London.

Lamiraux, F., S. Sekhavat and J. P. Laumond (1999). Motion planing and control for hilare pulling a trailer. *IEEE Transaction on Robotics and Automation* **Vol. 15**(No. 4), pp. 640–652.

Nakamura, Y., H. Ezaki, Y. Tan and W. Chung (2000). Design of steering mechanism and control of nonholonomic trailer systems. In: *Proc. Of 2000 IEEE International Conf. On Robotics and Automation*. San Francisco, CA. pp. 247–254.

Orosco-Guerrero, R., E. Aranda-Bricaire and M. Velasco-Villa (2002). Modeling and dynamic feedback linearization of a multi-steered general n trailer system. Submited to the 15th. IFAC World Congress, Barcelona.

Rouchon, P., M. Fliess, J. Levine and P. Martin (1993). Flatness, motion planning and trailer systems. In: *Proc. 32nd. IEEE Conf. On Decision and Control*. San Antonio, Texas USA. pp. 2700–2705.